UNCERTAINTIES PROPAGATIONS OF COMPOSITE MECHANICAL PROPERTIES FROM MICRO TO MACROSCALES

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ABSTRACT

This work deals with the modeling of the uncertainty through different scales of composite materials. Three different scales are investigated: micro (fibres and matrix), meso (ply) and macro (laminate). Homogenization methods are used on the micro and meso scales. The macro scale is modeled using laminates theory. The main result shows that the uncertainty, large at the micro-scale, becomes small in meso-scale because of mean effect. On the other way, dispersion of high stress zones on meso scale is demonstrated.
In this study, virtual materials have been generated in order to obtain mechanical distribution.

1. INTRODUCTION

Composite materials present mechanical variations due to their natural morphology. These variations are very dependent on the scale. Thus, in laminated composite, three scales are generally considered: i) micro: fibres and matrix, ii) meso: a ply, iii) macro: several plies (laminate).
The microscopic nature of the elementary components (fibres and matrix) can imply a variability of their properties because it is very difficult to control the manufacturing processes. For example, composites manufactured from the biomass, have a variability of some pourcents at the structural scale, whereas the fibres (hemp, flax,…) have a very strong dispersion of their properties [1].
The objective of this work is to show that the strong dispersion of the properties of the elementary components is strongly reduced by the scalings. In this study, a multi-scale modelling (micro, meso and macro) is developed in order to better understand and describe the implied mechanisms. This method will make it possible to optimize the behavior of materials.

2. METHODOLOGY

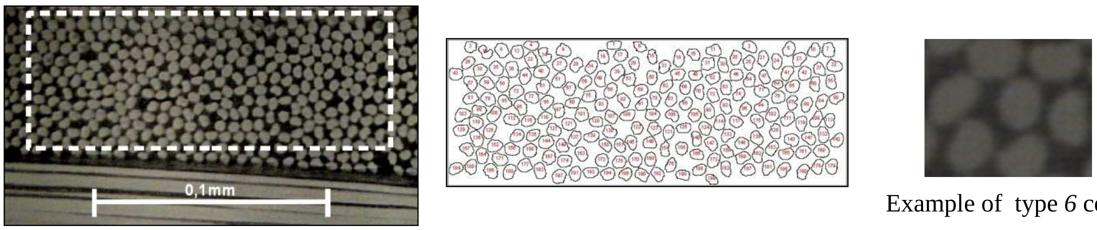
The method consists in integrating on a higher scale the results of the lower scale in order to obtain a structural material which depends on the microscopic behavior. The latter is built from morphological considerations (distribution of fibres in matrix). Cells of type *4*, *5*, *6* and *7* are then considered, with type *i* means *i* fibres surrounding a central one (see section 3). Their mechanical behavior is calculated numerically (FEM) by a homogenization method.
On the meso-scale, various kinds of cells highlighted from morphological analysis (*4/5/6/7* fibres cells) are randomly distributed with respect to their number. For each kind of cells, homogeneized mechanical properties calculated on micro-scale are affected. Finally, the mechanical properties of the ply (meso-scale) are then estimated by homogenization method using FEM.

At the macro scale, the various plies are stacked in order to obtain a laminated composite. The objective of this scale is to show that an angular variation of plies can influence the mechanical properties of the laminate. This variation will be strongly dependent on the number of plies and the degree of misalignment.

3. MICRO-SCALE

3.1 Morphological analysis

A morphological study has been done in order to describe, as well as possible, the distribution of fibres in cells. To this end, a micrography of a composite material with carbon fibres T800 was analyzed, figure 1-a. A thresholding filter allowing to transform fibres into particles was applied, figure 1-b. Figure 1-c gives an example of cell composed of 6 fibres around a central one.



Example of type *6* cell.

(a) (b) (c)

Figure 1: (a) micrography, b) transformation of fibres into particles and c) cell.

Three parameters are analyzed in order to estimate realistic cell, Figure 1-c: i) the number of close fibres, ii) their distance to the central fibre, iii) the angle between two consecutive fibres surrounding central fibre. The following figure presents the distribution of cells for a given number of close fibres, figure 2.

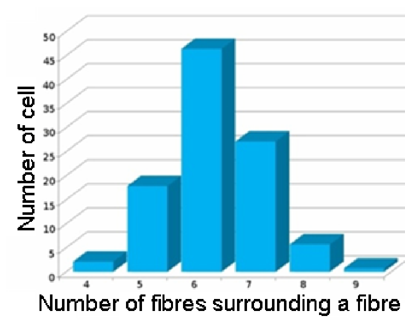


Figure 2: Distribution of the number of fibres surrounding a fibre.

3.2 Geometry and meshing of the cells

Following the morphological analysis of the UD composite, four cells were retained. These cells are respectively composed of 4, 5, 6 and 7 carbon fibres and an epoxy resin matrix. Each cell is modelled with ANSYS package by taking as random variables the number and the spatial distribution of fibres (see previous section), the diameter of fibres, the material properties. Table 1 presents the various variables used in models.

Variables	Law of probability	Mean	Standard deviation	Coefficient of variation (%)
E_f	uniform	73 (GPa)	8.43 (GPa)	11.5
E_m	uniform	3.45 (GPa)	0.40 (GPa)	11.5

v_f	uniform	0.22	0.025	11.5
v_m	uniform	0.3	0.035	11.5
Ø of fibres	uniform	10 (μm)	0.58 (μm)	5.8

Table 1: Definitions of the random variables.

(E_f, E_m) and (v_f, v_m) represent the young modulus and the poisson ratio for fibre and matrix, respectively.
The cells are parallelepipeds of identical length l_y and width l_x (32 μm). The thickness l_z is set to 3 μm.
The fibres are spatially distributed using the Random Sequential Adsorption algorithm (RSA) [2]. This algorithm is used for generating microstructure in the X-Y plane, figure 3. The first step in this algorithm is to generate, so unifom, coordinates (x_1, y_1) from center of the straight section of the first fibre. In order to avoid that a fibre cuts the cell edges, the four following relations must be verified simultaneously:

$$0 \le x_1 - r_1 - \delta \qquad x_1 + r_1 + \delta \le l_x$$
$$0 \le y_1 - r_1 - \delta \qquad y_1 + r_1 + \delta \le l_y$$

where r_1 and δ are the radius of the first fibre and tolerance necessary to obtain a regular mesh, respectively.
If these criteria are not satisfied, a new pair (x_1, y_1) is generated. In the favourable case, a new pair (x_2, y_2) is generated by verifying that the new fibre does not overlap the previous fibres and it does not cut the cell edges. If there is no overlap, a new coordinate pair is generated. This process is repeated until the number of fibres in the cell and the number of cells are met. Figure 3 shows an example of cell.

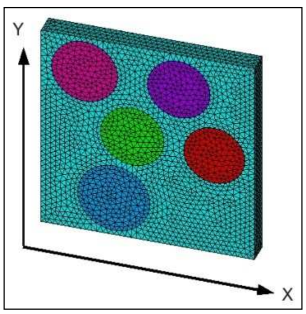
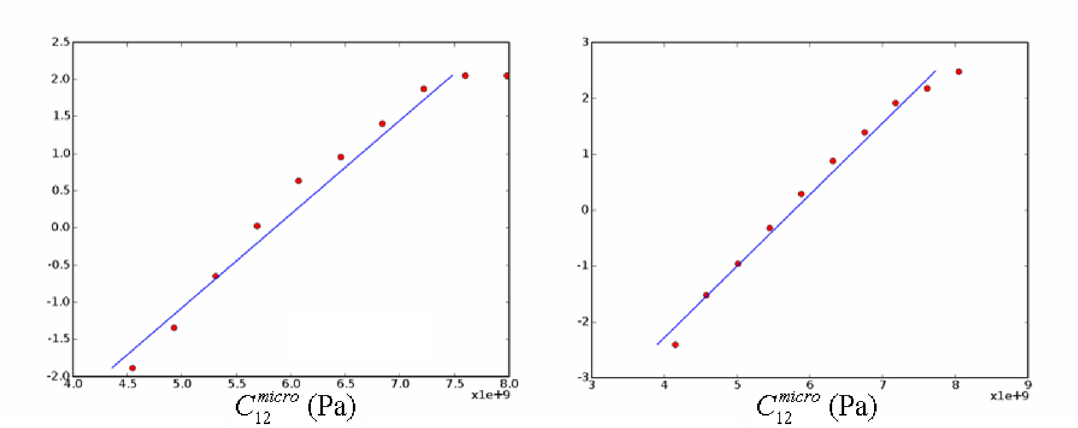


Figure 3: Meshing example of a cell.

The various cells are meshed with ten nodes tetrahedral elements (solid92). After a convergence study, the size of the elements has been fixed to 0.75 μm. Now, the objective is to estimate the equivalent stiffnesses of every cell. To this end, a homogenization method based on the Hill lemma [3,4] was implemented in ANSYS. This method requires to perform 21 cases of homogenous load so that all the terms of the stiffness matrix could be identified. The results are presented in the following section.

3.4 Results

Each type of cell was generated several times to obtain the distribution of C_{ij}^{micro}. Figure 4 shows the lines of Henry obtained during the estimate of C_{12}^{micro} for a different number of simulation.

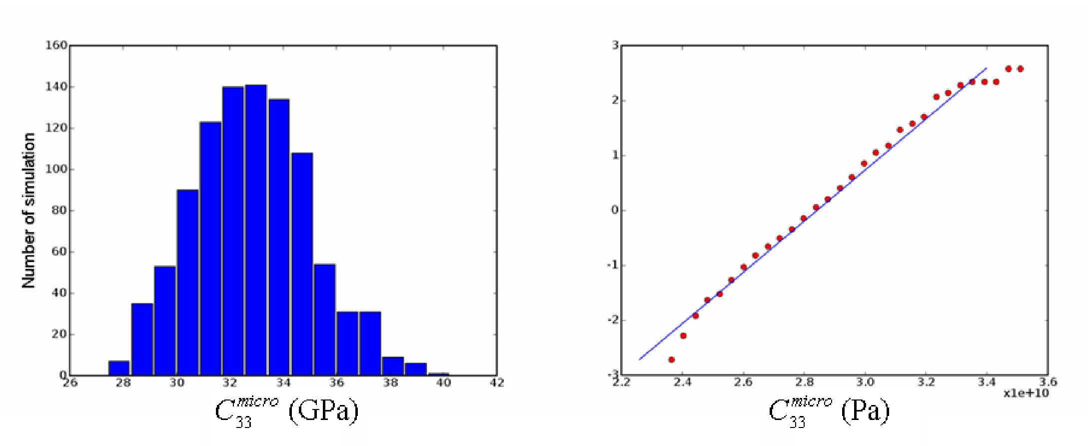


(a) (b)

Figure 2 : Distribution of C_{12}^{micro} for a two numbers of simulations (N): N=100 (a), N=963 (b).

It is noted that the distribution of C_{12}^{micro} tends towards Gaussian when the number of simulations increases. For reasons of computing time, the number of simulations is fixed to 963 for all calculations of C_{ij}^{micro}.

Figure 5 presents the distribution of C_{33}^{micro} for cells with 5 fibres as well as the Henry's line.



(a) (b)

Figure 3. Distribution of C_{33}^{micro} (a) and Henry's line (b).

It is noted that this distribution is almost Gaussian. This remark is also valid for the other terms of the stiffness matrix. Moreover, 12 components of the stiffness matrix have distributions of null mean and standard deviation lower than 0.01 GPa, confirming the transverse isotropy of the cell. Table 2 presents means and standard deviations for several stiffnesses obtained from the cell with 5 fibres.

Stiffness	Mean	σ	Stiffness	Mean	σ
C_{11}^{micro}	24.17	1.78	C_{33}^{micro}	32.87	2.19
C_{12}^{micro}	6.25	0.79	C_{44}^{micro}	10.85	7.18

C_{13}^{micro}	7.14	0.81	C_{55}^{micro}	10.85	7.18
C_{22}^{micro}	24.17	1.78	C_{66}^{micro}	8.96	6.19
C_{23}^{micro}	7.14	0.81			

Table 2: Mean and standard deviation of C_{ij}^{micro} (GPa) for 5 fibres cells.

It is noted that the relations of transverse isotropy are well checked.

$$C_{13} = C_{23} \quad C_{11} = C_{22} \quad C_{44} = C_{55} \quad C_{66} = \frac{1}{2(C_{22} - C_{12})}$$

The continuation of the method consists in using previous stiffness distribution to model the mechanical behavior of a ply (meso-scale). The following section describes the methodology.

4. MESO-SCALE

4.1 Meso-scale requirements

Looking at what became micro-scale mechanical properties dispersion on meso-scale is the dimensions 6 mm x 6 mm x 0.3 mm, Figure 6. It is made of volumes representing the homogeneized cells described on the microscopic scale. These cells are organized in way to satisfy as well as possible the physical requirements :

- the cells are randomly distributed following the distribution given in Figure 2, and issued from a morphological analysis,
- the ‘match’ volume generated by extending cells along y dimension, Figure 6, contains cells of comparable nature including for each of them a constant number of fibres,
- the continuity of fibres will be satisfied for two adjacent cells.

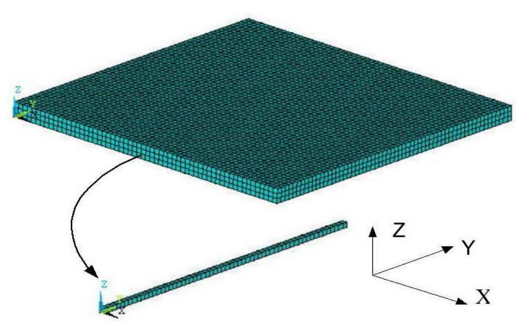


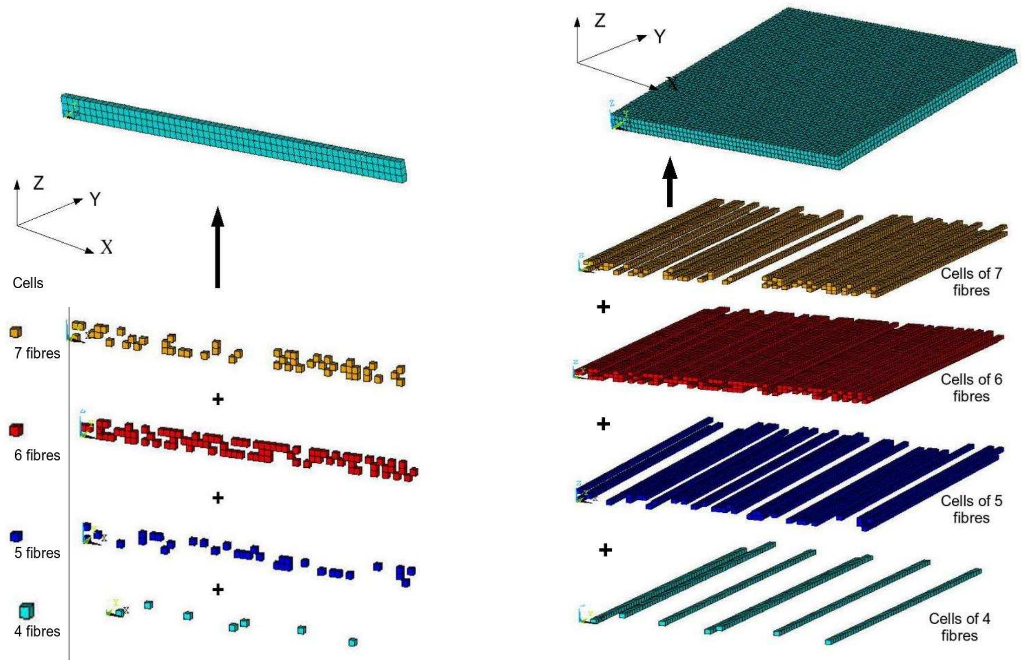
Figure 6: Meso-scale modelisation : ply and ‘match’ volume.

4.2 Generation of the cells

In present work, only the first two requirements are satisfied. The cells are generated using Ansys software in the following way.

1) only a section of cells, Figure 7, is randomly built respecting the distribution of Figure 2.
2) cells are extended along y direction to give volumes as described in Figure 6. Each of them contains cells of comparable nature which are selected and gathered.
3) the mechanical properties are finally affected by nature of cells following the distributions obtained at the microscopic scale.

The ply (or layer) is meshed with *3* FE in the thickness and *60* FE along the edges**.** The various cells are then meshed using the finite elements hexahedric with 20 nodes SOLID 182 available in Ansys. With these elements, the mechanical properties can be directly introduced as C_{ij}^{micro} elastic moduli.



a) Random distribution of cells in the first section b) Random distribution of cells in the ply

Figure 7: Distribution of cells.

4.3 Elastic modulus at meso-scale

The next step consists in estimating C_{ij}^{meso}, equivalent elastic modulus for the ply. They are obtained numerically using the homogenization method based on the Hill's lemma [3,4], see previous section. The procedure, illustrated on Figure 8, is repeated several times to involve different arrangement of cells while respecting physical requirements. C_{ij}^{meso} distributions can then be obtained.

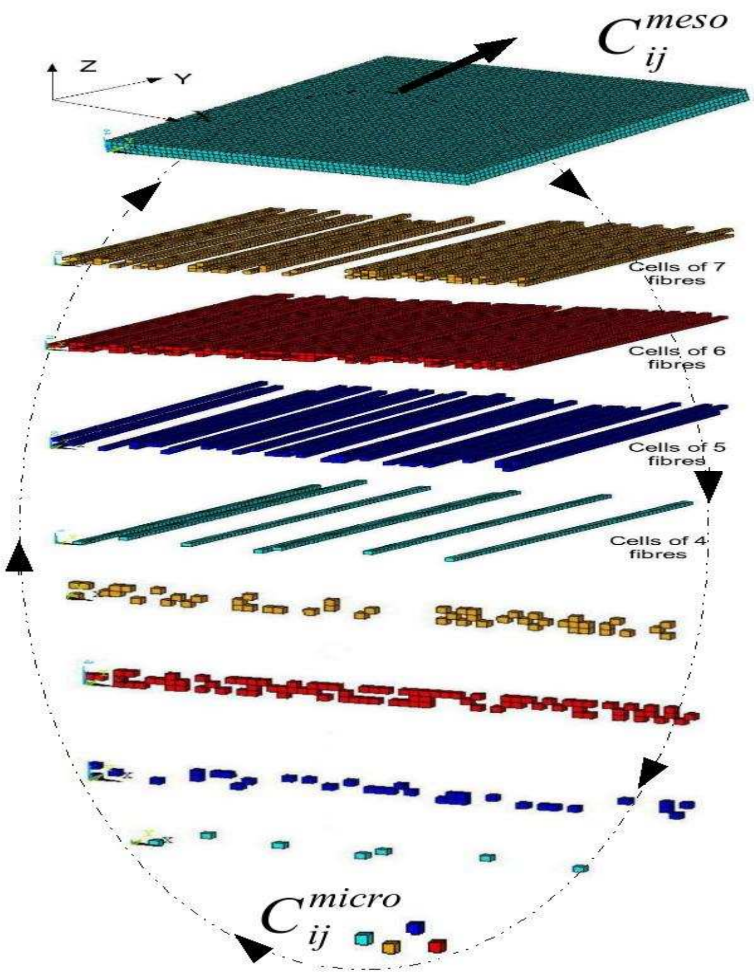


Figure 8: Scaling procedure from microscale to meso-scale.

4.4 Results

About two hundred calculations of C_{ij}^{meso} have been performed using FE method, each of them consuming near one hour of CPU time. Table 3 presents C_{ij}^{meso} mean value, standard deviation and coefficient of variation obtained from this sample. Obviously, a significant reduction of variability at the time of scaling from micro scale to meso-scale is shown by these first results.

C_{ij}	Mean	Standard deviation	Coefficient of variation (%)
C_{11}^{micro}	24170	1780	7.3
C_{11}^{meso}	33000	63	1.91
C_{22}^{micro}	24170	1780	7.3
C_{22}^{meso}	33100	64	1.95
C_{33}^{micro}	32870	2190	6.7

C_{33}^{meso}	40400	71	1.76

Table 3: Comparison of C_{ij} modulus at meso and micro scales (MPa).

In addition, a tensile test has been performed on the ply for different arrangements of cells. Stress field analysis shows that high stresses zones randomly located appear, Figure 9. This is a first explanation why the elastic properties of composite materials are often reproducible whereas those with rupture are very dispersed.

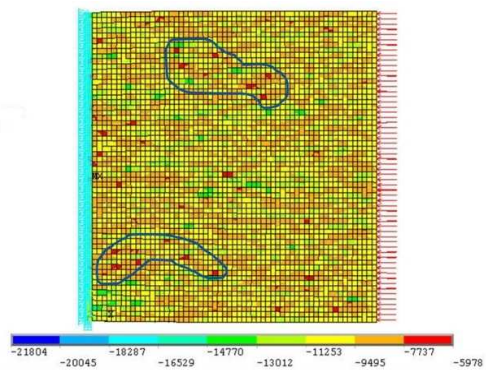


Figure 9: High stresses zones for a tensile test.

5. Macro scale

The multilayered structure made up of stacked layers is now considered. On this macro scale, the angle of orientation for a ply is one of the important sources of variability in laminated material. The objective of this part is to show that the dispersion of the mechanical behavior of the laminate is strongly related to the amplitude of variation of the angle but also to the number of plies considered. The present laminate is composed of unidirectional plies in order to obtain ideally a material for which all the fibres are directed in the same direction. The estimate of the longitudinal Young modulus E_L is carried out using the laminate theory.

For present simulations, from 1 to 10 plies have been considered with an angular variation for each one of 5°, 10° and 20°. The evolution of E_L's coefficient of variation according to the number of plies is plotted on Figure 10 for different angular variations.

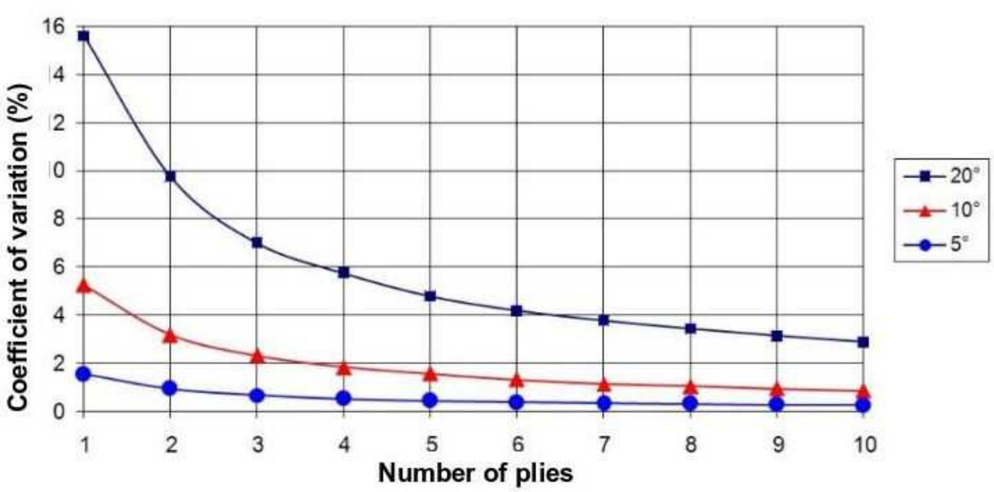


Figure 10. Evolution of E_L's coefficient of variation.

For a given angular variation, reproducibility of results when the number of plies increases can be underlined. Indeed, the probability that the whole of the plies is on average to be shifted of 5°, 10° or 20° is all the more weak as the number of plies increases.
The next stage of modeling will integrate, moreover, the variability of C_{ij}^{meso} calculated on the meso-scale.

6. Conclusions

A multi-scaled approach to see the evolution of the mechanical properties's dispersion from micro-scale to macro-scale has been developped in this study.

It reveals that significant dispersion observed on micro-scale (cell made up of fibre/matrix) for composite materials is largely reduced on meso-scale (ply made up of previous cells) due to the great number of cells distributed in the volume of a ply. On the other hand, high dispersion for zones with stress concentration has been demonstrated.
In addition, it has been proved that the effect of bad orientation of plies for a laminate (macro-scale) was compensated by the number of plies used : more there are plies, minus is the effect.

Finally, this study confirms that the elastic properties of composite materials are often reproducible whereas those with rupture are very dispersed. This last point, of great interest, will be largely investigated in future works.

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