NONLINEAR EDGE EFFECT ANALYSIS OF LAMINATED BEAMS WITH PIEZOELECTRIC LAYERS

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INTRODUCTION

The state of stress in the vicinity of edges of laminated structures is three-dimensional, with nonzero through-thickness stresses. To conserve equilibrium, large gradients of interlaminar stresses will occur in these regions. This effect is called edge effect. A detailed review of analytical and numerical approaches for investigating the edge effect is given by Kant and Swaminathan [1]. This effect in composite laminated structures with piezoelectric layers, as new smart materials should be considered. These materials have the potential of application in a wide field of engineering structures.

The coupled piezoelectric analysis of the free edge effect has been performed by Davi and Milazzo [2], who investigated a $[\pm 45^\circ]_s$ laminate under several loading conditions using boundary element formulation. Artel and Becker [3] analyzed the influence of piezoelectric coupling on interlaminar stresses and electric field near the free edge, using finite element method. Erturk and Tekinalp [4] introduced new types of elements, for finite element approach, and analyzed the interlaminar stresses of piezoelectric composite laminated beams. As it is seen, yet there are no analytical investigations including the edge effect in composite laminated structures with piezoelectric layers. In this paper an analytical solution is obtained to determine the interlaminar stresses in the vicinity of the ends of a general cross-ply laminated beam, which has piezoelectric layers. The von-Kármán nonlinearities are taken into consideration.

MATHEMATICAL FORMULATION

It is intended to determine the interlaminar stresses in a general cross-ply laminated beam with dimensions *l*, *b*, and *h* as its length, width, and height, respectively. Some layers have piezoelectric properties and electric voltage is applied to them. Also a transverse load $q(x)$ is applied to the upper surface of the beam. The displacement field using a third-order shear deformation beam theory may be represented as

$$\begin{aligned} u_1(x,y,z) &= u_0(x) + z\psi_x(x) + z^2\varphi_x(x) + z^3\eta_x(x), \\ u_2(x,y,z) &= 0, \\ u_3(x,y,z) &= w(x) + z\psi_z(x). \end{aligned} \tag{1}$$

Substituting Eqs. (1) into the von-Kármán nonlinear strain-displacement relations, mechanical strains for moderate displacements are obtained as

$$\begin{aligned} &\varepsilon_x = \varepsilon_1^0 + z\psi_x' + z^2\varphi_x' + z^3\eta_x', && \varepsilon_y = \gamma_{yz} = \gamma_{xy} = 0, \\ &\varepsilon_z = \psi_z, && \gamma_{xz} = \psi_x + w' + z(2\varphi_x + \psi_z') + 3z^2\eta_x. \end{aligned} \tag{2}$$

where only $\varepsilon_1^0 = u_0' + (w')^2/2$ is nonlinear. Using the principle of minimum total potential energy, equilibrium equations can be obtained in terms of stress resultants. Then the reduced piezo-elastic constitutive law for a cross-ply lamina is utilized, to derive the governing equations of equilibrium. With a parameter change, the nonlinear terms in these equations are handled as linear terms. The final equations are a set of ordinary differential equations with constant coefficients. The solutions to these equations are presented as:

$$\begin{aligned}
\psi_x(x) &= \sum_{i=1}^{8} a_{1i} e^{\lambda_i x} K_i + a_{19} + a_{1,10} x, & \varphi_x(x) &= \sum_{i=1}^{8} a_{2i} e^{\lambda_i x} K_i + a_{29} + a_{2,10} x, \\
\eta_x(x) &= \sum_{i=1}^{8} a_{3i} e^{\lambda_i x} K_i + a_{39} + a_{3,10} x, & w(x) &= \sum_{i=1}^{8} a_{4i} e^{\lambda_i x} K_i + a_{49} + a_{4,10} x, \\
\psi_z(x) &= \sum_{i=1}^{8} a_{5i} e^{\lambda_i x} K_i + a_{59} + a_{5,10} x, & \varepsilon_1^0(x) &= \sum_{i=1}^{8} a_{6i} e^{\lambda_i x} K_i + a_{69} + a_{6,10} x.
\end{aligned} \tag{3}$$

Finally, to improve the accuracy of the interlaminar stresses, local equilibrium equations are integrated.

NUMERICAL RESULTS

As some examples, the edge effect of [0°/90°]$_s$ and [90°/90°/0°/90°] laminated beams are investigated here. The employed material has mechanical properties of T300/Epoxy. The beam is simply supported at two ends, but the midplane con not move at both ends. The electric voltage is applied to the upper layer of the beam through its thickness. The piezoelectric properties of PZT-5A are chosen for this layer. Fig. 1 shows the distribution of normal interlaminar stress along the length at the midplane of the beams.

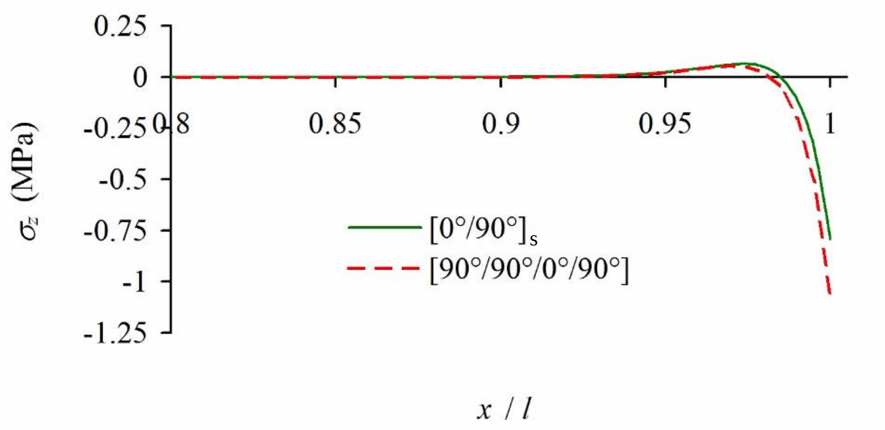

Fig. 1: Distribution of interlaminar normal stress along the length at the midplane of the beams

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