STRENGTH ANALYSIS OF COMPOSITE STRUCTURES WITH UNCERTAINTY ASSESSMENT

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ABSTRACT

This paper presents a progressive multiscale failure approach for composite materials which predicts the failure of a laminate from elementary unidirectional ply properties (behaviour and strength). This approach was compared successfully with tests results performed on laminates subjected to tensile loadings [1]. Nevertheless, for compressive loadings, it has been demonstrated that it is necessary to take into account the effect of the global buckling on the predicted final failure. An original method, specific for the determination of the buckling of composite structures, has been proposed [2] and has permitted to improve the predictions of the final failure of laminates subjected to compressive loadings.

Moreover, the effects of dispersion and uncertainty, concerning material and geometrical parameters, on prediction of laminate failure have been integrated and the predictions have been compared with experimental data. Indeed, uncertainty on material properties has been taken into account because experimental values are known with a limited confidence due to experimental dispersion, or because they are obtained by inverse identification. Moreover, the effects of uncertainty on geometrical parameters on the prediction of buckling of the laminated structures have also been studied. This approach permits to determine the interval of confidence in failure prediction (even for compressive loadings), the sensitivity of the different parameters and is also able to perform a transport of probability of rupture with a reasonable time of computation. Therefore, the overall information could help to design composite structures.

1. INTRODUCTION

Due to their high specific properties, the use of fibre-reinforced composites has spread increasingly, during the past few years, for the design of high performance structures in a large range of industrial applications. While prediction of laminate composite failure is a key point for the design of engineering structures, there is still a lack of confidence into existing failure criteria, especially for compressive loadings where the final failure could also be due to global buckling of the specimen. Moreover, the dispersion on the experimental values is important for composite materials and leads to the use of conservative allowable [3] for design of aeronautical structures. Therefore, it induces a loss of competitiveness of composite solutions as compared with metallic materials, widely studied for many years.

The aim of the present study is thus to propose an alternative method which permits to estimate confidence on strength of composite structures from uncertainty on input data (such as mesoscopic material properties, geometry of the specimen, boundary conditions...).

First, the multiscale progressive failure approach, which permits to predict laminate failure from the knowledge of mesoscopic properties, is briefly presented and predictions on laminates are compared with experimental data. For laminates subjected to compressive loading, it is necessary to take into account the buckling of the composite structures. The method to predict buckling is described, and the previous predictions obtained only with the material modelling are updated. Finally, an alternative method, which permits to perform a strength analysis of composite structures with uncertainty data is detailed and is compared with experimental data from the literature.

2. MATERIAL MULTISCALE PROGRESSIVE FAILURE APPROACH

A mesoscopic progressive failure approach [1] has been proposed in order to be predictive for different stacking sequences and takes into account the effects of plies failure on macroscopic behaviour. The principle of the presented multiscale progressive failure approach can be shared into four sub problems: (i) choice of the mesoscopic behaviour, (ii) definition of a mesoscopic failure criterion, (iii) proposition of the degradation model of the failed ply in a laminate, and (iv) finally the definition of the final laminate failure. In the following, these four considerations are discussed in details.

The mesoscopic behaviour, expressed in the material axes, is described by a thermo-damage-viscoelastic behaviour:

$$\sigma = \tilde{C} : (\varepsilon - \varepsilon^{th} - \varepsilon^{ve}) \quad \text{with} \quad \varepsilon^{th} = \alpha \Delta \mathrm{T} \tag{1}$$

where σ is the stress, $\tilde{C}$ the elastic stiffness tensor, ε the total strain, ε^{th} the thermal strain and ε^{ve} the viscoelastic strain (modelled using a viscoelastic spectral behaviour [4]). This viscoelastic model is able to describe creep and relaxation tests and to take also into account the influence of the loading rate on the macroscopic behaviour and final failure, contrary to the non linear elastic behaviour models widely used.
A failure criterion has been proposed which is based on Hashin's assumptions [5]. Two failure modes are considered: a fibre failure mode in Eq. (2) and an interfibre failure mode Eq. (3). For each failure mode, the failures in tension and in compression have been distinguished, because the failure mechanisms are very different.

$$\mathrm{FF:} \begin{cases} f_1^+ = \left(\dfrac{\sigma_{11}}{\tilde{X}_t(d_2)}\right)^2 = 1 \quad \text{with} \quad \tilde{X}_t = X_t\, e^{-h_{1r} d_2} + X_t^{yarn}(1 - e^{-h_{1r} d_2}) \quad \text{if } \sigma_{11} \geq 0 \\ f_1^- = \left(\dfrac{\sigma_{11}}{X_c}\right)^2 = 1 \quad \text{if } \sigma_{11} < 0 \end{cases} \tag{2}$$

$$\mathrm{IFF:} \begin{cases} f_2^+ = \left(\dfrac{\sigma_{22}}{\tilde{Y}_t}\right)^2 + \left(\dfrac{\tau_{12}}{\tilde{S}_c\,(1 - p\,\sigma_{22})}\right)^2 = 1 \quad \text{if } \sigma_{22} \geq 0 \\ f_2^- = \left(\dfrac{\sigma_{22}}{\tilde{Y}_c}\right)^2 + \left(\dfrac{\tau_{12}}{\tilde{S}_c\,(1 - p\,\sigma_{22})}\right)^2 = 1 \quad \text{if } \sigma_{22} < 0 \end{cases} \quad \text{with} \quad \begin{cases} \tilde{Y}_t = Y_t\,(1 - d_f) \\ \tilde{Y}_c = Y_c\,(1 - d_f) \\ \tilde{S}_c = S_c\,(1 - d_f) \end{cases} \tag{3}$$

where X_t, X_c, Y_t, Y_c, S_c are respectively longitudinal tension and compression strengths, the transverse tension and compression strengths, the in-plane shear strength and X_t^{yarn} is the strength of dry yarn fibres (i.e. unidirectional ply without matrix).
The three original points of this failure criterion are: (i) a better description of the strength of the UD ply under shear and transverse loadings, (ii) the use of a degradation variable d_f to represent the degradation of the interfibre strengths due to premature single fibres failures (statistical effect on fibre strength) and (iii) a coupling between the state of degradation of the matrix (due to matrix damage) and the longitudinal tension strength of the UD ply.
Then, the progressive degradation model is based on damage models [6]. After the first failure of a ply in the laminate (failure criterion value is higher than 1), the effective elastic compliance of the failed ply is increased as it is written in Eq. (4):

$$\tilde{S} = S^0 + d_1 H_1 + d_2 H_2 \qquad \begin{cases} FF: \quad d_1 = \alpha \left\langle \sqrt{f_1^{\pm}} - 1 \right\rangle^+ \quad \dot{d}_1 \geq 0 \\ IFF: \quad d_2 = \beta \left\langle \sqrt{f_2^{\pm}} - 1 \right\rangle^+ \quad \dot{d}_2 \geq 0 \end{cases} \tag{4}$$

where S^0 is the elastic (initial) compliance, d_1H_1 and d_2H_2 are tensors that represent respectively the effect of fibre failure and interfibre failure on the compliance of the broken ply. For each degradation model, a distinction is made between the kinetics of degradation with the scalar variables d_i and the effect of failure on the mesoscopic behaviour with the effect tensors H_1 and H_2.

The definition of the laminate failure depends on the industrial application. For moderate gradient structures, fibre failures (tension and compression) and transverse compression failure (inducing an explosive wedge effect) are considered as ultimate failure mechanisms for laminates. An important loss of macroscopic rigidity is also considered as catastrophic for the laminates (especially for $[\pm\theta]_s$ laminates).

The predictions of this multiscale failure approach [1] have been compared successfully with experimental results from the literature on laminates with different stacking sequences and constituted with different materials (Eglass/Epoxy and Carbon/Epoxy). The Figure 1 presents the comparison between the predicted macroscopic failure envelope (Σ_{xx}, Σ_{yy}) of a $[90/\pm30]_s$ laminate in Eglass/LY556 and experimental data [7].

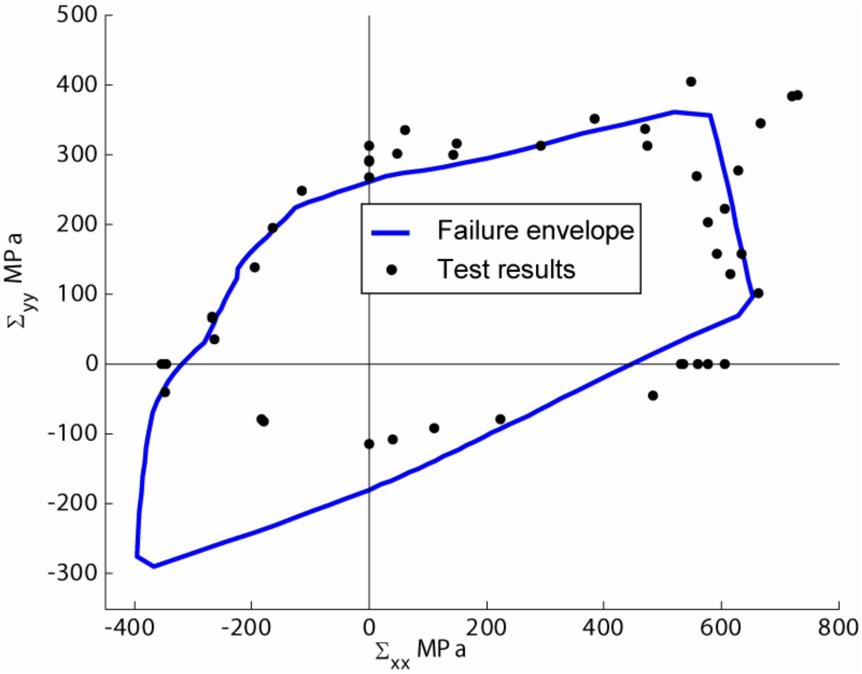

Figure 1 : Macroscopic failure envelope (Σ_{xx}, Σ_{yy}) of a $[90/\pm30]_s$ laminate in Eglass/LY556

The predictions of the failure approach are in very good agreement with experimental data for tensile/tensile loadings or tensile/compressive loadings, nevertheless the predicted final failure for bi-compressive loadings overestimate the experimental strength of the thin tested tubes because of the global buckling of the structure. It is an absolute necessity to take into account this structural instability to predict accurately the failure of composite structures subjected to compressive loadings.

3. FAILURE UNDER COMPRESSIVE LOADINGS

One of the main conclusions of the WWFE [8] is that all the failure approaches overestimates the laminate strength under compressive loadings because the structural buckling is neglected. It is necessary to consider the laminate as a structure in order to predict accurately the final failure for compressive loadings.

Buckling is classically defined as a loss of stability initiated by some imperfections (material, geometry). The prevision of the buckling and post-buckling, with FE modelling, is still a very difficult issue. Methods found in literature could be divided into two classes: (i) numerical methods, difficult to implement and use in a practical way, but know all the possible post buckling modes and (ii) methods which try to model the imperfections leading to buckling.
The second method, which is based on physical quantities: the spatial variability of mechanical properties of each ply (mainly due to the fabrication process), have been chosen. The proposed method is very simple to use in an element finite code, each Gauss point in the structure have different material properties (see Figure 2). Moreover, this method, called *stochastic buckling*, is well adapted to perform simulations with a material non linear behaviour.

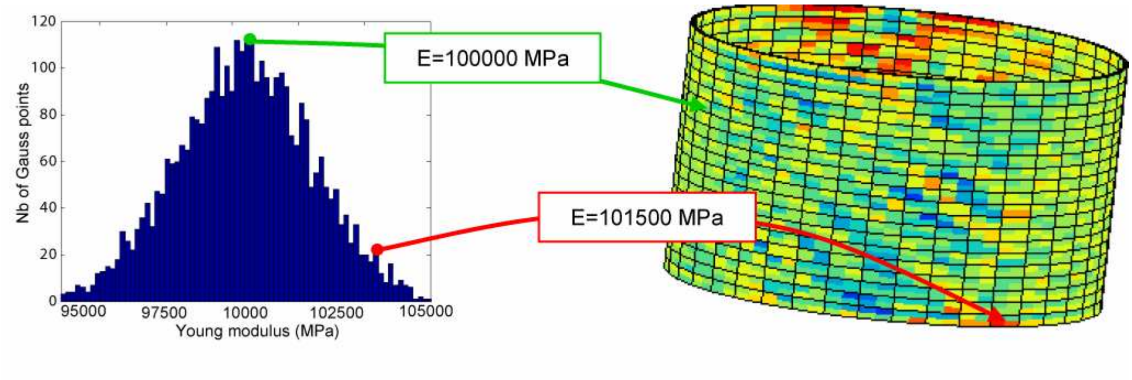

Figure 2 : Principle of the proposed stochastic buckling method.

It is worth mentioning that these imperfections are used as a numerical artefact in order to create local imperfections which lead to buckling of the structure. The robustness of this method has been demonstrated by studying the effects of the random distribution on the final results (buckling load and post-buckling behaviour). Moreover, this method has been validated using analytical solutions on laminated thin plates [2].

The multiscale progressive failure approach has been implemented in a finite element code, and it is thus possible to predict final failure of composite structures by taking into account both the material aspect (ply failure and degradation of the failed ply) and structural instabilities (such as buckling). Therefore, the previous prediction of the macroscopic failure envelope (Σ_{xx}, Σ_{yy}) of a $[90/\pm30]_s$ laminated tube in Eglass/LY556 has been updated by taking into account the effect of the buckling on the final failure predictions (see Figure 3).

Obviously, the failure prediction is similar for tension/tension loadings where the final failure is due to plies failure in fibre mode. For uniaxial compressive loading of the tube (Σ_{xx}:Σ_{yy}=-1:0), the material failure (ply failure of ±30° in transverse compression) leads to the rupture of the specimen prior the buckling. The predicted rupture of the material envelope is thus the same than the one predicted from the finite element simulation.

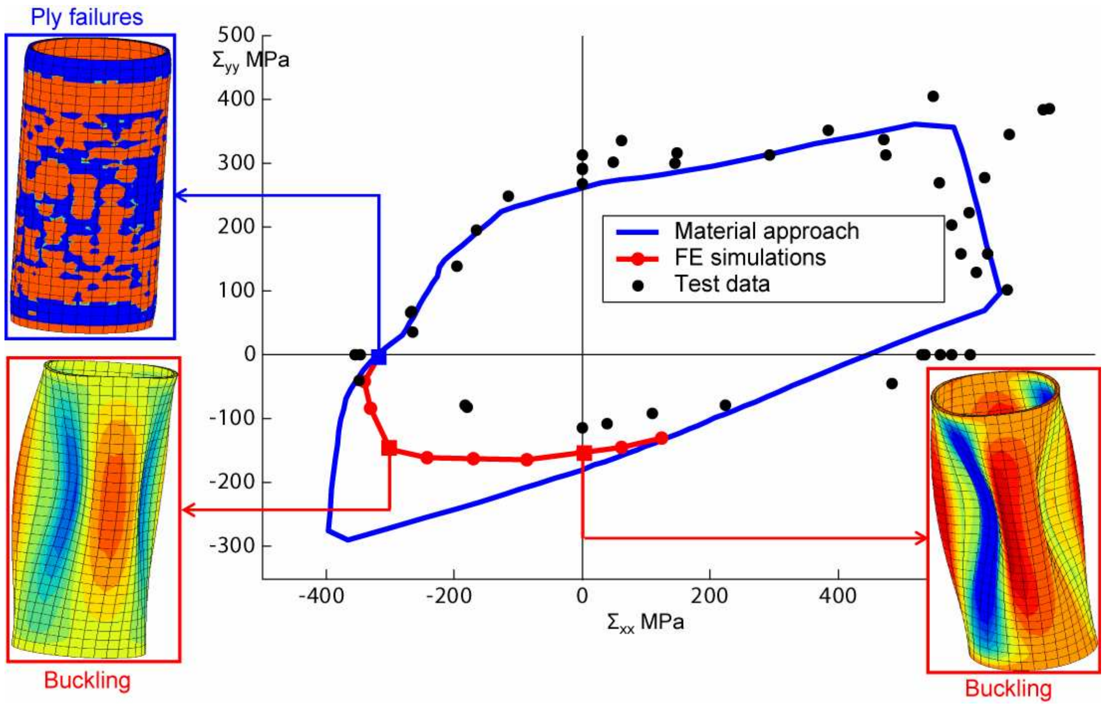

Figure 3 : Biaxial failure envelope of a [90/±30]s Eglass/LY556 laminated tube subjected to biaxial loadings by taking into account the global buckling

Nevertheless, for bi-compressive loadings, the global buckling (the modes of buckling are reported on the Figure 3) of the structure occurs prior the first ply failure and leads to the rupture of the specimen. For external pressure loading (Σ_{xx}:Σ_{yy}=0:-1), the final failure is also due to global buckling. Therefore, the corrected macroscopic failure envelope is in better agreement with experimental data. Indeed, the error of the prediction for loading ratio (Σ_{xx}:Σ_{yy}=-1:-1) bas been roughly decreased by a factor 2.
While the accuracy of the prediction has been improved, the predicted failure loadings remain not so close to the experimental data and not conservative. The dispersion of the experimental data, reported on the Figure 3, is rather important when the tests have been repeated. Because there is important dispersion on composite material, it seems necessary to perform strength analysis by taking into account the uncertainty on the input data. Obviously, the material parameters of the modelling are known with a certain level of confidence due to dispersion, but there are also geometrical tolerances on the dimensions of the specimens (length or thickness) which could have a strong influence on the prediction of buckling.

4. STRENGTH ANALYSIS WITH UNCERTAIN DATA

4.1 Principle of the proposed procedure

The aim of this kind of approach is to estimate the confidence on the simulated results as a function of the uncertainty on the input data of the modelling. Obviously a direct method, which consists in performing a finite element simulation of each possible set of data in the bounds of the uncertain domain, leads to tremendous time of calculation even for a little number of uncertain data. For instance, for N=10 (number of stochastic

variables) by evaluating the response at 3 different values for each input data (min, max and average), it is necessary to perform 3^{10}=59049 computations.
Therefore, the strategy, proposed in this paper, is based on the transport of uncertainties through response surfaces (also called in the following Meta Model MM). A response surface consists in building an analytical relation (most generally polynomial) between stochastic variables and the outputs using a number of runs of the complete modelling (usually a FE calculations). The aim is to use this approximation in order to determine statistical indicators (such as mean, standard variation, extreme values …) thanks to Monte-Carlo simulations or a formal transport with reasonable time of computation. Various methods could be found in the literature based on polynomials or more complex functions [9,10,11]. The following is focused on a polynomial expansion.

The general principle of the proposed strategy of strength analysis with uncertainty data is presented on the Figure 4. The details of this strategy are given in the following.

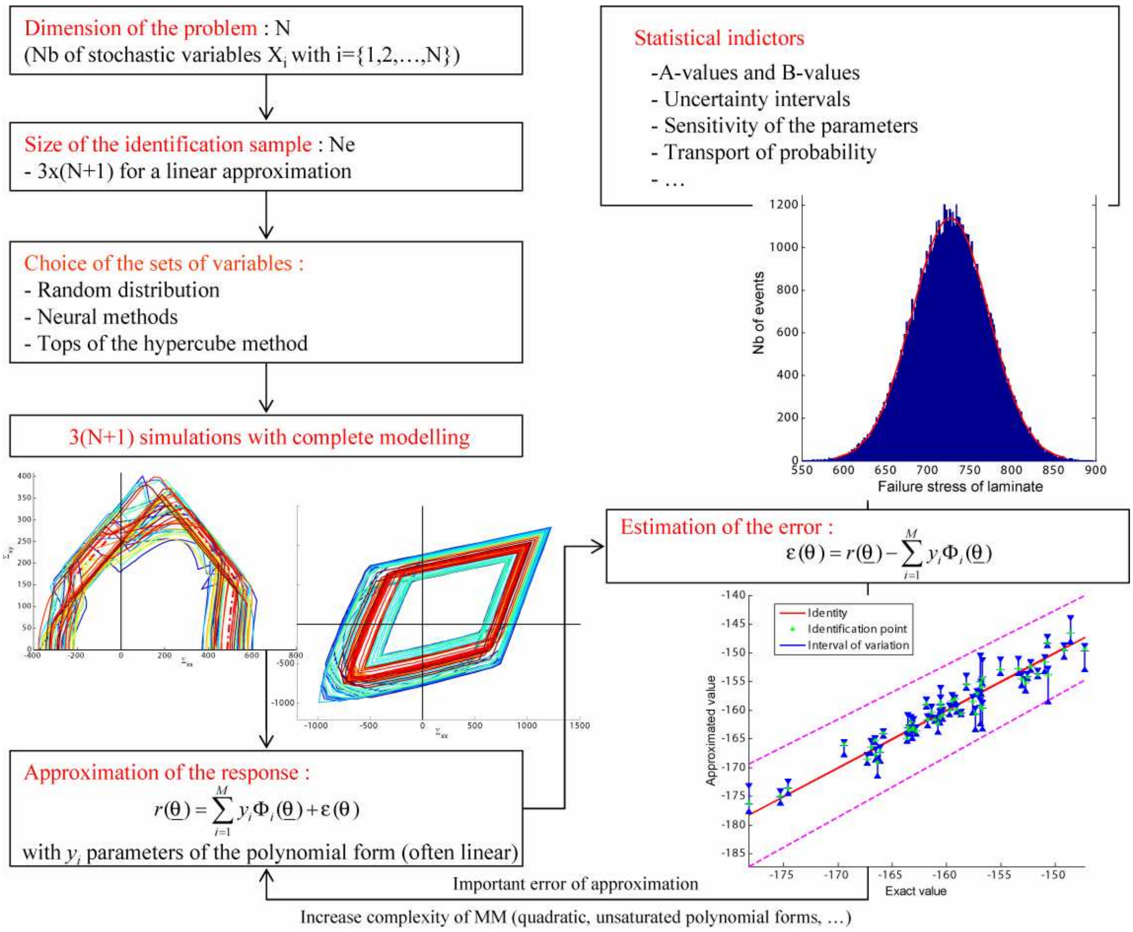

Figure 4 : Principle of the proposed strategy of strength analysis with uncertainty data

When the dimension of the problem is determined (fixed by the number of the stochastic variables), it is necessary to determine the choice of the sets of variables that must be tested in order to identify the approximation. Two strategies have been developed: one based on the neural methods [12] that permit to occupy as far as possible the whole hypercube defined by the range of the stochastic variables, the other one that consists in occupying the tops of the hypercube as far as possible.
The sets of variables being chosen, FE simulations are performed on a cluster of PC by using the natural parallelization of the problem.

The quantities under interest (responses of the complete modelling, in the present case final failure of the laminate) are expanded on functions of the stochastic variables as follows:

$$r(\underline{\theta}) = \sum_{i=1}^{M} y_i \Phi_i(\underline{\theta}) + \varepsilon(\theta) \qquad (5)$$

where $\underline{\theta}$ is the vector formed by the N stochastic variables, $\Phi_i(\underline{\theta})$ is the i^{th} term monomials of approximation and the coefficients y_i are the parameters (to be identified) of the approximation function. The use of a classical M order polynomial is the simplest choice. In this case, the determination of the best polynomial function, that relates the outputs to the stochastic variables, is made in the sense of the least square method.

In order to estimate the quality of the proposed MM the generalization error (error made on other points than those used for the identification) is used. The error evaluation of the polynomial approximation is performed using various techniques of cross-validations (bootstrap [13], leave-one-out [14]). It consists in identifying X times the approximation using sets of calculations randomly chosen through the N available ones (N randomly or (N-M) choices among the N for the bootstrap and the leave-M-out methods respectively). These techniques, which allow estimating the generalization error, improve considerably the reliability of the approximation.

A progressive strategy has been developed. Indeed, if the errors estimated are low, the approximation is thus validated. If the estimators of errors indicate that the quality of the approximation is not sufficient, the process is performed again by increasing the complexity of the metamodel.

Finally, by using the identified surrogate model, it becomes possible to determine statistical indicators with a reasonable time of computation.

4.2 Comparison with experimental data

The strength analysis method with uncertain data has been applied to the prediction of the macroscopic failure (Σ_{xx}, Σ_{yy}) of a $[90/\pm30]_s$ laminate in Eglass/LY556, previously presented.

The intervals of uncertainty of the 17 stochastic parameters are reported on the Table 1.

The 15 material stochastic variables are: the 5 elastic properties (E_{11}, E_{22}, G_{12}, ν_{12}, ν_{23}), the 3 thermo-elastic parameters (α_L, α_T, T_0), the α^{ve} viscoelastic parameters, (which evolves between 0 and 1, in order to perform simulations with a material behaviour evolving between linear elastic and highly non linear due to the viscosity of the matrix), the 5 uniaxial strengths of the UD ply (X_t, X_c, Y_t, Y_c, S_c) and the β kinetics parameter of the degradation of the mechanical properties of a failed ply in interfibre mode. The interval of uncertainty of the material parameters have been determined in order to border the available experimental data [7,15] at the ply scale, as it is reported on the Figure 5.

Moreover, the necessity to take into account the buckling in order to predict accurately the final failure of the tested specimens in compression has been demonstrated. Therefore, 2 geometrical stochastic variables are considered: the total thickness of the specimen t, and the length of the tube. The intervals of uncertainty of the geometrical variables considered are the tolerance recommended in aeronautical industries.

The dimension of the problem is N=17, it is thus recommended to perform at least 3(N+1)=54 finite element simulations (in case of linear regression), for each loading case of the macroscopic failure envelope, in order to identity the response.

Parameters	Values	Parameters	Value	Parameters	Values
E_1	53480±2000 MPa	α_L	8.6±1 10^{-6}/°C	X_t	1140±120 MPa
E_2	17700±1000 MPa	α_T	26.4± 3 10^{-6} /°C	X_c	-570±115 MPa
G_{12}	5830±1000 MPa	T_0	80±40 °C	Y_t	40±8 MPa
υ_{12}	0.278±0.04	α_{ve}	0.5±0.5	Y_c	-135±7 MPa
υ_{23}	0.4±0.06	t	2±0.6mm	S_c	61±12.5 MPa
		L	180±5mm	β	7±3

Table 1: Intervals of uncertainty for material and geometrical stochastic variables

From the knowledge of the interval of uncertainty on the stochastic variables (both material and geometrical parameters), it is possible to determine the intervals of uncertainty on the macroscopic failure envelope, as it is reported on the Figure 5 for $[90/\pm30]_s$ laminated tubes in Eglass/LY556 subjected to combined Σ_{xx} and Σ_{yy}.

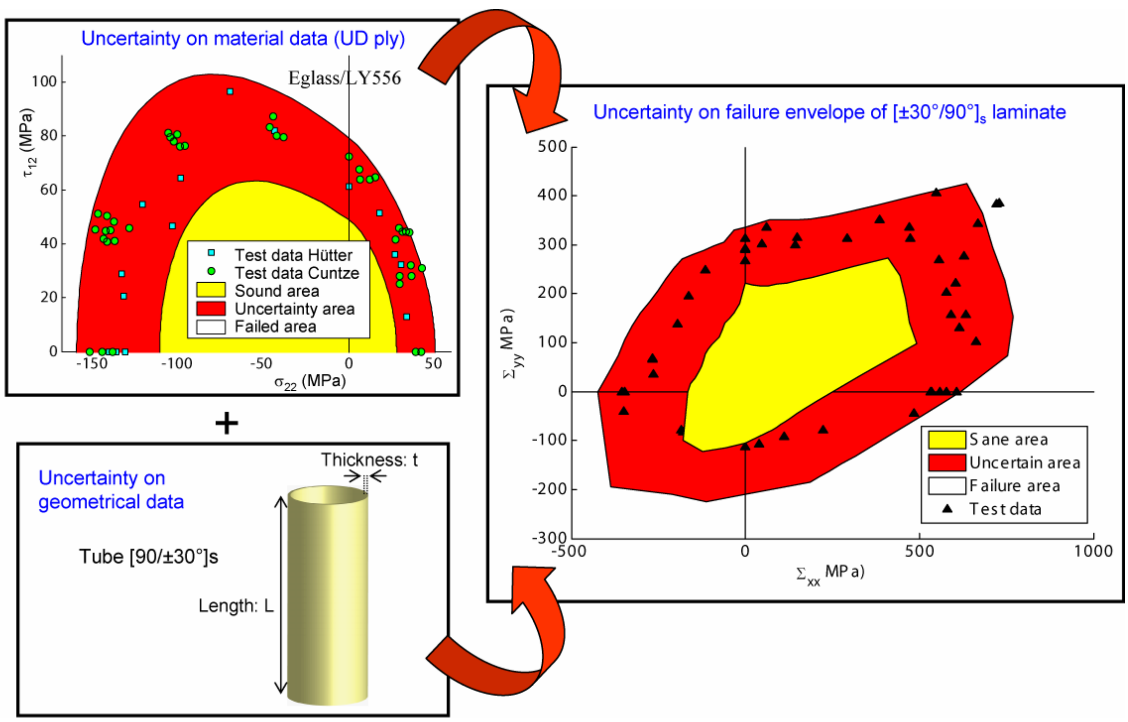

Figure 5 : Prediction of the uncertainty interval of the failure envelope of a $[90/\pm30]_s$ laminated tubes in Eglass/LY556

Three different areas are reported on the Figure 5: (i) a sane area (yellow area) where no failure is possible, (ii) a failure area (white area) where the laminate is necessarily broken and (iii) an uncertain area (red area) where it is not possible to conclude because of the uncertainty on the input data.

While the intervals of uncertainty are large, the uncertainty in the predicted macroscopic failure envelope is rather similar and in good agreement with test data from the literature. Moreover, it is worth mentioning that the interval of uncertainty for bi-compressive loading is more important without taking into account the buckling, because for this kind of loadings, the global buckling for the specimen occur prior the material failure.

This kind of approach permits also to determine the sensitivity of the parameters on the final failure prediction, what should permit to improve the design of composite structures. Indeed, for the uniaxial tensile loading (Σ_{xx}:Σ_{yy}=1:0), the most influent parameter is the longitudinal compression strength because final failure is due to fibre failure in compression in the 90° plies due to Poisson effect. In order to improve the confidence in the failure prediction for this loading, it could be interesting to perform other tests to refine the knowledge of this strength. For the bi-compressive loading (Σ_{xx}:Σ_{yy}=-1:-1), the most influent parameters are the longitudinal modulus and the thickness of the specimen, because the final failure is due to the global buckling of the specimen.

Finally, by assuming a statistical distribution on the input data (whatever the chosen probabilistic distribution); a transport of probability could be performed to estimate the density of probability of the final failure of the laminate for different loadings. Obviously, it is also possible to determine the A and B-values of the final failure of the laminate, what provides less conservative predictions than performing simulations with the A or B-values.

6. CONCLUSIONS

A multiscale progressive failure approach has been proposed in order to predict the failure of laminate for different stacking sequences from the knowledge of the mechanical properties of the unidirectional ply. The predictions have been compared successfully with experimental data from the literature. Nevertheless, the proposed material approach overestimates the final failure of laminates subjected to bi-compressive loadings due to structural instabilities not taken into account with a material approach.

Therefore, in order to predict structural instabilities, a method, specific for composite structures, has been proposed and permits to predict easily global buckling of structures even for simulations with a material non linear behaviour. Laminated tubes have been described as a structure and finite element simulations have been performed in order to improve the prediction of the material failure envelope for $[90°/\pm30°]_s$ laminated tubes.

The effects of dispersion and uncertainty on input data (material properties (rigidity, strengths…) and geometrical parameters (thickness and length)) on the prediction of laminate failure have been taken into account in predictions of macroscopic failure envelope and compared with experimental data from the literature. This kind of approach is able not only to predict the uncertainty on the final failure of a laminate, but also the sensitivity of the different stochastic variables, what could be useful to design composite structures. Moreover, it is also possible to estimate the probability of macroscopic failure, with a reasonable computational cost, if the probabilistic distribution on the input data (material and geometrical properties) has been previously determined.

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