A NEW ELASTO-PLASTIC MATERIAL MODEL FOR COATED FABRIC IN MEMBRANE STRUCTURES

T. D. Dinh[a*], A. Rezaei[a], M. Van Craenenbroeck[b], S. Puystiens[c], L. De Laet[b], M. Mollaert[b], D. Van Helmelrijck[c], W. Van Paepegem[a]

[a] *Department of Materials Science and Engineering, Ghent University, Technologiepark-Zwijnaarde 903, 9052 Zwijnaarde, Belgium*
[b] *Department of Architectural Engineering, Free University Brussels, Pleinlaan 2 B- 1050 Brussels, Belgium*
[c] *Department of Mechanics of Materials and Constructions, Free University Brussels, Pleinlaan 2 B- 1050 Brussels, Belgium*

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Abstract

Coated fabric is a special type of textile composite: the fabric architecture is not impregnated with a stiff resin, but only covered with a low stiffness coating material. This results in very flexible textile membranes, which are mainly used in architectural applications. Examples are retractable roofs, shading elements in adaptable facades, refugee shelters, large tent structures for events and festivals. During first installation, the membrane is pretensioned in such a way that ideally, an equi-biaxial tensile stress state is reached in the membrane.

Coated fabrics typically have simple plain weave fabric architecture, but often with a very different crimp angle in warp and weft direction. The warp direction is almost straight, while the weft yarns have a strong undulation, passing under and over the warp yarns. Further, since the coating material is only used to make the membrane waterproof, the impregnation of the fibres is non-homogeneous. All of these result in a very peculiar material behaviour: (i) the responses in warp and weft are strongly nonlinear. Because of the poor impregnation of the fibres, they can move relative to each other, and during the first loading (pre-tensioning) stage, large permanent (plastic) deformations occur, (ii) the strongly different crimp angle in warp and weft results in a strongly orthotropic behaviour, both in the initial linear as nonlinear (plastic) response, (iii) the crimp interchange between warp and weft leads to a strong dependence of the material response on the biaxial load ratio, (iv) in architectural applications where the membrane is sequentially folded and unfolded (e.g. retractable roofs), the coated fabric shows a hysteresis behaviour with accumulation of permanent strains after each loading cycle, till a certain steady-state condition is reached after 5 to 10 loading cycles.

This article proposes a new orthotropic elasto-plastic material model for coated fabrics. First uniaxial and biaxial tension tests have been performed with Digital Image Correlation for full-field strain mapping. Based on these tests, an orthotropic elasto-plastic material model is proposed which incorporates the observed nonlinearities and permanent strains in the constitutive behaviour of these materials. In total, eleven parameters are involved in the model of which seven are determined from uniaxial tests and the remainder from biaxial tests. The developed material model is then implemented in the finite element code ABAQUS® as a user material subroutine and validated with the data obtained from the biaxial tension tests.

In order to minimize the error between the experimental data and theoretical predictions, a parameter identification method is employed to optimise the boundary value problem in ABAQUS.

The proposed approach can be generally used for different kinds of coated fabric. In this study polyvinyl chloride (PVC) coated polyester fabric is considered. The model not only shows a good performance in computation, but the obtained results are also in very good agreement with experimental data. These features enable the developed model to provide a more realistic simulation of tension membrane structures, in comparison to the available models for these materials.

1. Introduction

Tension membrane structures have been widely used for the last five decades. In these structures, coated fabrics are used due to high ratio of strength to weight. However, design of the tension membrane structure is a challenging task due to the complicated mechanical responses of coated fabric under biaxial loads in which coated fabric exhibits a severely nonlinear relationship between strain and stress, orthotropic nature, irreversible strains and load ratio and load history dependence [1]. Some material models have been proposed for coated fabrics [1-6], however, many parameters are involved and demand high computational resources which prevent them from being widely used in realistic applications**.** At the moment, mostly linear elastic models are used for the numerical simulation of coated fabrics. Due to this oversimplified assumption, the safety factor in the design stage for fabric membrane is very high, ranging from five to ten [7]. From the aforementioned problems, it is feasible to demand a material model which is not only sophisticated enough to capture the salient features of coated fabrics, but also computationally inexpensive.

2. Experiments

Since the mechanical properties of a coated fabric are highly dependent on the weaving method and the type of coating, it is essential to conduct experiments to investigate the response of coated fabrics under loading conditions and later to use them to assess the validity of the proposed material model. Uniaxial and biaxial tensile tests are two types of tests which have been widely used in fabric research.

In this study, the PVC coated fabric utilised in the experiments has a thickness of 0.83 mm and a surface mass density of 1050 g/m^2. In warp direction, the density of reinforcement is 12 yarns/cm and 13 yarns/cm in fill direction. During the course of the experiments, strains are measured by means of digital image correlation device (DIC).

2.1 Uniaxial tensile tests

In this test, the specimen has a rectangular shape with the width of 50 mm, and the length of 200 mm, and it is cut out along the warp and the fill directions. In this test, cyclic load is applied, and the load is commenced with a loading cycle in which the maximum load is equal to 200 N. In subsequent cycles, the maximum load is increasing by 200 N, until a maximum load of 1400 N is reached. This way, both the initial and stabilised behaviour are determined. The relationship between strains and stresses in warp and fill directions is shown in figure 1.

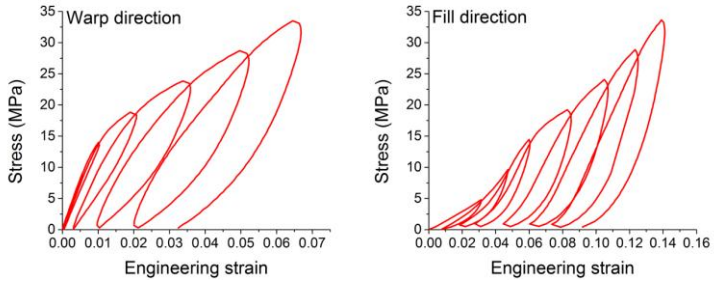

Figure 1. Stress strain curves obtained from uniaxial test in warp and fill directions.

2.2 Biaxial tensile tests

The behaviour of the coated fabric under biaxial stress state is investigated on a cruciform sample. The biaxial experiments were performed with repetition of load cycles (3 cycles for each load ratio), to explore both the initial and the stabilised behaviour of the coated fabric, at limited load levels ($\sigma_{\max} = 24\,\text{MPa}$). During the course of experiments, the applied load and the elongation are recorded at the arms of the biaxial testing device, and the strains in the centre of the cruciform are determined by means of DIC. The stresses are subsequently calculated by dividing the applied load by the cross section of the arms. Therefore, the stress-strain curves showed in figure 2 do not represent the true stress at the centre of the sample, but the applied stress at the arms. In figure 2, results of the biaxial tests are shown in which the sequencing load ratios 1:1, 2:1, 1:2, 1:0, 0:1 and 1:1 are performed.

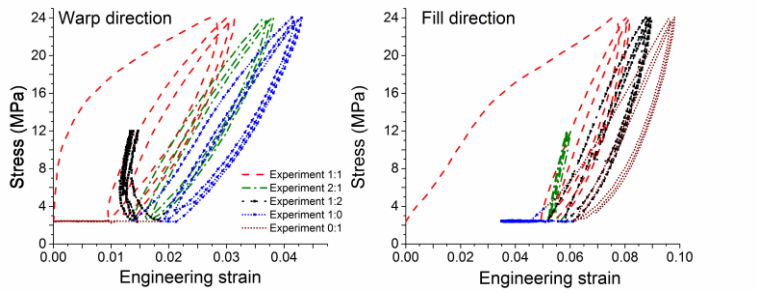

Figure 2. Stress strain curves for the biaxial test with different load ratios.

3. Proposed material model for PVC coated fabric

In this section, w and f are used to designate the warp and fill directions, respectively. Based on the obtained results from the uniaxial tensile tests, the linear hardening elasto-plastic models are separately proposed for warp and fill directions, (figure 3). The proposed model is implemented in ABAQUS software by using the implicit return mapping method.

3.1 Elastic step

Stresses are determined from strain increments by using plane stress formulas.

$$\sigma_{n+1}^{w,trial} = \sigma_n^w + \frac{E_w}{1-\nu_{12}\nu_{21}}\Delta\varepsilon_{n+1}^w + \frac{E_w\nu_{12}}{1-\nu_{12}\nu_{21}}\Delta\varepsilon_{n+1}^f \qquad (1)$$

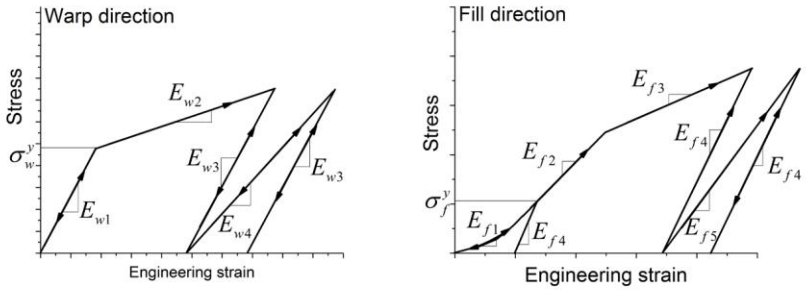

Figure 3. Hypothesis of the proposed model.

$$\sigma_{n+1}^{f,trial} = \sigma_n^f + \frac{E_f \nu_{21}}{1-\nu_{12}\nu_{21}} \Delta\varepsilon_{n+1}^w + \frac{E_f}{1-\nu_{12}\nu_{21}} \Delta\varepsilon_{n+1}^f \tag{2}$$

where E_w, E_f denote the Young's moduli in warp and fill directions, respectively. Moreover, to capture the load ratio and load history dependent characteristics of the coated fabric, the load level α and load ratio β are introduced and defined as follows:

$$\alpha_i = \sigma_i / \sigma_{\max} \quad (i = w, f) \tag{3}$$

$$\beta = \sigma_w / \sqrt{\sigma_w^2 + \sigma_f^2} \tag{4}$$

Herein $\sigma_{\max}$ is the maximum stress that the material experienced in the first load cycle of a certain load ratio.

In case β is kept constant and once the load level $\alpha_i\,(i = w, f)$ gets the value of 1, it will maintain this value. The Young's moduli in warp and fill directions are defined as follows

$$\begin{cases} E_w = E_{w1} & if\ \sigma_w < \sigma_w^y \\ E_w = E_{w3} & otherwise \end{cases} \tag{5}$$

$$\begin{cases} E_f = E_{f1} & if\ \sigma_f < \sigma_f^y \\ E_f = E_{f4} & otherwise \end{cases} \tag{6}$$

where σ_w^y and σ_f^y are the yield stresses in warp and fill directions, respectively.

In addition, if the material has been tensioned, the Young's moduli in warp and fill direction would be defined as follows (figure 4):

$$E_w = E_{w4} \quad if\ \alpha_w < 1 \tag{7}$$

$$E_f = E_{f5} \quad if\ \alpha_f < 1 \tag{8}$$

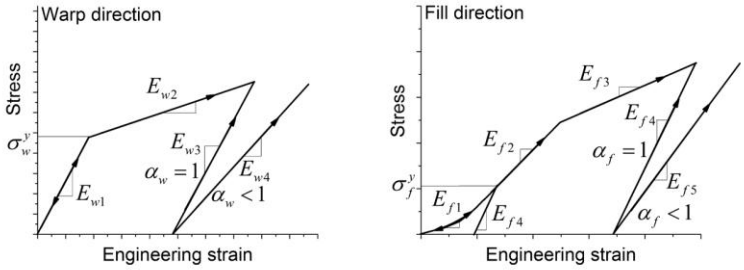

Figure 4. The behaviour of the model after the first load cycle.

3.2. Yield functions

In this study, we respectively define two different yield functions for warp and fill directions.

$$f_{n+1}^{w,trial} = \sigma_{n+1}^{w,trial} - \left(\sigma_y^w + r_n^w\right) \tag{9}$$

$$f_{n+1}^{f,trial} = \sigma_{n+1}^{f,trial} - \left(\sigma_y^f + r_n^f\right) \tag{10}$$

Herein, $f_{n+1}^{w,trial}$ and $f_{n+1}^{f,trial}$ are the yield functions, σ_y^w and σ_y^f are the yield stresses r_n^w and r_n^f are the hardening terms and defined as follows:

$$r_n^w = H_w \varepsilon^{p,w},\ r_n^f = H_f \varepsilon^{p,f} \tag{11}$$

where $\varepsilon^{p,w}, \varepsilon^{p,f}$ are the accumulated plastic strains and H_w, H_f are the plastic moduli in warp and fill direction, respectively.

3.3. Return mapping

In the model, we use the return mapping algorithm in the plastic corrector step. There are two cases that can occur:

i. Plastic deformation only occurs in warp or fill direction

Without losing the generality, we assume that plastic deformation only occurs in warp direction. Plastic strain increments can be calculated as follows:

$$\Delta\varepsilon^{p,w} = f_{n+1}^{w,trial} / (k_{11} + H_w) \tag{12}$$

Stresses are then updated as follows:

$$\sigma_{n+1}^w = \sigma_{n+1}^{w,trial} - k_{11}\Delta\varepsilon_{n+1}^{p,w} \tag{13}$$

$$\sigma_{n+1}^f = \sigma_{n+1}^{f,trial} - k_{21}\Delta\varepsilon_{n+1}^{p,w} \tag{14}$$

ii. Plastic deformation occurs in both warp and fill direction

By solving the following equations

$$\begin{cases} f_{n+1}^{trial,w} - \left(k_{11} + H_w\right)\Delta\varepsilon^{p,w} - k_{12}\Delta\varepsilon^{p,f} = 0 \\ f_{n+1}^{trial,f} - k_{21}\Delta\varepsilon^{p,w} - \left(k_{22} + H_f\right)\Delta\varepsilon^{p,f} = 0 \end{cases} \tag{15}$$

Plastic strain increments are obtained as:

$$\begin{cases} \Delta\varepsilon^{p,w} = \dfrac{-f_{n+1}^{f,trial}k_{12} + f_{n+1}^{w,trial}\left(k_{22} + H_f\right)}{\left(H_f + k_{22}\right)\left(H_w + k_{11}\right) - k_{12}k_{21}} \\ \Delta\varepsilon^{p,f} = \dfrac{-f_{n+1}^{w,trial}k_{21} + f_{n+1}^{f,trial}\left(k_{11} + H_w\right)}{\left(H_w + k_{11}\right)\left(H_f + k_{22}\right) - k_{12}k_{21}} \end{cases} \tag{16}$$

Stresses are then updated as follows:

$$\sigma_{n+1}^{w} = \sigma_{n+1}^{w,trial} - k_{11}\Delta\varepsilon^{p,w} - k_{12}\Delta\varepsilon^{p,f} \tag{17}$$

$$\sigma_{n+1}^{f} = \sigma_{n+1}^{f,trial} - k_{21}\Delta\varepsilon^{p,w} - k_{22}\Delta\varepsilon^{p,f} \tag{18}$$

4. Verifications and validations

4.1. Uniaxial tests

By fitting with the experimental data obtained from uniaxial tensile tests, the parameters are determined as follows:

In warp direction

$$\begin{cases} E_{w1} = 1364\,MPa & if\ \sigma_w < 15\,MPa \\ E_{w2} = \dfrac{E_{w3}H_w}{H_w + E_{w3}}MPa & if\ \sigma_w > 15\,MPa\ and\ \Delta\sigma_w > 0 \\ E_{w3} = 1130\,MPa & otherwise \end{cases} \tag{19}$$

and the plastic modulus in warp direction: $H_w = 448\,MPa$.

In fill direction, we have

$$\begin{cases} E_{f1} = 10^4\left(14.1309\left(\varepsilon_f\right)^2 + 0.04\varepsilon_f + 0.0115\right)MPa & \sigma_f < 5\,MPa \\ E_{f2} = \dfrac{H_{f1}E_{f4}}{H_{f1} + E_{f4}}MPa & 5\,MPa \le \sigma_f < 15\,MPa\ and\ \Delta\sigma_f \ge 0 \\ E_{f3} = \dfrac{H_{f2}E_{f4}}{H_{f2} + E_{f4}}MPa & \sigma_f \ge 15\,MPa\ and\ \Delta\sigma_f \ge 0 \\ E_{f4} = 825\,MPa & otherwise \end{cases} \tag{20}$$

and the plastic modulus in fill direction can be defined as

$$\begin{cases} H_{f1} = 593\,MPa & 5\,MPa \le \sigma_f < 15\,MPa \\ H_{f2} = 236.82\,MPa & \sigma_f \ge 15\,MPa \end{cases} \tag{21}$$

The stress-strain curves obtained from the FEM model and experimental data are plotted in figure 5. As can be seen from these graphs, good agreement is observed between FEM results and experimental data.

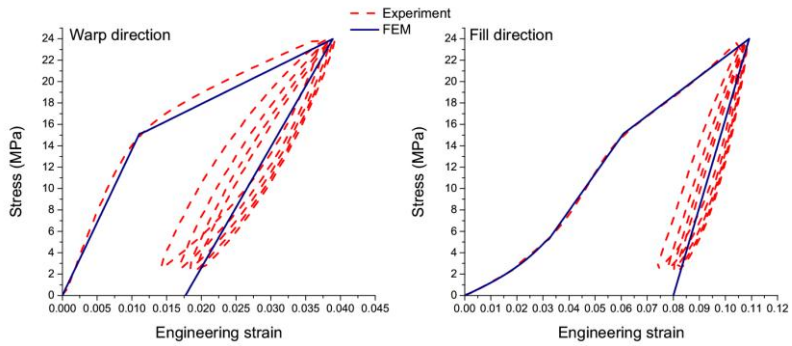

Figure 5. Stress-strain curves for uniaxial test in warp and fill directions.

4.2. Biaxial tests

The finite element analysis for the biaxial test can be performed on a quarter of the cruciform. The mesh is composed by 5041 quadrilateral membrane elements (M3D4). As mentioned in section 3, in order to take into account the interaction between yarns as well as the load ratio dependency, two Poisson's ratios and two Young's moduli are needed.

While the extracted parameters from the uniaxial tests are identified straightforwardly, the ones from the biaxial tests are not easy to determine. In order to minimise the discrepancies between numerical simulation and experimental data, a parameter identification method is implemented, in which the boundary value problem in ABAQUS is combined with the multi-objective genetic algorithm routine in MATLAB. The parameter identification attempts to find $\nu_{12}, \nu_{21}, E_{w4}$ and E_{f5} which can yield the best fit to the given stress-strain history. To obtain this target, the following objective functions should be minimised.

$$f_1(\nu_{12}, \nu_{21}, \mathrm{E}_{w4}, \mathrm{E}_{f5}) = \sum_{i=1}^{N} \min\left[\left(\varepsilon_{11_i}^{\mathrm{w},ex} - \varepsilon_{11_i}^{\mathrm{w},sim}\right)^2 + \left(\sigma_{11_i}^{\mathrm{w},ex} - \sigma_{11_i}^{\mathrm{w},sim}\right)^2\right] \tag{22}$$

$$f_2(\nu_{12}, \nu_{21}, \mathrm{E}_{w4}, \mathrm{E}_{f5}) = \sum_{i=1}^{N} \min\left[\left(\varepsilon_{11_i}^{f,ex} - \varepsilon_{11_i}^{f,sim}\right)^2 + \left(\sigma_{11_i}^{f,ex} - \sigma_{11_i}^{f,sim}\right)^2\right] \tag{23}$$

The state variables in these objective functions are subjected to the below linear constraints

$$0 \le \nu_{12} \le 1; 0 \le \nu_{21} \le 1; 0 \le E_{w4} \le E_{w3} = 1130\,MPa; 0 \le E_{f5} \le E_{f4} = 825\,MPa \tag{24}$$

where N is the number of data points from the simulation, $\varepsilon_{11_i}^{ex}$, $\varepsilon_{11_i}^{sim}$, $\sigma_{11_i}^{ex}$, $\sigma_{11_i}^{sim}$ are respectively the strains and stresses in warp direction in the experiment and the simulation,

$\varepsilon_{22_i}^{ex}$, $\varepsilon_{22_i}^{sim}$, $\sigma_{22_i}^{ex}$, $\sigma_{22_i}^{sim}$ are the counterparts in fill direction. From this identification parameter procedure, $\nu_{12} = 0.09076$, $\nu_{21} = 0.46923$, $E_{w4} = 852.241\,\text{MPa}$ and $E_{f5} = 618.402\,\text{MPa}$ are determined.

In figure 6, the stress strain curves in warp and fill direction are plotted for both the experimental and the simulation results. Good agreement between them can be seen. Especially, the proposed model can successfully capture nonlinearities in warp and fill directions as well as the orthotropic effect. Moreover, the permanent strains and load ratio dependence are captured quite precisely in both directions.

5. Conclusions

An elasto-plastic model for coated fabrics is proposed based on the experimental data obtained from uniaxial tensile tests and validated by the biaxial tests. Poisson's ratios are used to capture the interaction between warp and fill yarns as well as between yarns and coating. The obtained results show that the proposed model can capture the nonlinear, orthotropic and load ratio-dependent behaviour as well as permanent strains. Consequently, we can reduce the uncertainties in the design of tension membrane structures. In addition, during the numerical simulation, good performance in computation is observed. From this point, the proposed model is eligible to simulate the large scale structures. However, in this work, time dependent behaviour is not considered and nonlinear shear responses are also excluded. These characteristics will be taken into account in further developments of this model.

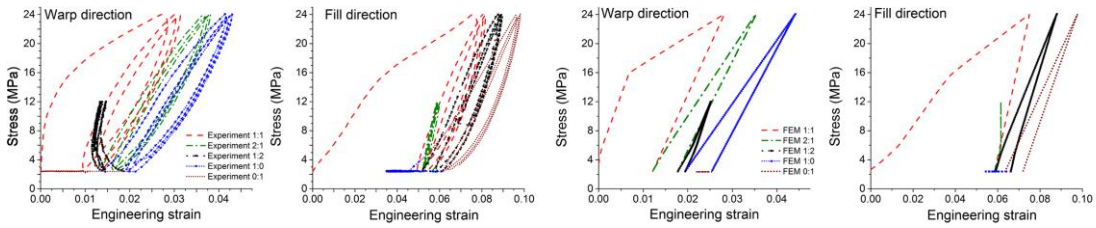

Figure 6. FEM results and experimental data from the biaxial tests.

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