IDENTIFICATION OF ORTHOTROPIC MATERIAL BEHAVIOUR USING THE ERROR IN CONSTITUTIVE EQUATIONS

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ABSTRACT

The aim of this work is to allow the simultaneous identification of the material parameters governing an orthotropic elastic behaviour from one single specimen. The chosen mechanical test is a non-standard geometry. Two heterogeneous experimental configurations have been designed and numerically tested in the aim to compare their identifiability through the obtained results. The adopted identification technique is an inverse method based on the minimization of an energy norm. It is formulated on the basis of the error in constitutive relation. This error estimator is expressed according to the variational principle.

1. INTRODUCTION

The determination of parameters governing a constitutive law is a challenge that becomes difficult when the number of parameters is significantly high. Such a case occurs when anisotropic or/and non-linear constitutive laws are considered. The usual approach consists of performing several tests like tensile tests and fitting the model with the experimental data. However, the number of tests increases with the number of parameters. Moreover, parasitic effects can disturb the stress field which is usually expected to be uniform when homogenous tests are performed. These drawbacks can be avoided by performing heterogeneous tests on non standard specimens. In this case, only one specimen is tested and the stress field which occurs in the specimen is heterogeneous. Different techniques have been proposed to identify elastic properties using redundant or full-field kinematic data [1, 2]. In this work, an inverse approach based on variational principle is adopted for the identification of an orthotropic elastic behaviour from an experimental configurations. An expected result consists in determining which experimental configuration will increase the identification robustness with regard to unavoidable measurements noise.

This paper focus on the error in constitutive relation this error initially proposed in [3], in order to identify an orthotropic elastic behaviour, (constitutive equation gap). It is based on the minimization of an energy norm expressed through a variational principle [4, 5]. The error in constitutive relation constitutes an error estimator expressed as a function of the quadratic gap between computed strain and measured strain fields obtained by a whole-filed technique [4]. The optimization problem is solved using the Levenberg-Marquardt algorithm [6]. This first order minimization method requires the computation of a sensitivity matrix $\partial Xi/\partial Pj$, containing the derivatives of different data components (Xi) with respect to the material parameters to be identified (Pj). Several techniques

can be used to determine the sensitivity matrix. Among them, the analytical method presented in [7] or the adjoint method [8]. Some authors [9] propose a mixed technique based on the semi-analytical method using the finite difference and analytical calculation. It must be noticed that the finite difference requires computation of the strains for small perturbations of each identified parameters. In this work, the chosen method is the semi-analytical to evaluate the gradient and thus to compute the matrix sensitivity. The direct problem is solved by the FE code Abaqus [10].
The present work focuses on the numerical aspects inherent to the identification technique. Numerical simulations are carried out to assess the robustness and the stability of the identification procedure with respect to the choice of initial parameters and the identification robustness when processing noisy strain measurements. Since the measurement noise corrupting the used displacement field is unavoidable, the stability of the identification procedure has been assessed by introducing two types of data noises. The first is a random error simulated by a Gaussian white noise and the second is a bias error simulating a shift measurement error induced by an out-of plane displacement for instance.

2. ERROR IN CONSTITUTIVE RELATION FOR ELASTICITY: PRINCIPLES

Consider a structure (Ω), with an elastic material. The reference problem's equations can be described in two groups :

- Admissibility conditions : kinematic constraints, equilibrium and compatibility equations ;
- constitutive relations :

$$\sigma = C : \varepsilon(u) \tag{1}$$

Where :
σ denotes the stress tensor ;
ε denotes the strain tensor associated to the displacement u ; C denotes the fourth order elasticity Hooke tensor.
It is worth noting that constitutive relations are often the least reliable equations describing a mechanical problem. Accordingly, to solve the problem, one must find an approximated solution displacement/stress satisfying the most reliable equations : admissibility conditions. Namely, the pair of solution (σ,u) may satisfy the kinematic constraints, the equilibrium and the compatibility equations.
Let (u_{KA}, σ_{SA}) be a displacement-stress pair, where u_{KA} is a kinematically admissible displacement field (KA) and σ_{SA} is the statically admissible stress field (SA). Whether the constitutive relation (1) is satisfied, the pair (u_{KA}, σ_{SA}) is the exact solution of the problem. Hence, one can write that :
$\sigma_{SA} = C : \varepsilon(u_{KA})$, where $u_{KA} = u_{exact}$ and $\sigma_{SA} = \sigma_{exact}$
Generally, this pair does not satisfy the constitutive relation (1), consequently one obtains $\sigma_{SA} - C : \varepsilon(u_{KA}) \neq 0$, and then the pair (u_{KA}, σ_{SA}) is only an approximated solution of the considered problem. The quantity

$$\sigma_{SA} - C : \varepsilon(u_{KA}) \tag{2}$$

is referred to as the error in constitutive relation associated with the admissible pair (u_{KA}, σ_{SA}) at each point in the whole structure (Ω) [3]. This quantity is chosen to evaluate the quality of the pair (u_{KA}, σ_{SA}) and it allows quantifying the variation in the exact solution in terms of the constitutive relation. The error in constitutive relation has a main advantage : it carries the uncertainty on the constitutive relation considered as the least reliable by solving the problem (1). In elasticity, the energy norm over the whole structure is adopted to evaluate this error estimator ; the global absolute error in constitutive relation is defined by :

$$e_{CR} = \|\sigma_{SA} - C : \varepsilon(u_{KA})\|_{\Omega} \tag{3}$$

where :

$$\|x\|_{\Omega}^2 = \int_{\Omega} (x : C^{-1} : x) d\Omega \tag{4}$$

The contribution of the whole structure to the relative error ϵ can be considered as a global measure of the solution quality with the pair (u_{KA}, σ_{SA})

Consider in elastic problems the sum of potential and complementary energies with the displacement field u, and the stress filed σ [4](no volume or body forces) :

$$\psi(\varepsilon(u), \sigma, C) = \frac{1}{2}\int_{\Omega} (\varepsilon(u) : C : \varepsilon(u)) d\Omega + \frac{1}{2}\int_{\Omega} (\sigma : C^{-1} : \sigma) d\Omega - \int_{\partial\Omega} (u.\sigma.n) ds \tag{5}$$

As a well-known consequence of the variational principles in elasticity, for a given stiffness elasticity tensor C and well posed boundary conditions:

$$\min_{C, u_{KA}, \sigma_{SA}} \psi(\varepsilon(u), \sigma, C) = 0 \tag{6}$$

In absence of body forces and by taking into account the kinematic constraints, the function $\psi(\varepsilon(u), \sigma, C)$ may be rewritten for the static admissible stress field σ_{SA} and the kinematic admissible displacement field u_{KA} in order to introduce the gap in constitutive relation as follows [4] :

$$\psi(\varepsilon(u_{KA}), \sigma_{SA}, C) = \frac{1}{2}\int_{\Omega} (\sigma_{SA} - C : \varepsilon(u_{KA})) : C^{-1} : (\sigma_{SA} - C : \varepsilon(u_{KA})) d\Omega \tag{7}$$

REMARK : from the equations (3), (4) and (7), the function $\psi(\varepsilon(u_{KA}), \sigma_{SA}, C)$ can be reformulated by the error in constitutive relation as follows :

$$\psi(\varepsilon(u_{KA}), \sigma_{SA}, C) = \frac{1}{2} e_{CR}^2 \tag{8}$$

This aspect highlights the positivity of the function $\psi(\varepsilon(u_{KA}), \sigma_{SA}, C)$. For the stiffness tensor C identification, it leads then to define the function of the error in constitutive relation $\psi(\varepsilon(u_{KA}), \sigma_{SA}, C)$ on the basis of the problem's data. Using the conditions of admissibility, the function (7) is expressed in two equivalent forms as proposed by [4] and given in equations (9) and (10) :

$$\psi(\varepsilon(u_{KA}), \sigma_{SA}, C) = \frac{1}{2}\int_{\Omega} (\sigma_{SA} - \sigma_{KA}) : C^{-1} : (\sigma_{SA} - \sigma_{KA}) d\Omega \tag{9}$$

$$\psi(\varepsilon(u_{KA}), \sigma_{SA}, C) = \frac{1}{2}\int_{\Omega} (\varepsilon(u_{SA}) - \varepsilon(u_{KA})) : C : (\varepsilon(u_{SA}) - \varepsilon(u_{kA})) d\Omega \tag{10}$$

Where: $\left(\varepsilon(u_{SA}), \sigma_{KA} \right)$ defined by $\left(\varepsilon(u_{SA}) = C^{-1}\sigma_{SA}, \sigma_{KA} = C\varepsilon(u_{KA}) \right)$

3. IDENTIFICATION BY MINIMIZATION THE ERROR IN CONSTITUTIVE RELATION

The only 2D problem based on the plane stress assumption is developed here. The general 3D case can be easy deduced by extrapolation of the such formulation.
Using the conventional notation for contracted indices $[xx \to x, yy \to y, xy \to s]$, the in-plane constitutive relation for orthotropic materials, is given by :

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{pmatrix} = \begin{pmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{pmatrix} \tag{11}$$

The aim of this work is to identify the four stiffness parameters governing the orthotropic linear elastic behaviour Q_{xx}, Q_{yy}, Q_{xy} and Q_{ss} using full-field strain obtained from one single heterogeneous test. It should be noted that for such tests, no closed-form relations between loads and displacements are available. The material parameters are thus estimated through on an inverse approach based on an iterative minimization of a quadratic error. It is expressed as an energy norm between the experimental data and the calculated values.
The objective function is formulated by variational approach based on an energy norm of the error in constitutive relation (equation 10), adopted by [11], and the error between the boundary measurements and the simulated boundary conditions. This approach requires the measured and the computed discrete or continuum strain field of the studied zone, noted hereafter as the zone of interest. One must notice that the objective function expressing the error as an energy norm represents the gap for which the material parameters solution may minimize the incompatibility between the calculated strain fields (ε^{num}) and the measured ones (ε^{exp}).
By using Neumann and Dirichlet conditions, the objective function is defined as follows [4] :

$$\begin{cases} \psi(C) = \dfrac{1}{2}\displaystyle\int_{\Omega} (\varepsilon^{num}(C) - \varepsilon^{exp})^T C (\varepsilon^{num}(C) - \varepsilon^{exp}) d\Omega \\ \textit{under the constraint} \\ g(C) = B \displaystyle\int_{d\Omega f} \|(\sigma.n - f_m)\|^2 ds = 0 \end{cases} \tag{12}$$

Where :
f_m is the boundary measurements load
B is parameter which allow, depending on the relative confidence in those terms, to balance the two terms defining the function $\Phi(C)$, the constitutive relation and the error associated with the boundary measurements.
$\varepsilon^{exp} = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{pmatrix}^{exp}$ represents the actual strain fields obtained by an optical technique of full-field measurement.

$C = \begin{pmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{pmatrix}$ represents the in-plane stiffness matrix to be identified. It is deduced from the Hooke's tensor thats to the plane stress assumption.

$\varepsilon^{num} = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{pmatrix}^{num}$ represents the strain fields calculated by finite element computation with the given in-plane stiffness matrix C.
Ω denotes the volume where the error in constitutive relation is calculated, namely, the zone of interest.
The parameters governing the constitutive law can therefore be identified by the minimization of the function $\psi(C)$ under the constraint $g(C)$ (equation 12), [5]. The identification problem is defined as follows :
Find the in-plane stiffness matrix C that satisfies :

$$\begin{cases} C^{opt} = \arg min\Phi(C) \\ \Phi(C) = \dfrac{1}{2}\displaystyle\int_{\Omega} \|L(\varepsilon^{num}(C) - \varepsilon^{exp})\|^2 d\Omega + B\int_{d\Omega f} \|(\sigma.n - f_m)\|^2 ds \\ C = L^T L \end{cases} \tag{13}$$

The decomposition of the matrix C in a $L^T L$ product (Cholesky's decomposition) in the equation (13) is introduced in order to improve the numerical conditioning of the problem. And used as a ponderation terms.

4. INVERSE PROBLEM RESOLUTION

The resolution of the problem (13) is based on the Levenberg-Marquardt algorithm [6]. The latter requires the calculation of the direction and the amplitude of the descent using the gradient (G) and the hessian matrix (H) of the objective function. The evaluation of these quantities is carried out according to a semi analytical scheme [9]. To this end, one must determine the first derivative of the calculated strain with respect to the unknown parameters. This derivative is called the sensitivity matrix M and is estimated for each iteration :

$$M = \frac{d\varepsilon(C)}{dP} \tag{14}$$

Where (P) is a vector whose components are the unknown material parameters (Q_{ij}) and (ε) is the strain component calculated by finite element computation.
The choice of the approximation method is determined by the type of the calculation model, the degree of complexity of the constitutive equation, the number of parameters to be identified and the existence of several local minima [7]. In this work, the minimization algorithm of the inverse problem performed according Levenberg-Marquardt method is illustrated by the following overview :

$$\left| \begin{array}{l} P^0 : \text{given (initial parameters)} \\ \text{as long as } (\|\Delta P\| < \xi), (\|\Delta P\|^2 = \Delta P.\Delta P^T) \\ J = \dfrac{dL}{dP}(\varepsilon^{num}(C) - \varepsilon^{exp}) + LM \\ G = \dfrac{1}{2}\displaystyle\int_{\Omega} (J^T L(\varepsilon^{num}(C) - \varepsilon^{exp})) d\Omega + 2Bg(C)\dfrac{dg(C)}{dP} \\ H = \dfrac{1}{2}\displaystyle\int_{\Omega} (J^T J) d\Omega + 2B\dfrac{dg(C)}{dP}\dfrac{dg(C)}{dP} \\ \Delta P = -(H + \lambda I)^{-1} G \\ P^{k+1} = P^k + \Delta P \end{array} \right. \tag{15}$$

Where :
G, H, J, M are respectively the gradient, hessian matrix, jacobian matrix and the sensitivity matrix of the function $\Phi(C)$;
ξ is the threshold scalar fixed at (10^{-5}) ;
λ is the regularization parameter.
A convergence criteria is set-up in order to stop the search procedure. In our case, the test consists to fixe a threshold value ξ and assessing if the condition of stationarity $\|\Delta P\| < \xi$ is satisfied.

5. SENSITIVITY ANALYSIS

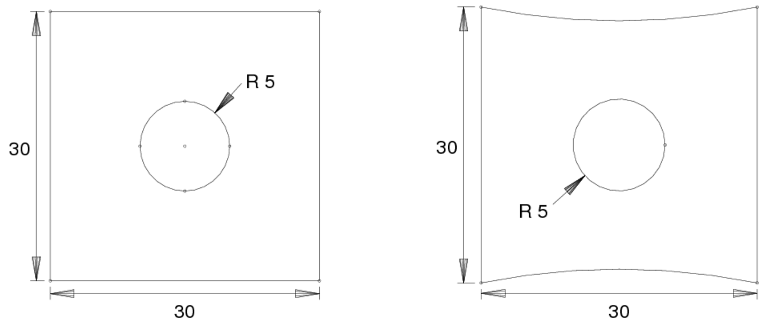
The most important aspect of a gradient method, such Levenberg-Marquardt, is the determination of the sensitivity matrix, which expresses the sensitivities of the strains with respect to the material parameters. The sensitivity matrix (M) can be determined using several techniques [7–9], (as discussed in the introduction). Among them, the finite difference method that requires the computation of strains for small variations of each material parameter, as given in equation (16) is adopted. Nevertheless, this technique is time consuming since the number of necessary FE simulations will increases linearly with the number of the unknown material parameters. The sensitivity matrix M is defined as follows :

$$M_{ij} = \frac{\varepsilon_i(C + \delta P_j e_j) - \varepsilon_i(C)}{\delta P_j} \tag{16}$$

Where δP_j is the small variation (perturbation) of the parameterP_j and j is the unitary vector .
The matrix (M) formulated in equation (16) regroups the sensitivity coefficients of the strain components with respect to the unknown material parameters (P) for each Gauss point of the FE mesh. The components of the matrix (M) are calculated for each iteration (k) of the identification algorithm using the finite difference method. Numerically, the perturbation value is a posteriori chosen so as to get a high accuracy. This parameter is often chosen within a range of values : $10^{-5}P_j \leq \delta P_j \leq 10^{-3}P_j$ [12]. In the framework of this study, the value of this small variation has been chosen as $0,1\%$ of the considered parameter $(P^{(k)})$ at the iteration (k).

6. RESULTS AND DISCUSSION

In this first part of the work, the identification procedure is performed upon simulated data. Indeed, the main purpose is to assess the robustness and the accuracy of the identification based on the error in constitutive relations. Note that the experimental part of this work is currently performed and will be published in the near future. The finite elements simulation has been carried out using the Abaqus FE code [10]. The goal here is to retrieve the mechanical properties introduced in the finite element model considered as the reference values. Mechanical tests chosen here are: uniaxial tensile test on two specimens having different geometries (Fig.1). For such tests, the whole set of the unknown parameters is expected to be involved in the overall response. They can therefore be identified if a suitable strategy is adopted, since no closed-form solution is available. Numerical computations are carried out on an orthotropic linear elastic behaviour. The objective of the following sections is to study the capabilities of the identification procedure in terms of mesh and guess values sensitivities as well as its stability when processing data with noise corrupting strain measurement.



(a) Holed plate (b) Wasted holed plate

Figure 1: Geometry of non-standard specimens and dimensions in mm

6.1 Mesh sensitivity

The study of the spatial convergence constitutes an important numerical aspect for the finite element analysis. Indeed, an optimisation of the finite element size may lead to reach a reliable description of stress/strain gradient in the zone of interest. To this end, the spatial convergence study has been achieved for the four considered configurations.
For each studied configuration, the corresponding FE model has been meshed with four different element sizes. Computations were performed using the in-plane stress assumption and four-node quadrilateral and isoparametric continuum elements (CPS4). For a sake of clarity, only the results for holed plate (Fig. 1.a) are given in the present paper. The identified parameters are reported in table 1. As it can be noticed, the identified stiffness values are exactly the same as the reference ones, the identification procedure becomes insensitive to the mesh refinement. However, a compromise has to be adopted between a coarse mesh density with poor stress/strain gradient and a refined mesh density with high consuming time. A size of 0.8 mm of CPS4 element is chosen for the four studied configurations.

	Q_{xx}(GPa)	Q_{yy}(GPa)	Q_{xy}(GPa)	Q_{ss}(GPa)
Reference values	25.93	10.37	3.11	4.00
Initial values	50	25	1.5	5.3
Type of mesh case : 1		Size ≈ 5.6 mm		
Identified values	25.93	10.37	3.11	4.00
Type of mesh case : 2		Size ≈ 2.2 mm		
Identified values	25.93	10.37	3.11	4.00
Type of mesh case : 3		Size ≈ 0.9 mm		
Identified values	25.93	10.37	3.11	4.00
Type of mesh case : 4		Size ≈ 0.15 mm		
Identified values	25.93	10.37	3.11	4.00

Table 1: Mesh influence on the identified values

6.2 Sensitivity to the initial set of parameters

The described above identification procedure (Section 3.) being iterative, requires initial guess values to start the identification algorithm. To assess the sensitivity of the identification results to initial guess values, three sets of initial parameters value have been used. Only the numerical results obtained from these three initial parameters sets and

relative to the holed plate configuration(Fig. 1.a) are presented here. These are reported in table 2.
For the three cases, the identified values are identical to the reference values. The obtained results show that the identification algorithm converges to the same optimal solution that minimizes the function of the error in constitutive relation. This can be explained by the convexity of both members expressing the objective function (12) formulated through a variational principle.

	Q_{xx} (GPa)	Q_{yy} (GPa)	Q_{xy} (GPa)	Q_{ss} (GPa)
Reference values	25.93	10.37	3.11	4.00
Initial values : case 1	50	25	1.5	5.3
Identified values	25.9	10.3	3.11	4.00
Initial values : case 2	12	7	4.5	2.5
Identified values	25.9	10.3	3.11	4.00
Initial values : case 3	42	5	5.8	1.5
Identified values	25.9	10.3	3.11	4.00

Table 2: Identified parameters : Sensitivity to the initial values

6.3 Stability analysis

In practice, the identification procedure uses actual strain fields measured by an optical full-field technique. These experimental measurements are always corrupted by several sources of noises (electronic noises, image acquisition, light intensity, out of plane displacements, etc). To study the stability and the robustness of the identification procedure when processing noisy data, two types of errors have been considered to simulate experimental errors :

1. Gaussian white noise : this noise is generated from random numbers whose elements are normally distributed with mean value equal to 0 and the standard deviation equals to 5×10^{-4}. It is added to each component of the actual strain maps. Repeating the process 30 times using a renewed noise, a distribution of identified parameters is obtained.
2. Bias noise : this type of noise can simulate systematic but parasitic strains resulting from an apparent out-of-plane displacement of the zone of interest. It is introduced by adding a constant value of 5×10^{-4} to each component of the actual strain fields.

The identification procedure taking into account both types of noises is carried out on the two configurations (Fig. 1) and for the three sets of initial parameters considered in the previous section.

1. **Gaussian white random**
   Considering this type of noise, the mean values of identified parameters are reported in Table 3, this result shows hence that the identification algorithm is insensitive to the initial values, even in the presence of a random noise.

2. **Bias noise**
   For this type of noise, The identified values are reported in Table 4. The relative error value calculated for the whole identified parameters is more important than

that resulting from data containing a random noise. Indeed, the relative error for the Q_{xy} parameter is the most important, indicating hence that this parameter has small effect on the overall mechanical response of the two studied configurations. Conversely, the shear stiffness parameter is the most stable, confirming the importance of the shear stress/strain gradients in these test configurations. It seems thus that the studied configurations can be widely adopted to determine reliably the shear parameter.

Amplitude of the noise $= 5 \times 10^{-4}$	Q_{xx}(GPa)	Q_{yy}(GPa)	Q_{xy}(GPa)	Q_{ss}(GPa)
Reference values	25.93	10.37	3.11	4.00
Holed plate				
Initial values : case 1	50	25	1,5	5.3
Identified values	25.91	10.22	3.07	3.99
Initial values : case 2	12	7	4.5	2.5
Identified values	25.9	10.22	3.07	4.00
Initial values : case 3	42	5	5.8	1.5
Identified values	25.9	10.23	3.07	4.00
Wasted holed plate				
Initial values : case 1	50	25	1.5	5.3
Identified values	25.89	10.20	3.06	3.99
Initial values : case 2	12	7	4.5	2.5
Identified values	25.89	10.21	3.07	4.00
Initial values :case 3	42	5	5.8	1.5
Identified values	25.89	10.22	3.07	3.99

Table 3: Mean identified parameters from strain fields containing a random noise whose amplitude is 5×10^{-4})

Amplitude of the noise $= 5 \times 10^{-4}$	Q_{xx}(GPa)	Q_{yy}(GPa)	Q_{xy}(GPa)	Q_{ss}(GPa)
Reference values	25.93	10.37	3.11	4.00
Holed plate				
Identified values	24.55	11.12	2.74	4.01
Wasted holed plate				
Identified values	24.61	11.08	2.73	4.01

Table 4: Identified parameters from strain fields containing a bias noise whose amplitude is 5×10^{-4})

7. CONCLUSIONS

An identification procedure based on an inverse analysis has been proposed to determine the material parameters governing an orthotropic elastic constitutive law using full-field strain. The identification technique developed in this paper is based on a minimization of an energy norm formulated on the basis of error in constitutive relations expressed through a variational principle. The stability and the robustness of the algorithm have been analyzed. It has been demonstrated that, due to the variational formulation of the error in constitutive relation, the identification algorithm converges to an optimal solution in absence of data noise. This convergence is consistent and can be explained by the convexity of the formulated error in constitutive relation. Indeed, the error in constitutive relation is expressed as a sum of the total potential and the complementary energies.

In the case of presence of measurement noise corrupting displacement fields, the robustness of the developed identification has been assessed for two types of noise : Gaussian white noise (random) and a bias error. For the random error (Gaussian white noise), the identified mean values are close to the reference parameters and present low discrepancies. For the bias error, the identified results show the importance of this type of noise induced by out-of-plane displacements for instance. Indeed, these displacements introduce parasitic kinematic fields that can be interpreted by the optical system of measurement (CCD camera) as structural displacements and will derive strain components. Particular attention will be taken for experimental validation of the identification method (in progress) to improve the quality of measurement.
The expected results aiming at determining which experimental configuration will increase the identification robustness with regard to unavoidable measurements noise is thus achieved. The work in progress will be devoted to validate experimentally the obtained results and to assess the best parameters identifiability.

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