STRESS ANALYSIS OF COMPOSITE PLATES WITH SQUARE CUTOUT

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ABASTRACT

Panels with variously shaped cutout are often used in engineering structures. The understanding of the effects of cutout on the load bearing capacity and stress concentration of such plates is very important in designing of structures. However, little research has been focused on stress analysis of plates with special shaped cutout. This study investigates problems associated with the stress concentration in perforated composite plates with quasi-square shaped cutouts. Analytical solution for stress analysis of composite plates with central cutout is presented. Analytical and numerical studies were conducted to investigate the effects of variation in cutout orientation and bluntness, material properties and loading direction on the location and the value of the maximum stress in flat plate subjected uni-axial tension load. This study expands the Lekhnitskii's solution for circular and elliptical cutout to square-shape cutouts. The solution is capable of considering large variety of cutout shapes and loading conditions analytically. Based on results presented herein, the maximum normalized stress of perforated composite plates can be significantly changed by using proper material properties, loading angle, cutout bluntness and orientation

1. INTRODUCTION

Different cutout shapes in structural elements are needed to reduce the weight of the structure or provide access to other parts of the structure. In some cases, structural elements are being damaged during their service life. It is well known that the presence of a cutout or hole in a stressed member creates highly localized stresses at the vicinity of the cutout. The ratio of the maximum stress at the cutout edge to the nominal stress is known as stress concentration factor (SCF).

Many researchers have outlined the importance of SCF in isotropic and composite materials. In all of these works, circular and elliptical cutouts are considered. Folias and Wang[1] presented most of the previous works on stress concentration in this subject. They presented a series solution for stress filed around a circular holes in a plates with arbitrary thickness.

Ko[2] used anisotropic plate theory to evaluate the stress concentration factor for laminated composite plates with central circular cutout. Daoust and Hoa[3] investigated the influence of bluntness curvature and material properties on the state of stress around a triangular cutout in an infinite composite plate subjected to a tensile load. Most of these works are based on the theory developed by Savin[4] and Lekhnitskii[5] for anisotropic perforated plates. Wu and Mu[6] investigated the SCF for isotropic plates under uni-axial and bi axial loads. The SCF were also determined for isotropic and orthotropic cylindrical shells with circular cutout. Free vibration of rectangular plates with different cutouts are investigated by Chai[7] and Huang [8] . Little research has been focused on stress analysis of plates with special shaped cutout.

In our previous works[9][10], stresses in perforated composite plates with circular, rectangular, and triangular shaped cutouts are investigated.

This study investigates problems associated with the stress concentration in perforated composite plates with quasi-square shaped cutouts. Analytical solution for stress analysis of composite plates with central cutout is presented. Numerical evaluation using commercial finite element code, were conducted to evaluate the analytical results. Since the cutout is small, its effect will be negligible at a distance of a few diameters from its edge. Thus point at such distance may be regarded as at infinity. In the present studies, the material behavior is assumed to be linearly elastic.

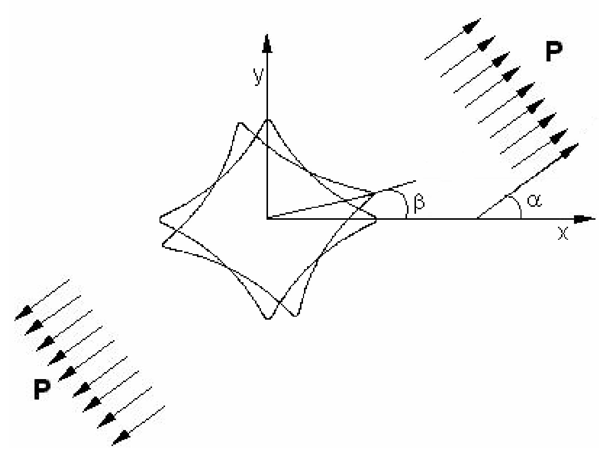

Figure 1. An infinite plate with a quasi-square central cutout subjected to off axis uni-axial tension.

2. MATERIALS AND METHODS

An infinite composite plate subjected to off axis uni-axial load is considered in this study. A central cutout of nominal diameter D is located at center of the plate (figure 1). D is the diameter of the circumscribe circle of the shape. The cutout size is small enough, that its effect will be negligible at a distance of a few diameters from its edge. Assume, the major axis of cutout is directed at angle β with horizontal axis and a uniform tensile load is applied at angle of α (figure1). The material properties used in this analysis presents in table1.

Material	**E_1** (Gpa)	**E_2** (Gpa)	**G_{12}** (GPa)	ν_{12}	**$n=E_1/E_2$**
Woven Glass/epoxy(7781/5245C)	29.7	29.7	5.3	0.17	1
CE9000 Glass/epoxy	47.4	16.2	7	0.26	2.93
Graphite/epoxy	181	10.3	7.17	0.28	17.57
Graphite/epoxy(GY-70/934)	294	6.4	4.9	0.23	45.94

Table1: Material properties.

2.1- Basic Relations

The stress components can be expressed in term of a single stress function which is called Airy's stress function.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \;,\quad \sigma_y = \frac{\partial^2 F}{\partial x^2} \;,\quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{1}$$

Based on Likhnitskii's theory of elasticity of anisotropic bodies, the stress function can be represented by an analytical function with unknown coefficients. The unknown coefficients can be determined by applying corresponding boundary conditions of the cutout. By substituting eq.(1) in equilibrium equation, a forth order differential equation in F is obtained.

$$R_{22}\frac{\partial^4 F}{\partial x^4} - 2R_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + (2R_{12} + R_{66})\frac{\partial^4 F}{\partial x^2 \partial y^2} - 2R_{16}\frac{\partial^4 F}{\partial x \partial y^3} + R_{11}\frac{\partial^4 F}{\partial y^4} = 0 \tag{2}$$

R_{ij} are the coefficient of the reduced compliance matrix. These coefficients are function of material properties. For plane stress and plane strain problems the stress function is governed by the same differential equation with different coefficients which derived from corresponding assumption. Lekhnitskii[5] proved, this equation can be transferred to four linear operators of the first order D_k.

$$D_1 D_2 D_3 D_4 F(x,y) = 0 \quad , \quad D_k = \frac{\partial}{\partial y} - \mu_k \frac{\partial}{\partial x} \tag{3}$$

where μ_k are the roots of the following characteristic equation.

$$R_{11}\mu^4 - 2R_{16}\mu^3 + (2R_{12} + R_{66})\mu^2 - 2R_{26}\mu + R_{22} = 0 \tag{4}$$

It can be proved, in general, the eq.(4) has four distinct roots with positive imaginary part. The general expression for the stress function F depends on μ_k, the roots of characteristic equation. Since the stresses are real, the general, expression for the stress function (eq. (5)) involves the real part of some functions.

$$F(x,y) = 2\,\mathrm{Re}\left[f_1(z_1) + f_2(z_2)\right] \tag{5}$$

f_1 and f_2 are arbitrary function in terms of complex variable $z_k = x + \mu_k y$ for k=1,2. With this approach, the problem is reduced to find two complex functions f_1 and f_2 such that the boundary conditions on the cutout edge are satisfied. Once the stress function is obtained, the stresses can be determined from eq.(6). The in-plane stresses can be expressed as function of two complex stress potentials.

$$\begin{aligned} \sigma_x &= 2\,\mathrm{Re}\left[\mu_1^2 f_1''(z_1) + \mu_2^2 f_2''(z_2)\right] \\ \sigma_y &= 2\,\mathrm{Re}\left[f_1''(z_1) + f_2''(z_2)\right] \\ \tau_{xy} &= -2\,\mathrm{Re}\left[\mu_1 f_1''(z_1) + \mu_2 f_2''(z_2)\right] \end{aligned} \tag{6}$$

Different forms of functions f_1 , f_2 were introduced [3], [11] for evaluating the stress function F and stress components. In this study [12], a stress function F similar the one presented by Daost and Hoa[3] is used.

Lekhnitskii[3] presented a stress function for an anisotropic infinite plate with circular and elliptical cutout. Leknitsikii's solution is limited to circular, elliptical cutouts in isotropic and orthotropic plates. In order to use this approach to other shape cutouts, establishing a relationship between any cutouts and a circular cutout is necessary.

A simple mapping function is used to model different shape cutouts. A wide variety of cutout shapes can be analyzed, once the mapping function is determined for different cutout boundaries. The geometrical representation of square-shaped cutouts in xy-plane is presented by following simple equations[10].

$$x = \lambda(\cos\theta + w\cos(3\theta)) \quad , \quad y = -x \tag{7}$$

The parameter λ, which is a positive and real number, controls the size of the cutout. Parameter **w** is the bluntness factor which changes the radius of curvature at the corner of the cutout. Figure 3 presents different cutout shapes generated using these equations.

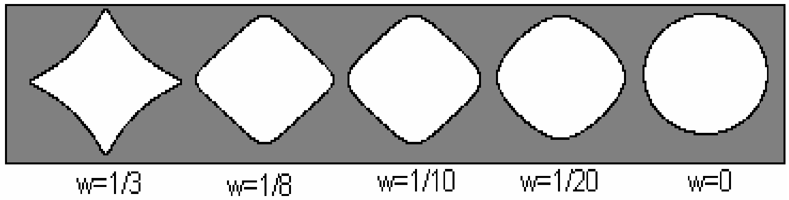

Figure 2. Different bluntness for quasi-square cutout

2.1- Validation of the Results

To check the accuracy of the result, the existing cases in literatures are modeled and compared with corresponding results[10]. As an example, Table 2 presents a comparison between the present results and results presented by Daoust and Hoa[3] for a glass epoxy plate with triangular cutout.

A finite element model is created for each case too. For each case mesh sensitivity is investigated and acceptable mesh is chosen. The stress resulting from closed form solutions were compared to those of FE models. Figure 3 present the FEM results and present study for an isotropic plate with square cutout as a function of bluntness parameter. The bluntness parameters **w** are the same as those are presented in figure 2. Good agreement is observed and provides confidence in the accuracy of the present results.

The normalized stresses are the stress in σ_x or σ_θ divided by the nominal or applied stress σ. In most cases, this normalized stresses represent theoretical stress concentration factor of the plate.

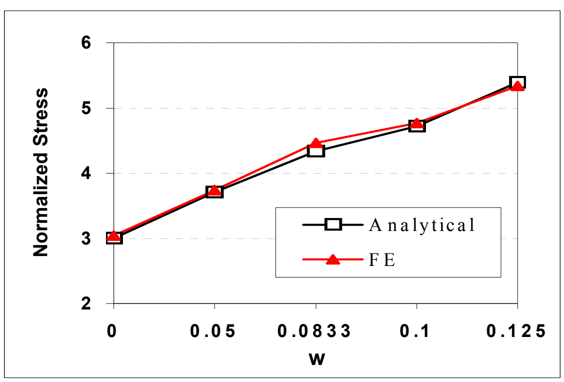

Figure 3: Analytical and FE stresses for isotropic plates with square cutout.

θ (degree)	x (length)	Ref[3]	Present study	%Discrepancy
60	0.333	0.426	0.426	0
110	-0.597	5.879	5.830	0.83
115.17	-0.638	12.131	12.390	2.13
120	-1.167	6.278	6.370	1.47

Table 2: Comparison of normalized stresses for triangular cutout w=1/3.

3. RESULTS AND DISCUSION

Stress distributions of flat composite plates with quasi-square central cutouts are investigated. The varying parameters such as, cutout orientation (rotation angle) and bluntness, material properties, and load direction which affect the stress distributions in the perforated plates are considered.

There are two values of maximum normalized stress for each cutout which are very important in design, desirable and undesirable one which can be seen in figure 4. The undesirable maximum normalized stress (undesirable stress) must be avoided in design. The desirable maximum normalized stress (desirable stress) of a specific shaped cutout is the one which yields to lower stress in plates. Based on load angle or cutout orientation each square cutout with a specific bluntness has these two desirable and undesirable stresses. As shown in figure 4, the desirable stresses are achieved when the cutout is oriented at angle 45. Similarly, the undesirable stresses are achieved at 0 or 90 degree.

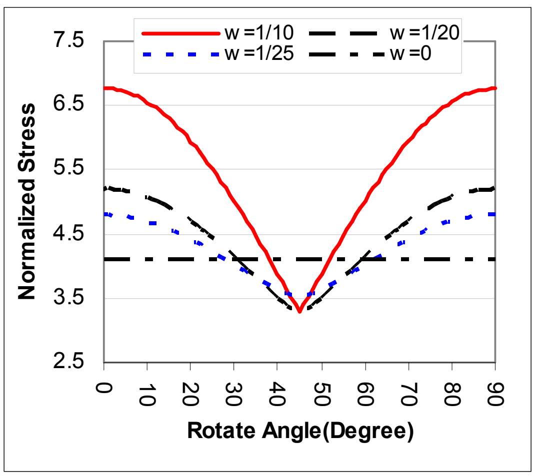

Figure 4: Effect of Rotation Angle on normalized stress

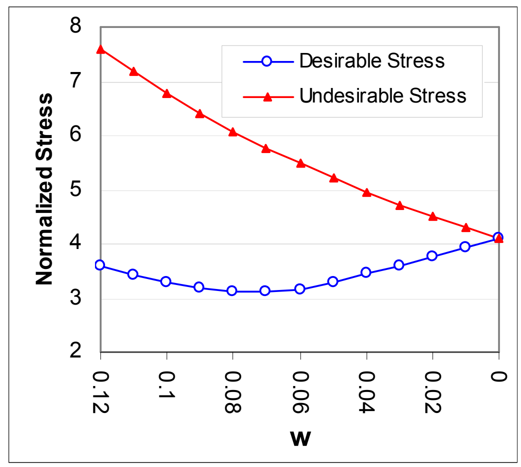

Figure 5: Effect of rotation angle and bluntness on normalized stress.

3.1- Effect of Cutout Orientation

Figures 4 and 5 illustrate the variation of desirable and undesirable stresses for square cutouts with respect to variation of cutout orientation (rotation angle) and bluntness for CE9000 glass/epoxy plates. It is clear, for a square cutout undesirable stress high for sharp corner and its value reduces when bluntness is decreases. However, the desirable stress is changed completely different than undesirable stress. For w=0 which represent a circular cutout, both desirable and undesirable stress are the same and close to 4 as it was expected. As demonstrated by this figure, the circular cutout is not optimum cutout and for w=0.073 the desirable stress has its lowest value equal 3.103 which is less than corresponding value (4.108) for circular cutout.

3.2- Effect of Fiber Angle

Effect of fiber angle of unidirectional glass/epoxy plate for different values of cutout bluntness is presented in Figures 6 and 7. As demonstrated by the figures, there is a specific fiber angle (60 degree) in which the undesirable stresses are minimum but the desirable stress shows different behavior for each cutout bluntness. The desirable stress has a maximum value of 3.5 and a minimum about 2.5. However, the undesirable stress varies between 3 and 7.

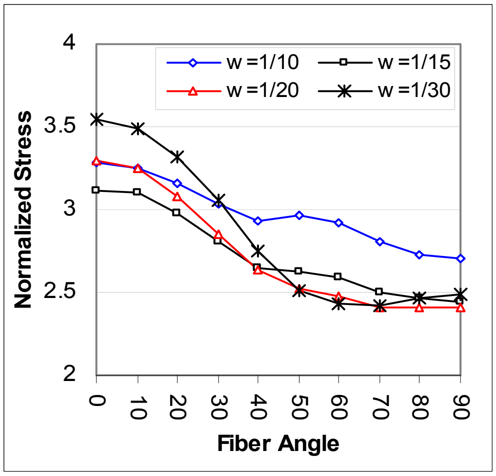

Figure 6: Effect of Fiber Angle on desirable stress.

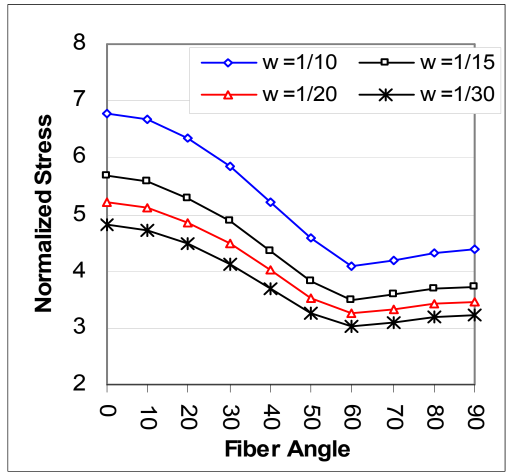

Figure 7: Effect of Fiber Angle on undesirable stress.

3.3- Effect of material properties

Figures 8 and 9 illustrate the effect of the material properties on maximum stress around square cutouts. As demonstrated by the graph, higher normalized stresses are achieved for materials with higher module ratios (E_1/E_2). This study is performed for *w*=1/20. In these figures, n is the ratio of E_1 to E_2 ($n=E_1/E_2$) for each material.

As shown in figures, there is a reduction in desirable and undesirable stresses up to specific fiber angle. This angle approaches to 90 degree for higher modulus ratios. For example, for n=1, this angle is 45 and for n=2.93 is 60 degree.

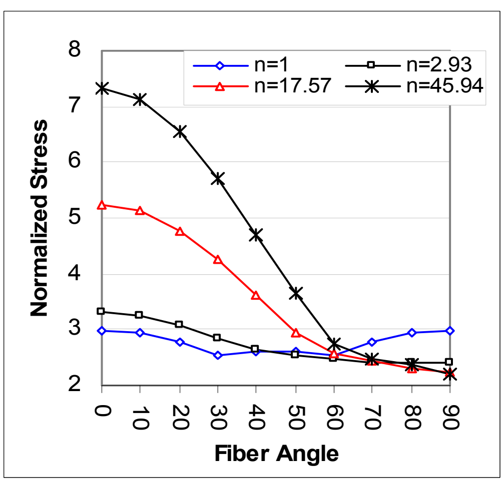

Figure 8: Effect of material properties on desirable stress for w=1/20

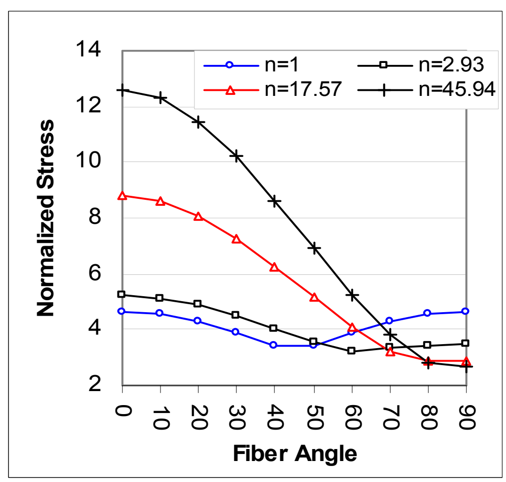

Figure 9: Effect of material properties undesirable stress for w=1/20

3.4- Effect of Load Direction

The effect of load direction on maximum normalized stress is considered in this section. In these cases the cutout rotation angle is zero. Figure 10 compares the normalized stresses of glass epoxy plates with square cutouts. Only four angles α = 0, 30, 60 and 90 are presented. It can be seen from the data, load direction has considerable effect on the value of the maximum stress in composite plates for higher cutout bluntness. Similar results are presented in figure 11. For plates with square cutout, change in the desirable stress is approximately happened at the same load angle (55 degree) for all bluntness parameter. It is shown that, loading angle has considerable influence on the maximum stress for higher bluntness parameters.

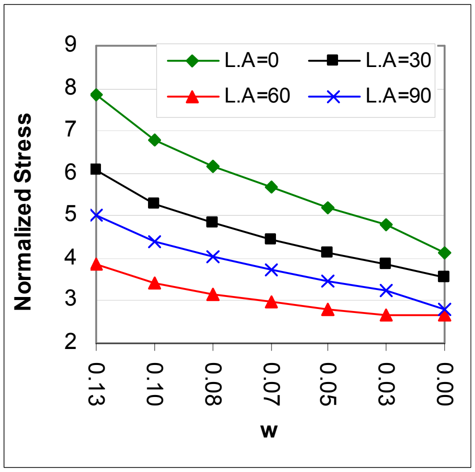

Figure 10: Effect of load angle and bluntness.

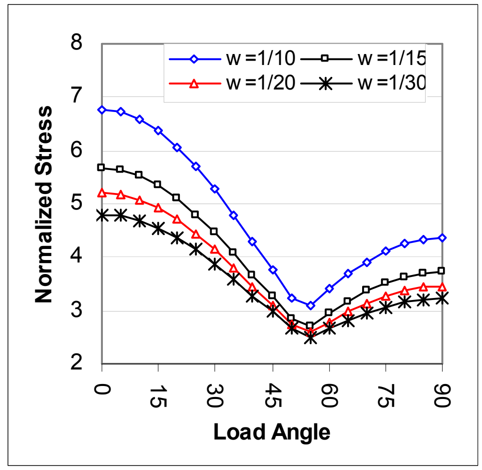

Figure 11: Effect of load angle on maximum normalized stress.

4. CONCULUSION AND RECOMMENDATIONS

The high stress concentration at the edge of a special shaped cutout is of practical importance in designing of the engineering structures. The stress concentration of composite perforated plates with centrally located square cutout was investigated. Analytical and numerical studies were conducted to investigate the effects of variation in cutout orientation and bluntness, material properties and loading direction on the location and the value of the maximum stress in flat plate subjected uni-axial tension load. This study expands the Lekhnitskii's solution for circular and elliptical cutout to square-shape cutouts. The solution is capable of considering large variety of cutout shapes and loading conditions analytically. As presented in previous sections, the maximum normalized stress of perforated composite plates can be significantly change by using proper material properties, loading angle, cutout bluntness and orientation.

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