OPTIMIZATION OF BLENDED COMPOSITE STRUCTURES: A PARAMETRIC STUDY

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ABSTRACT

The optimization of composite structures raises a complex problem because of the high number of parameters and often requires computer-assisted design tools. It is well known that genetic algorithms (GA) are particularly efficient in treating this type of problem. If a suitable compromise between calculating time and quality of configurations for GA methods is needed, it can be proved that working on simplified problems can be more efficient because the possible parameter values would be restricted and therefore converge on a near optimal solution in a reasonable number of iterations. The purpose of this study was to assess the influence of the main parameters on the optimization tool's behaviour in terms of the performance of solutions and the reliability of the process.

1. INTRODUCTION

The use of genetic algorithms (GA) to optimize laminated composite structures is widespread, as indicated by the profusion of publications on the subject. This type of method is particularly effective in the case of combinatorial problems, when the number of design variables (DVs) is high. However, considering the stochastic nature of this method, a high number of structural analyses (SAs) must be made to obtain a successful configuration. In practice, the duration of the optimization process is inevitably limited. It seems difficult to reach a satisfying compromise between the quality of the results and a reasonable calculation time. Several options were tested to reduce the calculation time and/or improve the reliability of such methods: construction of response surfaces, hybridization, specific operators, etc. One possibility is to simplify the problem to reduce its complexity: by considering a limited set of parameters among the most influential, it may be preferable to obtain a good estimate of a near optimal design. The objective of this work was to estimate the influence of various parameters on the quality of an optimization process and to deduce a number of methodological rules. We will therefore consider two simple problems. SAs are conducted using the finite element method (FEM). Both structure and material will be simultaneously optimized and the most influential parameters can be identified from design of experiment.

2. FRAMEWORK

Classically, optimizing composite laminates involves the number of plies, their nature, the respective orientations and the stacking sequence. In addition, the possible orientations are generally chosen in a list of discreet values to facilitate the production of the laminate.

The intensity and the direction of the loads evolve across the structure, which are currently decomposed into panels that will be sized according to the loads applied to them locally. It is also necessary to impose the continuity of the orientations (or materials) of the layers between two neighbouring panels (blending). To respect these constraints, Kristinsdottir and al. [1] used "key region" and "bigger or equal" rules. Soremekun and al. [2] propose a procedure in several stages, based on the definition and optimization of sublaminates.

We will use a guided design [3]: a stacking sequence common to all the panels is defined, then the various panels are subjected to controlled ply withdrawals. This method presents the

advantage of reducing the number of DVs with regard to the methods suggested above. Moreover, all the configurations defined automatically respect the continuity constraints. However, the use of this method means excluding numerous practicable configurations.
All the optimizations are governed by GA [4]. Considering the combinatorial nature of the problems considered, we used an integer coding which defines the global stacking sequence as well as the number of layers in each panel (see Fig. 1). We use a binary stochastic tournament selection method. The constraints are taken into account according to the principle of superiority of the feasible individual. In the case of mono-objective optimization, the performance is the value of the function to be minimized. With multi-objective optimization, the performance is determined from the SPEA method [4]. We retained an operator of uniform crossing. The mutation operator modifies a single integer (chosen at random), with a probability p_{mut}. In the multi-objective version, we used a generational replacement. In the mono-objective version, we used a multi-elitist procedure: the n_e better configurations of the current population were systematically copied out in the next population. The missing individuals were obtained by reproduction. We used n_e =N/2, N being the size of the population. Two problems will be considered. Symmetric laminates will be considered later.

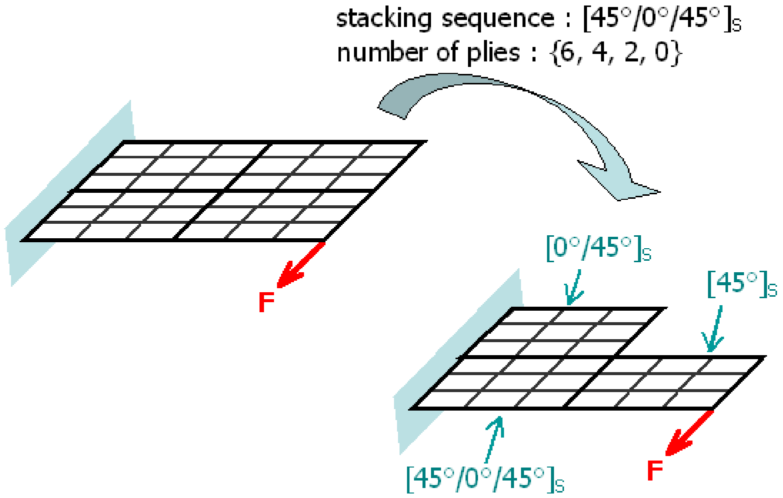


Figure 1: Example of guide-based optimization

2.1 Composite plate in membrane (P1)

We considered a rectangular plate measuring 1000×800 mm^2. The AD side was clamped, and the plate was subjected to a vertical load F=5 kN at point B (Fig. 2). The plate was discretized into 80 elements Q4 [5]. The behaviour was elastic linear, and the failure criterion was the first ply failure (FPF), predicted with the Hill criterion [6]. The total mass M of the plate is limited, as is the vertical displacement d_B of point B. To guarantee the load transfer, we prevent the total withdrawal of material along edge AB. The problem is formulated as:

$$\text{P1:}\begin{cases} \text{minimize} & (\mathrm{M}, \mathrm{d_B}) \\ \text{under constraint that} & \mathrm{e} \leq 2.6\,\mathrm{mm} \\ & \theta_k^p \in \Gamma \qquad k = 1, \mathrm{n_{pli,p}};\ p = 1, \mathrm{n_p} \\ & \theta_k^l = \theta_k^m \qquad \forall (l,m) \in (1,\mathrm{n_p} \times 1, \mathrm{n_p}) \\ & \mathrm{I_{TH}^{k,e}} \leq 1 \qquad k = 1, \mathrm{n_{pli,p}};\ e = 1, \mathrm{n_e} \end{cases} \tag{1}$$

$n_{pli,p}$ represents the number of plies of the panel p. n_p and n_e represent the number of panels and the number of elements, respectively. The influence of three groups of parameters, for which we consider two modalities, will be estimated:

- The constituent material of layers. Two materials will be compared: a glass/epoxy UD such that V_f = 60%, noted U, and a balanced glass fabric/epoxy (V_f = 50%) noted T. The characteristics of both materials are summarized in Table 1.

Table 1: mechanical properties of the materials

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	e (mm)	ρ ($kg.m^{-3}$)
U	45.0	12.0	4.5	0.3	0.13	2080
T	20	20	2.85	0.13	0.21	1900
	X^+ (MPa)	X^- (MPa)	Y^+ (MPa)	Y^- (MPa)	S (MPa)	
U	1250	600	35	141	63	
T	400	390	400	390	54	

- The set of possible orientations for layers. We associated sets of specific orientations with every available material, notably to integrate the symmetry of balanced fabrics. Table 2 presents these sets. The maximum number of layers was deduced from the maximal thickness allowed: 20 with U (i.e. e = 2.6 mm), 12 with T (i.e. e = 2.52 mm).

Table 2: orientation sets available

Material	U	T
Γ_{-1}	{0°, 45°,–45°,90°}	{0°,45°}
Γ_1	{0°,15°,–15°,30°,–30°,45°,–45°,60°,–60°,75°,–75°,90°}	{0°,15°,45°,60°,75°}

- The decomposition. The structure can be decomposed into 20 panels measuring 200 x 200 mm^2 (see Fig. 2) or into 80 panels (every finite element is then considered as a panel).

2.2 Composite plate in flexion/torsion (P2)

We considered a rectangular plate measuring 100 x 45 mm^2. The side AD was clamped. The plate was subjected to a vertical load F = 50 N on point B and to a vertical load F = 10 N on point C (see Fig. 3). The plate was discretized into 120 DKQ elements [5]. The behaviour was elastic linear, and the failure criterion was FPF, predicted with the Hill criterion. The total mass M of the plate as well as the vertical displacement of points B and C were minimized. An "equivalent" displacement is built:

$$d = \sqrt{d_B^2 + d_C^2} \qquad (2)$$

To guarantee the load transfer, we forbid the total withdrawal of material along edges AB and DC. The problem is formulated as:

$$P2: \begin{cases} \text{minimize} & (M, d) & \\ \text{under constraint that} & e \le 2.6\,\text{mm} & \\ & \theta_k^p \in \Gamma & k = 1, n_{pli,p};\ p = 1, n_p \\ & \theta_k^l = \theta_k^m & \forall (l, m) \in (1, n_p \times 1, n_p) \\ & I_{TH}^{k,e} \le 1 & k = 1, n_{pli,p};\ e = 1, n_e \end{cases} \qquad (3)$$

The parameters taken into account are the same as in problem (P1). The plate will be decomposed into 15 panels (i.e. eight elements per panel) or into 60 panels (i.e. two elements per panel).

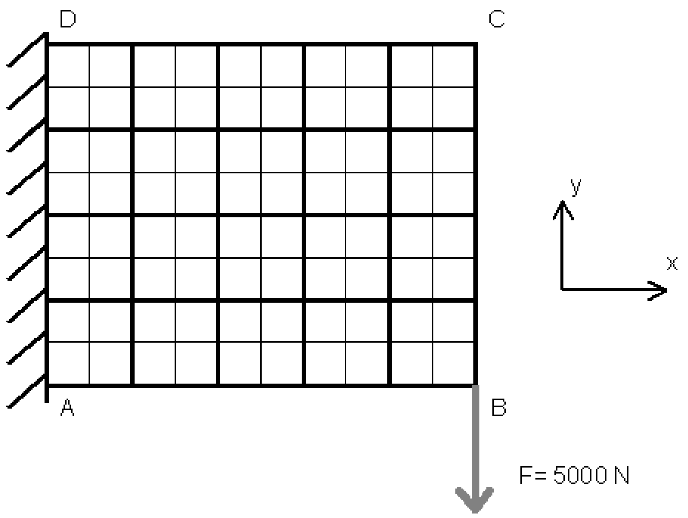


Figure 2: FE mesh and available decompositions, P1

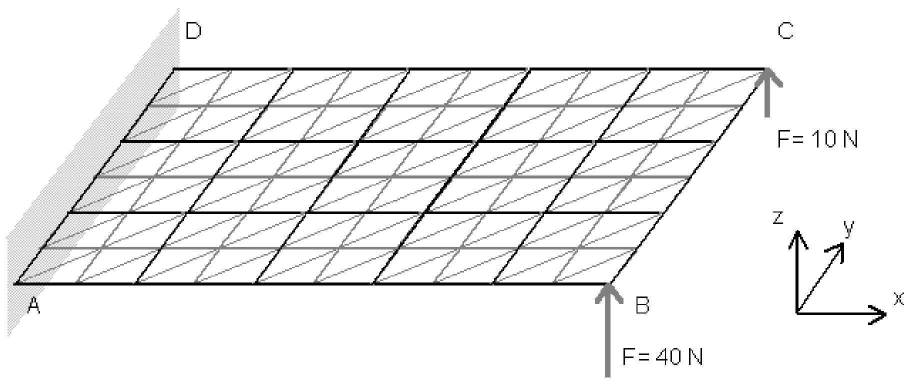


Figure 3: FE mesh and available decompositions, P2

3. MULTI-OBJECTIVE OPTIMIZATION

To estimate the respective influence of orientation sets (Γ), the constituent material (M) and the refinement of the decomposition (D), we used a complete factorial plan, representing eight experiments. Considering the stochastic nature of the GA, every experiment was conducted ten times. Finally, in every case, the parameters of the GA will be set to the values summarized in Table 3.

Table 3. GA parameters

p_{mut}	0.05
p_{sel}	0.7
p_{crois}	0.9
Population size	200
Number of elites	100
Max number of SAs	20,000
Max number of generations	200

3.1 Problem 1

The accumulation of the non dominated designs obtained on ten optimizations and for each of the experiments, is shown in Fig. 4. As a rough guide, the average duration of ten optimizations was approximately 1.5 h on an eMac computer with a 1.25-GHz processor and 1-Gb RAM. For better clarity, we eliminated the configurations for which $d_B > 20$.

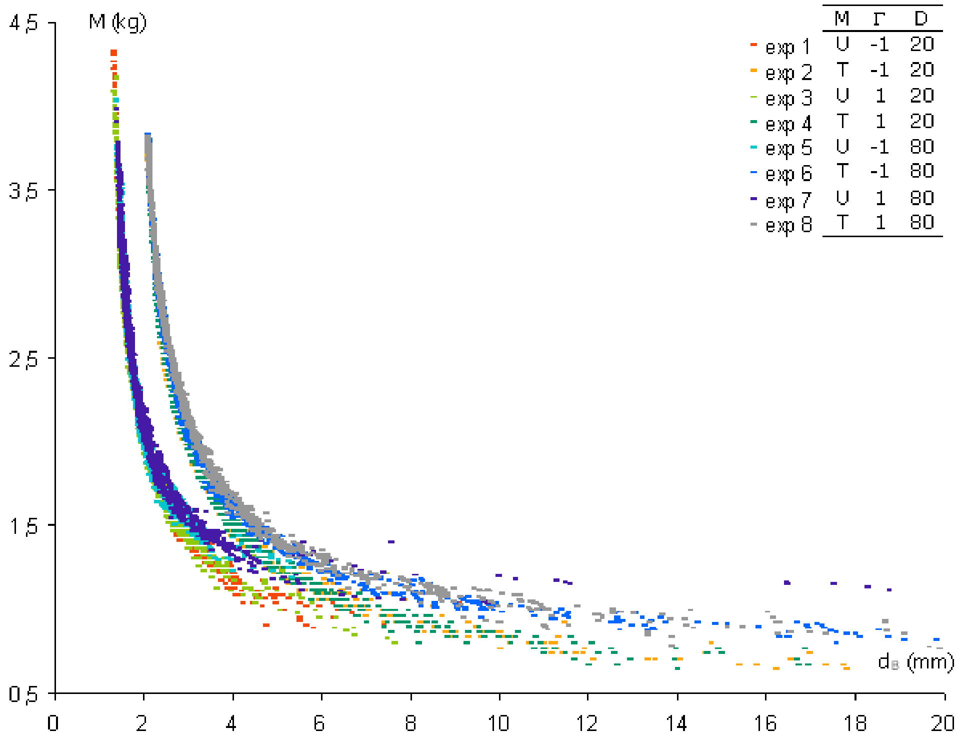


Figure 4: Nondominated designs, P1

3.1.1 The extrema

In the range of the stiffest configurations (d < 4 mm), two groups of solutions were distinguished, associated with both available materials. Depending on the material used, the variation of rigidity was on the order of 50%. The type of decomposition did not influence the rigidity: as for the stiffest configurations, the most predictable are those that had the greatest number of layers. We also see that the choice of the set of orientations had a negligible effect on rigidity. Finally, dispersion was low because GA worked on a simplified problem by manipulating designs with the maximum thickness: on one hand because the number of influential parameters (orientations) was reduced, and on the other hand because all the individuals were feasible.

In the range of the lightest configurations (d > 16 mm), two groups of configurations were distinguished. All the designs were made up of the same material (here T). These two groups were associated in terms of the level of decomposition. With equivalent rigidity, the configurations obtained with the "fine" decomposition (80 panels) were approximately 20% heavier than the configurations obtained with a coarser decomposition. This is paradoxical given that the "unrefined" configurations were included in the set of the "fine" configurations.

Finally, observations of the various curves obtained show that the set of orientations available influenced a priori not the optima values, and moderately the dispersal (i.e. the reliability of the optimization).

3.1.2 A compromise

Let us now consider the concave part of the pareto front, i.e. the configurations providing a certain compromise between mass and rigidity. For that purpose, we define a measure of quality:

$$\delta = \sqrt{\left(\frac{M}{M_{min}}\right)^2 + \left(\frac{d}{d_{min}}\right)^2} \quad (6)$$

which represents the distance between a compromise (M, d) and the origin of the space of the objectives. M_{min} and d_{min} are the minimal mass and the minimal displacement on all the optimizations, respectively. For each of the experiments, we determined the minimal value of δ. The following figure shows the effects of the three factors studied. We also show the standard deviations (represented by the black lines). The result here is similar to the previous study, i.e. the material is the most influential factor on the quality of the result. Note also that the best configurations are obtained by decomposing the structure into 20 panels. However, the effects of the decomposition and the set of orientations are minimal compared to the dispersal, and can thus be considered as not significant.

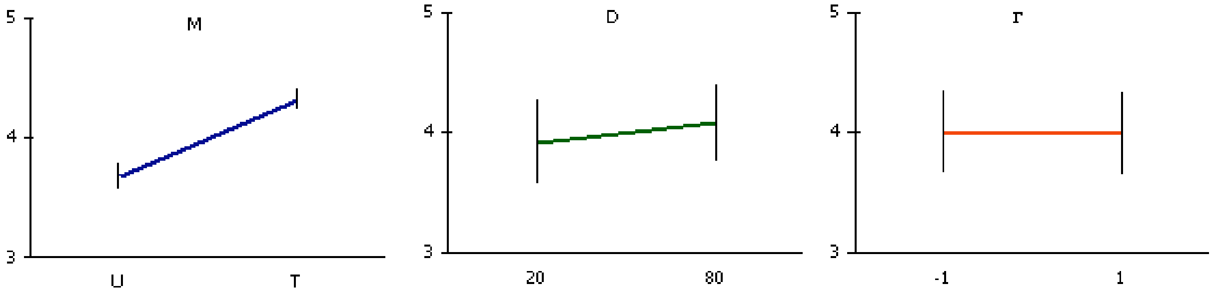


Fig. 5 Effects of material, sub-structuring and orientations set, minimization of δ, P1

3.2 Problem 2

The accumulation of the nondominated designs obtained on all the optimizations is presented in Figure 6. Ten optimizations lasted approximately 2 h, with results similar to problem (P1): the choice of the set of possible orientations seems to have a moderate effect on the response of the process. The material was by far the most influential parameter, because the maximal rigidity varied by a factor of 2 for an equivalent mass. Also, the minimal mass varied by 30% depending on the material.

The refinement of the decomposition seems to have little effect on the quality of the designs obtained: the pareto fronts obtained with 60 panels and with 15 panels were almost superposed. On the other hand, it should be noted that the front obtained with 60 panels occupies a smaller portion of the space of the objectives. It would seem that the algorithm investigates the search space in a less effective way.

Finally, let us note that the dispersal is greater for lighter designs. In this region of the space, numerous configurations do not respect one or several constraints.

3.2.1 A compromise

Here again, the most influential parameter is the material (see Fig. 7).

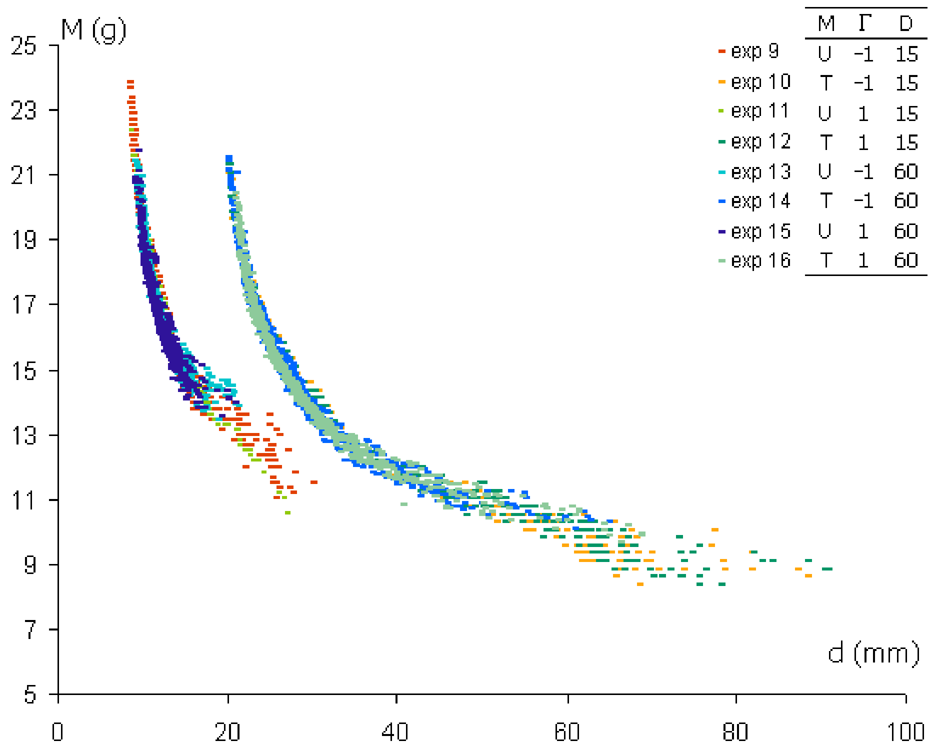


Figure 6: Nondominated designs, P2

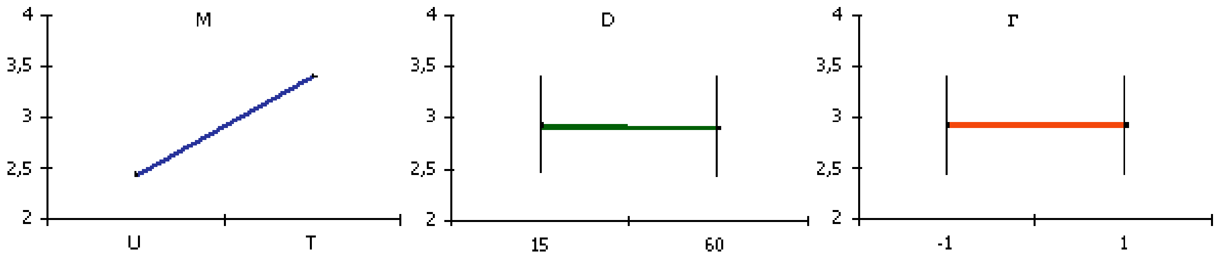


Figure 7: Effects of material, orientations set and sub-structuring, minimization of δ, P2

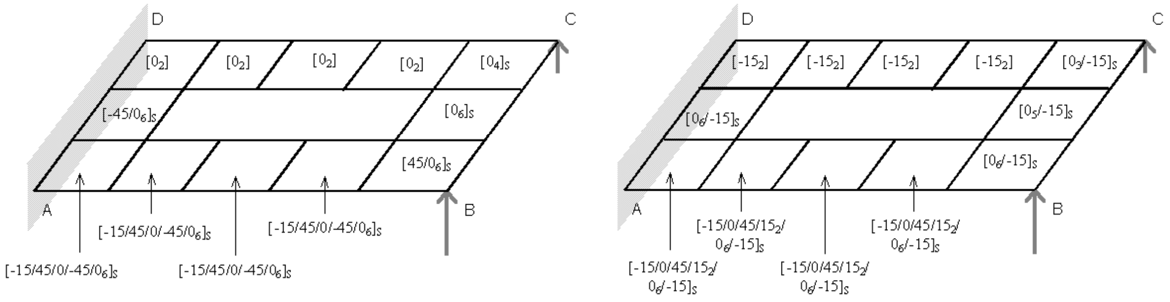
4. MONO-OBJECTIVE OPTIMIZATION

In addition to the parameter setting, the reliability of an optimization process is also linked to the problem considered and to the algorithms chosen. Therefore, the previous step was removed in mono-objective optimizations. At convergence, a mono-objective GA concentrated the main part of the research effort in restricted regions of the search space, contrary to a multi-objective GA, which had to distribute the search over the entire pareto front.

These optimizations validate the remarks previously made on a different implementation of GA and improve certain pareto compromises, thus providing an idea of the quality of the multi-objective optimizations performed.

4.1 Problems considered

We considered a plate in flexion or twisting such as the plate defined in section 2. We then used the material U. The observation of the results of problems P1 and P2 shows that the search for the lightest configurations presents the highest level of complexity, notably because of the high number of light configurations that do not respect the failure criterion. Figure 8 represents the best design (M= 10.546 g, d= 26.82 mm) and the stiffest design weighing less than 11.033g (M= 11.032 g, d= 25.319 mm).



a) b)

Figure 8: Extreme solutions to problem P2 (material: U)
a) lightest design obtained by MOGA
b) stiffest design with M < 11.03232 g obtained by MOGA

We therefore estimated the effect of the set of orientations and the decomposition on the resolution of the following problems:

$$P3 = \begin{cases} \text{minimize} & \mathrm{M} & \\ \text{under condition that} & \mathrm{d} < 40\,\mathrm{mm} & \\ & \mathrm{e} \le 2.6\,\mathrm{mm} & \\ & \theta_k^p \in \Gamma & k = 1, \mathrm{n_{pli,p}};\ p = 1, \mathrm{n_p} \\ & \theta_k^l = \theta_k^m & \forall (l,m) \in (1, \mathrm{n_p} \times 1, \mathrm{n_p}) \\ & \mathrm{I_{TH}^{k,e}} \le 1 & k = 1, \mathrm{n_{pli,p}};\ e = 1, \mathrm{n_e} \end{cases} \tag{7}$$

and

$$P4 = \begin{cases} \text{minimize} & \mathrm{d} & \\ \text{under condition that} & \mathrm{M} < 11.033\,\mathrm{g} & \\ & \mathrm{e} \le 2.6\,\mathrm{mm} & \\ & \theta_k^p \in \Gamma & k = 1, \mathrm{n_{pli,p}};\ p = 1, \mathrm{n_p} \\ & \theta_k^l = \theta_k^m & \forall (l,m) \in (1, \mathrm{n_p} \times 1, \mathrm{n_p}) \\ & \mathrm{I_{TH}^{k,e}} \le 1 & k = 1, \mathrm{n_{pli,p}};\ e = 1, \mathrm{n_e} \end{cases} \tag{8}$$

4.2 Problem P3: results

The average mass obtained on all the optimizations was 9.1 g, with a standard deviation on the order of 0.86 g. These results are close to the mass of the reference configuration stemming from the multi-optimization objective, demonstrating the efficiency of the SPEA method. In particular, the lightest solution was obtained by considering 15 panels and 12 orientations, represented in Fig. 9 (M = 7.95 g, d= 39.82 mm).
The diagram of the effects (Fig. 10) shows that the parameters have only a slight effect on the quality of the optimization results. In particular, a fine decomposition is not advantageous, and the configurations obtained are globally the heaviest.

4.3 Problem P4: results

This problem was more complex than the previous one, since GA did not succeed in producing feasible designs. To avoid this problem, we injected ten optimal solutions from problem P3 into the initial population, to provide the AG with light and feasible designs. This made it possible to obtain configurations respecting all the constraints. It should be noted, however, that we introduced a bias into the process. For this problem, the parameters seem to

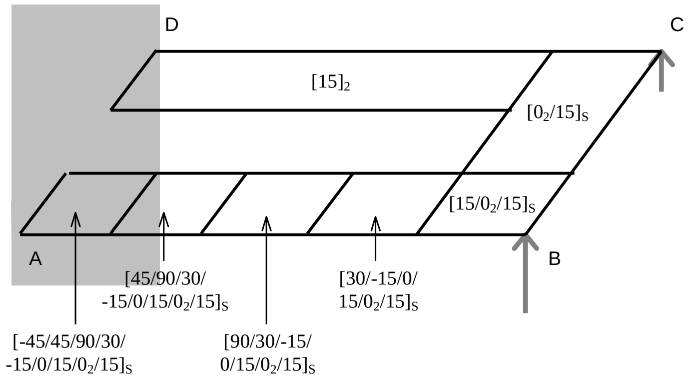


Figure 9: Lightest design obtained with GA (mat: U)

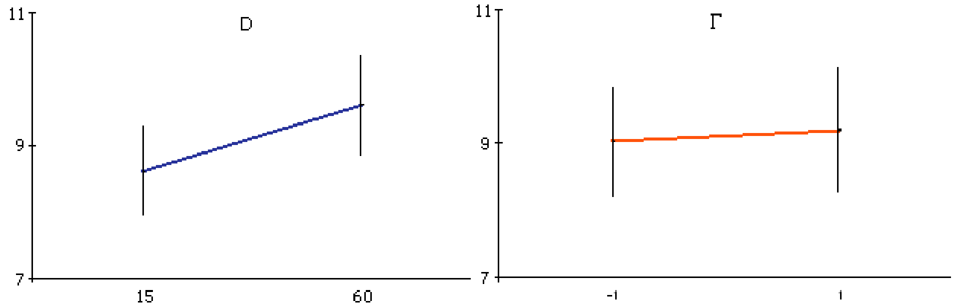


Figure 10: Effects of decomposition and orientations set, P3

have a more pronounced effect, and the best design (17.47 mm, 10.83 g) was obtained by using a fine decomposition (see Fig. 11). However, this configuration is not optimal: considering its mass, it is possible to add material on one of the panels. Furthermore, islands of material do not contribute to the rigidity, and it would be profitable to move them (Fig. 12).

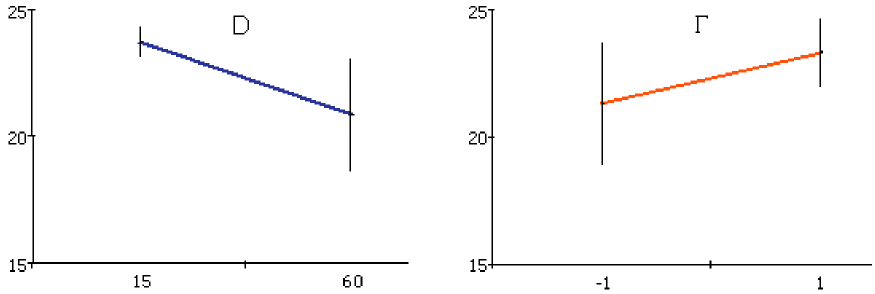


Figure 11: Effects of decomposition and orientations set, P4

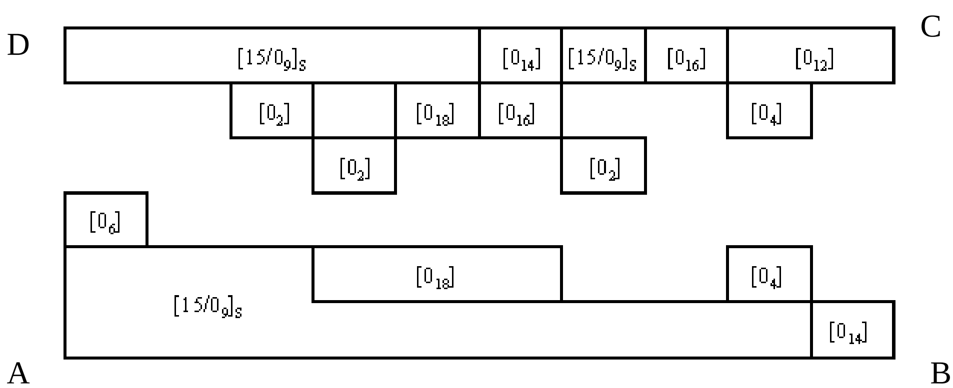


Figure 12: Stiffest design with M< 11.033 g obtained with GA (mat: U)

CONCLUSION

The quality of the result of an optimization process using GA depends on numerous factors, such as the functions treated (objectives, constraints), the implementation of the optimization tool (choosing the operators and setting the parameters), as well as the size of the problem. Typically, obtaining a successful design requires an often overwhelming calculation time, and it may be necessary to simplify the SA and the sets of possible values of the various design variables. The influence of the parameter setting on the performance of the configurations obtained by GA was studied on a set of simple but representative problems of real cases. Two variants of GA were tested. Several comments can be made from these results.

The constituent material greatly influences all the mechanical characteristics of a structure. The designer will therefore have to be particularly careful in these choices, depending on both the application and the objectives.

It is common to retain only a limited set of orientations, the objective being to simplify the implementation and to decrease costs. This study shows that using a larger list of orientations only contributes a moderate gain in terms of mechanical performance. Moreover, this choice can be penalizing from the point of view of the reliability of the optimization process.

Finally, the structure must be decomposed with caution. We could expect that a fine decomposition provides the best performance. Experience shows this not to be the case. Two reasons can be given to explain this contradiction:

- The combinatorial explosion of the number of possible configurations. The more limited the resources, the greater the probability of unsuccessful designs.
- The presence of additional local extrema, which are merely pitfalls for the algorithm.

The heart of the problem lies in defining the decomposition to a sufficiently fine degree while limiting the number of panels. This decomposition could be defined, for example, by defining an equivalent isotropic material. One preliminary analysis would probably allow the designer to make a relevant choice by taking into account gradients of constraints on the extent of the structure and the specificities of the material retained.

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