PROBABILISTIC METHODS FOR STRUCTURAL RELIABILITY AND RISK

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ABSTRACT

A probabilistic method is used to evaluate the structural reliability and risk of select metallic and composite structures. The method is a multiscale, multifunctional and it is based on the most elemental level. A multi-factor interaction model is used to describe the material properties which are subsequently evaluated probabilistically. The metallic structure is a two rotor aircraft engine, while the composite structures consist of laminated plies (multiscale) and the properties of each ply are the multifunctional representation. The structural component is modeled by finite element. The solution method for structural responses is obtained by an updated simulation scheme. The results show that the risk for the two rotor engine is about 0.0001 and the composite built-up structure is also 0.0001.

1. INTRODUCTION

The pursuit of achieving and retaining competitive advantages in world markets necessarily leads to proactive drives for better-cheaper-faster products to market. This becomes even more important in the high tech sector which includes aerospace vehicles. The awareness for conservation of natural resources also leads proactively to the reliable cost-effective useful-life extension of existing products. In the first case, it requires very effective use of available resources. In the second case, it requires formal methods to quantify the current strength of a specific structure/component and subsequent reliable evaluation of respective remaining strength. Both cases include a multitude of uncertainties. The first case includes uncertainties associated with lack of sufficient data of new potential material such as composites. The second case is full of uncertainties from (1) unknown assumptions and conditions of the initial design, (2) records of environmental exposure, and (3) material degradation of various factors, etc. In scenarios of multiple uncertainties described above, probabilistic methods offer formal approaches to quantify those uncertainties and their subsequent effects on material behavior, on service, and on reliabilities and risks. Therefore, the objective of this article is to describe a probabilistic method for predicting structural reliability and risk from material behavior to service life. That probabilistic method (1) is based on formulations that describe the physics in terms of primitive variables and (2) respective scatter ranges, at the lowest engineering manageable scale by a Multi-Factor Interaction Model (MFIM) [1], and (3) relies on computational simulation methods to propagate those uncertainties from that elementary scale through all intermediate scales where metrics for structural reliability and risk are specified. The method has evolved over the past three decades and has matured sufficiently to evaluate structural reliability and risk under various scenarios. The schematic in Figure 1 illustrates the concept. The method has several unique features. The two that are most useful to present results are (1) quantifiable reliability in terms of Cumulative Distributions Functions (CDF) and (2) sensitivity factors of the primitive variables that affect reliability at every point in the CDF curve. The method was illustrated by first applying it to a metallic engine rotor [2], and then to three different composite structures with progressive complexity [3].
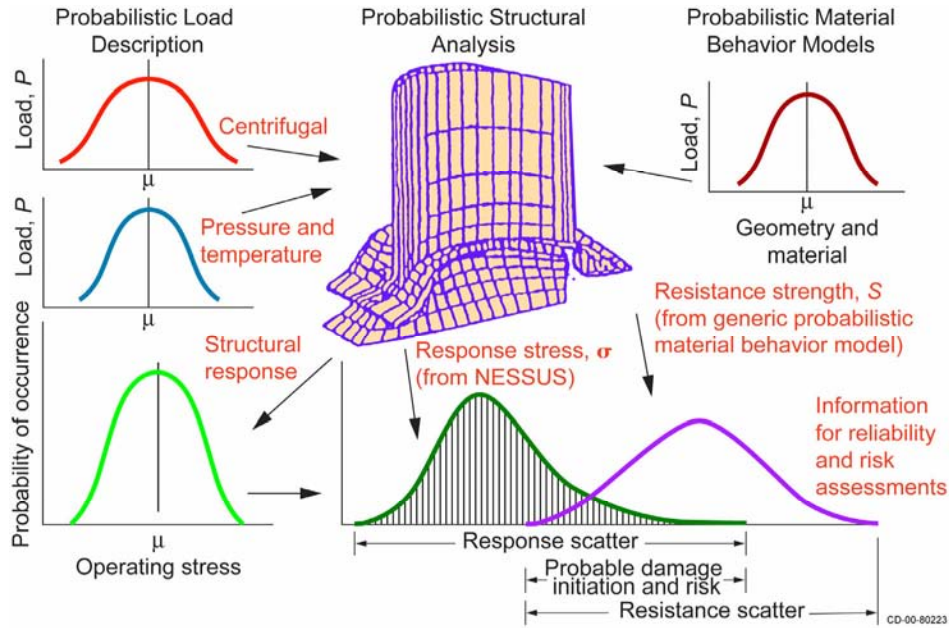
Figure 1.—Probabilistic simulation of component reliability and risk.

2. MULTI-FACTOR INTERACTION MODEL

The complex material behavior is generally affected by several factors. These factors herein are represented by a Multi-Factor Interaction Model (MFIM) of multiplicative form as is illustrated in Figure 2. The concept of this model is that the complex behavior of any material is assumed to be a surface in space. This surface is represented by an N dimensional vector from some origin. For illustration purposes, this figure
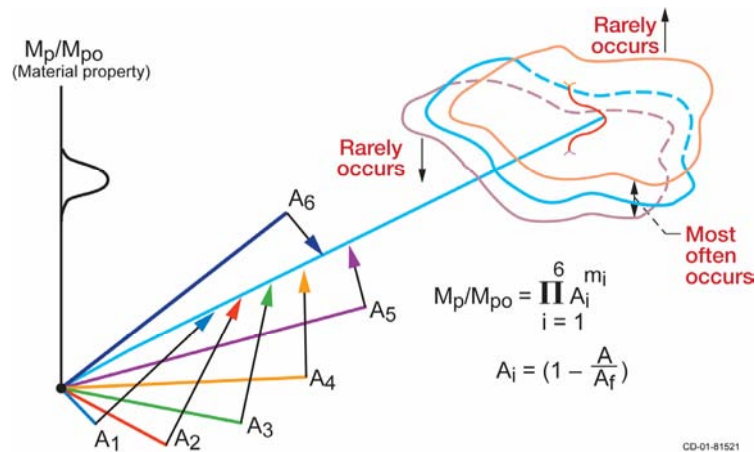

Figure 2.—Multifactor interaction model with substructuring.

indicates 6-vectors that define the shape of the assumed surface. The figure also describes the product form of the MFIM. Note the definitions of A. An expanded form of the MFIM is illustrated in Figure 3. Note in Figure 3 that each property is defined by P_o at the origin and by an unsubscripted P at its evolution as it is affected by the various factors. There are two conditions associated with the MFIM expanded form: (1) the exponents must be positive and that their value is unity at the origins and has nanotonic behavior from the origin to their failure value, and (2) all the factors within the parentheses have absolute values.

$$\frac{M_p}{N_o} = \left(1 - \frac{T}{T_f}\right)^{\ell} \left(1 - \frac{T_{gw}}{T_f}\right)^{M} \left(1 - \frac{\sigma}{S_f}\right)^{n} \left(1 - \frac{\sigma t}{S_f t_f}\right)^{p} \left(1 - \frac{\sigma_M N_M}{S_f N_{fM}}\right)^{q} \left(1 - \frac{\sigma_T N_T}{S_f N_{fT}}\right)^{r} \left(1 - \frac{\omega}{\omega_f}\right)^{s} \ldots$$

where:
- P - property
- T - temperature
- S - strength
- σ - stress
- N - number of cycles
- ω - load frequency

Subscripts:
- gw - wet glass temperature
- o - reference condition
- f - final condition
- M - mechanical load
- T - thermal cyclic load

M_p/M_{po} (Material property)

Rarely occurs

Rarely occurs

Most often occurs

$M_p/M_{po} = \prod_{i=1}^{6} A_i^{m_i}$

$A_{\ell} = \left(1 - \frac{A}{A_o}\right)$

Superscripts: ℓ, m, n, etc. are exponents for that material that property effect which describe respective behavior paths from the reference to the final value
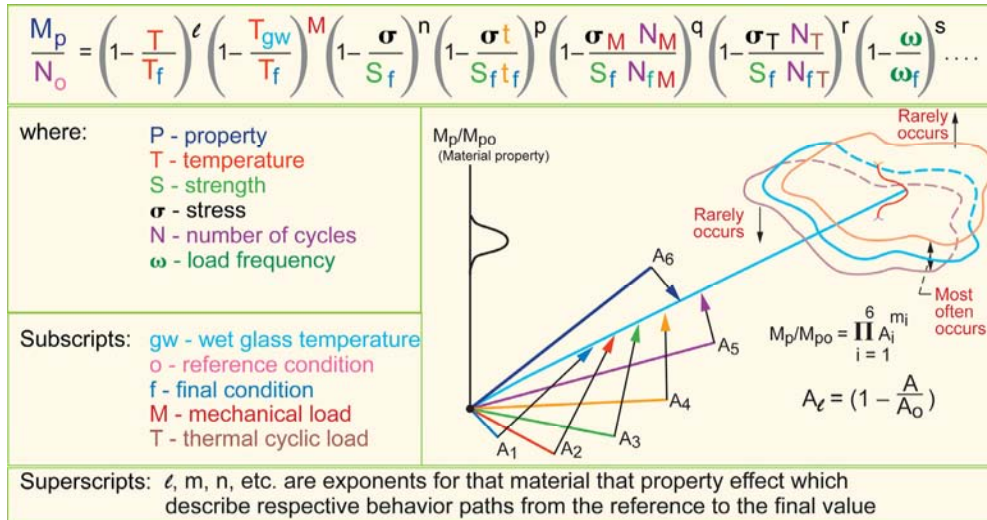
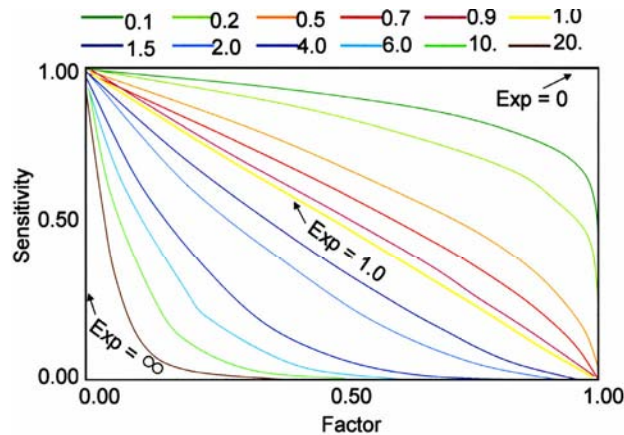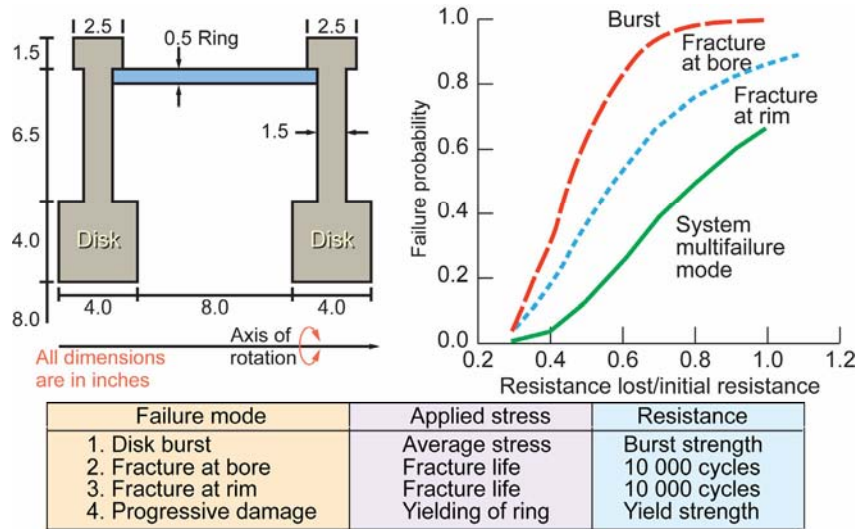Figure 3.—Time dependent multifactor interaction mode (MFIM).

Figure 4.—Multifactor interaction equation is very sensitive to factor exponent.

The MFIM describes behavior as is illustrated in Figure 4. Note that when the exponent is less than one the exponent describes "sudden death" type of function. When it is one it describes a straight line between the origin and the final value. When it is greater than one it describes an inverse death function (infant mortality), rapid degradation early on and then progressively decreasing values as the curve approaches its final value.

3. SIMULATED TWO STAGE ROTOR

A simulated two-stage engine rotor is illustrated in Figure 5 top left. At the right are the cumulative distribution functions for the four conditions noted in the table at the bottom. These results are noted for each of the three conditions and a combined one which coincides with the fracture at the rim. In the table down below is a list of the components that were evaluated. The last column in the table shows the resistance associated with each component failure. Risk is not appropriate when the CDF is available. The reason is that for any $0 <$ probability < 1 the values are known. For example, take the probability of 0.6, which corresponds to a resistance lost of 0.9. The remaining resistance is only 0.1 or 10 percent. Therefore, the margin of safety prior to

Figure 5 diagram labels: 2.5 | 0.5 Ring | 2.5 | 1.5 | 6.5 | 1.5 | 4.0 | Disk | Disk | 8.0 | 4.0 | 8.0 | 4.0 | Axis of rotation | All dimensions are in inches

Figure 5 graph: Failure probability (0.0–1.0) vs Resistance lost/initial resistance (0.2–1.2); curves: Burst, Fracture at bore, Fracture at rim, System multifailure mode

Failure mode	Applied stress	Resistance
1. Disk burst	Average stress	Burst strength
2. Fracture at bore	Fracture life	10 000 cycles
3. Fracture at rim	Fracture life	10 000 cycles
4. Progressive damage	Yielding of ring	Yield strength

Note: Yielding of ring affects all other modes through mutual interaction. 1 inch = 2.54 cm

Figure 5.—Rotor system survival probability under multiple failure modes.

Name	Sensitivity
Rotor speed (SPEED)	0.850827
Rotor density (ROTOR DENS)	0.438499
Rotor temperature (TEMPE)	0.170793
Fracture Correlation (C)	0.133702
Ring yield (RINGY)	0.073086
Fracture toughness (Kt)	0.060917
Initial defect (AO)	0.057976
Rotor modulus (E_ROT)	0.016011
Rotor tolorance (TOLER)	0.000000
Fracture parameter (RK1C)	−0.061872
Burst pressure (BURST)	−0.011983
Low cycle fatigue (A_LCF)	−0.005132
Ring modulus (E_RIN)	−0.002698
Ring density (RING DENS)	−0.000386
Cycle effects prior to initiation (NI)	−0.000008

Figure 6.—Probabilistic sensitivity factors of rotor system.

failure is only 10 percent. The remaining strength (resistance) is a better indicator compared to risk. The probabilistic sensitivity factors for the two-stage engine rotor are listed in Figure 6 in hierarchical order. The largest factor (value-wise) is that of the rotor speed. The second largest is the rotor density. The third largest is the rotor temperature. The rest of them vary between 0.01 and "0". The magnitude of the factors is indicative of the effect that the factor will have in the CDF of the response. The sum of the squares of the factors equals unity which means that the rotor speed has about four times the effect of the density and about twenty-five times the temperature effect. Other points to be noted in Figure 6 are the variables associated with fracture mechanics K_{1c}, A_o, N_l, K_t, A-LCF which have negligible affects on the reliability and risk of the fracture probability.

4. PROBABILITY EVALUATION OF COMPOSITES

Composites really constitute a multiscale structure with multifunctional material. These two features of composites are illustrated in the computational simulation cycle denoted in Figure 7. As can be seen in this figure, the multifunctional aspect starts at the material behavior space at the lower right where the local properties are described
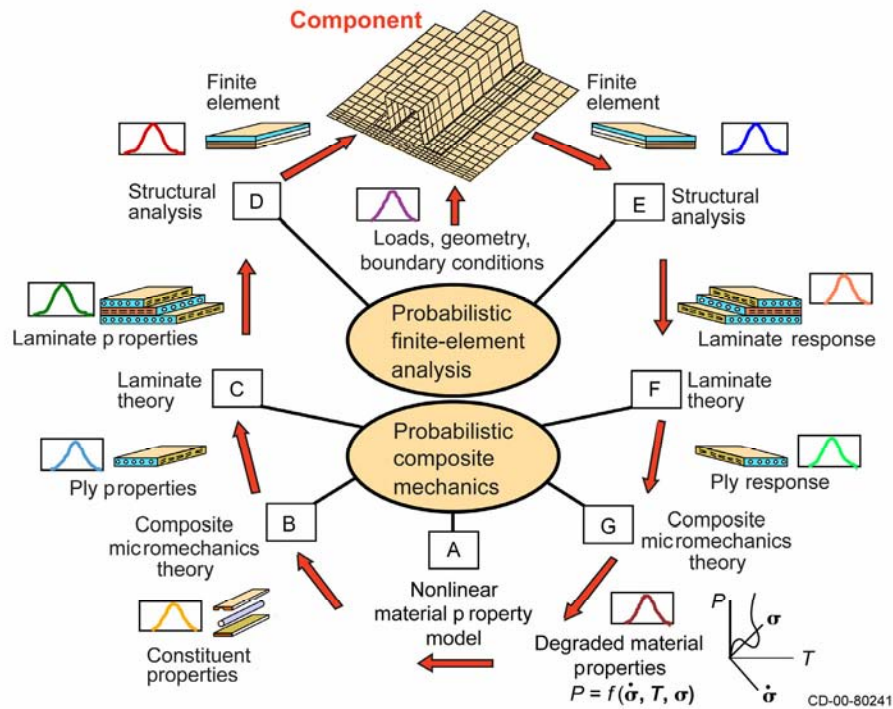
Figure 7.—Probabilistic assessment of composite structures computational simulation cycle.

probabilistically (A). Next there is a schematic of a fiber with two matrix slabs indicating the probabilistic simulation of micromechanics (B). The next scale is the probabilistic description of a mono-fiber ply indicated by (C). Next is the probabilistic description of a laminate which ends the composite probabilistic description. Next is the probabilistic description of a finite element (D), the beginning of the structural analysis with the probabilistic loads, structural geometry and boundary conditions. A to D on the left side of the chart represent an upward synthesis. After the finite element structural analysis, the downward decomposition starts with the probabilistic description of displacements, loads, stresses and strains in each finite element or node. With these probabilistic values as inputs, the probabilistic decomposition continues to F, G and to the probabilistic micromechanics description of the material behavior space. The probabilistic cycle continues until all the loads have been finished and structural fracture has occurred. This describes an incremental approach where any change can be included in the upward synthesis or the downward decomposition. As can be imagined, the input probabilistic material properties can be quite numerous. These are shown in the next Figure 8. This table has five columns; the first column has a description of each property; the second has reference values of each property, the third has corresponding mean properties, the fourth has the assumed scatter for each property, and the fifth has the distribution type of each property. Note that the distribution type can be different for each property. Only two distributions are shown in this table. This is strictly for convenience because after the simulation is completed changes in the scatter and in the distribution type can be introduced. Now the comparison can be made between the two probabilistic results of the CDF and of the sensitivities. If the two CDF's are about the same, and if the order of the sensitivities is about the same, then the assumed scatter and their respective distributions do not influence the probabilistic results and the conclusion is that no additional testing is required at the materials quality control level, which, in essence, is a considerable cost saver. Also, at the same

Random Variables	Ref.	Mean Value	Assumed Scatter	Distribution Type
Fiber:				
Normal modulus E_{f11}	32 mpsi	32 mpsi	8%	Normal
Normal modulus E_{f22}	2 mpsi	3 mpsi	8%	Normal
Poisson's ratio ν_{12}	0.2	0.23	8%	Normal
Poisson's ratio ν_{23}	0.2	0.25	8%	Normal
Shear modulus G_{f12}	2 mpsi	2.5 mpsi	8%	Normal
Shear modulus G_{f23}	2 mpsi	2.5 mpsi	8%	Normal
Tensile strength S_{fT}	400 ksi	400 ksi	8%	Weibull
Compressive strength S_{fC}	400 ksi	400 ksi	8%	Weibull
Matrix:				
Normal modulus E_m	0.4 mpsi	0.45 mpsi	8%	Normal
Poisson's ratio ν_m	0.4	0.41	8%	Normal
Tensile strength S_{mT}	7 ksi	6.7 ksi	8%	Weibull
Compressive strength S_{mC}	36.3 ksi	39 ksi	8%	Weibull
Shear strength S_{mS}	7 ksi	8.9 ksi	8%	Weibull
Fabrication Variables:				
Fiber volume ratio (fvr):	60%	60%	8%	Normal
Void volume ratio (vvr):	0.01%	0.01%	8%	Normal
Ply thickness:	0.0055 in.	0.0055 in.	5%	Normal
Ply misalignment:	0	0	0.9° stdv	Normal

(1 psi = 6.9 Pa)

Figure 8.—Statistical properties of random variables.

time it illustrates how the probabilistic simulation is "self corrective." The results described and their significance constitute a multifunctional and a concurrent multiscale probabilistic simulation.

5. PROBABILISTIC FATIGUE LIFE

Now after the three essential parts (MFIM, multifunctional and multiscale) have been described, we are ready to implement them in specific problems. A composite panel is subject to in-plane cyclic loads as is illustrated in Figure 9. As is seen in the figure, the panel is subjected to four different frequencies. The schematic of the panel with a cyclic load is at the top of Figure 9 while results are in the middle and the loading conditions are at the bottom. The small table at the right is mixed because it presents inputs in the first line and outputs in the next three lines. The first line indicates the frequency inputs while the other three lines represent respective frequency outputs. For example, at 25 cps the mean life is about 0.91; the scatter percent is about 24.85 and the panel relative life is 0.21 if the maximum fatigue life is assumed to be unity (1). The corresponding CDF is shown in the left schematic which has four different CDF's corresponding to each one of the frequencies. Each CDF is identified with the frequency that produced it. The 25 cps CDF is the solid curve to the right. Below the figure the dominant failure modes are listed as transverse tension in the 90° plies at low frequencies and 90° plies compression at higher frequencies. It is seen in Figure 9 that
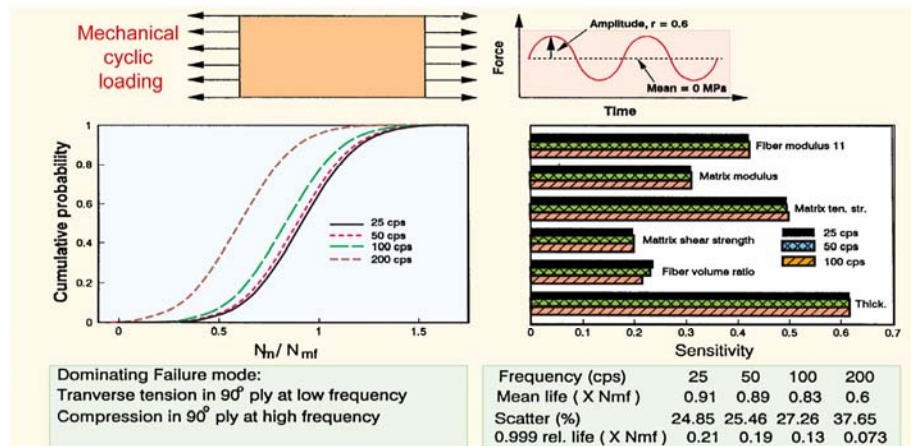
Figure 9.—Probabilistic simulation of fatigue life curve for graphite/epoxy (0/±45/90)s laminate.

the 200 cps curve is substantially to the left of the other three curves. This effect is also reflected in the small table which shows the corresponding degradation to be 0.6 mean life, 37.65 percent scatter and relative life 0.073. The other significant information in Figure 9 is the probabilistic sensitivity factors which are plotted horizontally in the middle right. Two major observations are important in this figure: (1) The frequencies have almost no effect on these factors. (2) The three dominant sensitivities are (in decreasing order of magnitude): panel thickness, matrix tensile strength, and fiber modulus. The next dominant factor is the matrix modulus and the last two factors are the fiber volume ratios and the matrix shear strength. The other 11 properties listed in Figure 8 had less than 0.1 effects and hardly worth testing to quantify their influence.

This problem demonstrates the benefits of probabilistic simulation in multiscale multifunctional composite structure. The wealth of information obtained is definitely worth the effort required to generate it.

6. PROBABILISTIC EVALUATION OF CERAMIC HOT COMPONENT

In order to demonstrate the versatility of probabilistic methods, we discuss its application to a ceramic high temperature aircraft engine liner. The liner is schematically illustrated in Figure 10 with loads, geometry and finite element model.
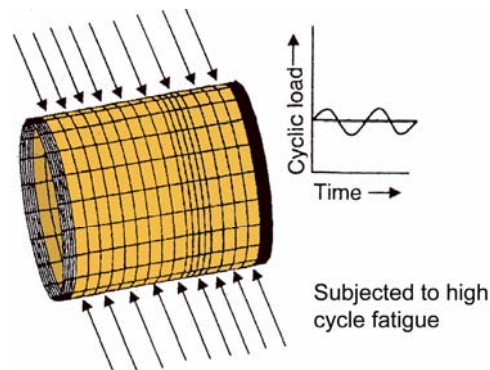
Figure 10.—Finite element model of engine hot section ceramic-matrix-composite component.

The multiscale simulation is comparable to Figure 7 and the input properties are the same as in Figure 8 with the exceptions that the fiber properties and the matrix are replaced with high temperature ceramic materials and the processing temperature is much higher. It is about 3000 °F. The use temperature is about 2800 °F. The high temperature effects of the ceramic materials were simulated by the use of the MFIM. Probabilistic results obtained for strength and stress at two different probabilities are shown in Figure 11 as functions of normalized cycle numbers. The most significant point to be observed in this figure is the greater spread between strength and stress at 0.01 to 0.001 probability. This effect illustrates the point that probabilistic evaluations must be used judiciously. The spread between the strength and stress at the 0.01 probability is relatively large and will lead to an erroneous conclusion especially at very low fatigue cycles.

The probability of survival which is comparable to that of Figure 8 is plotted in Figure 12 as a function of the normalized cycle numbers. This figure illustrates a very interesting point which is the nearly constant variation from about 0.25 to 0.65 of the normalized cycles. It indicates a relatively rapid degradation at low fatigue cycles to about 0.25 and then again at relatively high cycles at cycles greater than 0.65. Note that
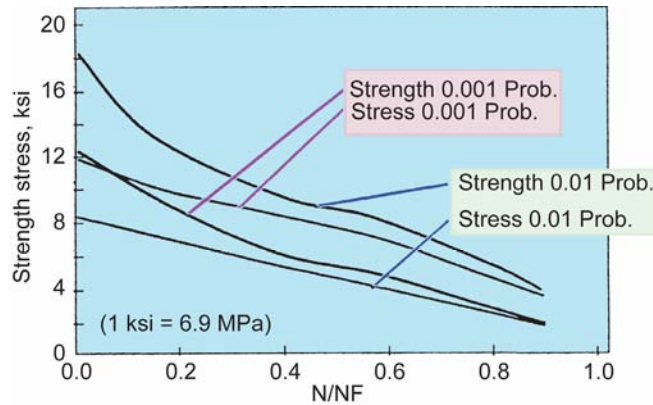
Figure 11.—Probabilistic evanuations of local strength and stress evolution in engine hot section components subjected to high cycle fatique environment.
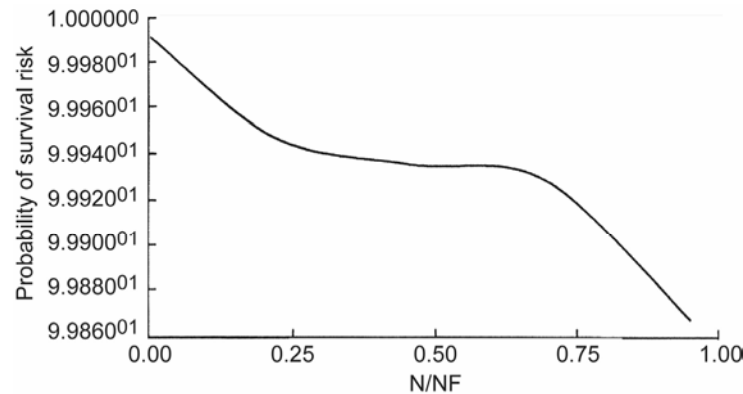

Figure 12.—Survival probability of engine hot section components subjected to high cycle fatigue.

the survival probability scale varies from about 0.9999 down to about 0.9986. This is a relatively small change of about 4/10,000. This small variation is very interesting indeed. It leads to conclusion that ceramic matrix composites operating at high temperatures are not fatigue prone.

7. PROBABILISTIC EVALUATION OF A BUILT-UP COMPOSITE STRUCTURE

The last select example to illustrate the probabilistic evaluation of a multiscale multifunctional composite structure is the built-up structure depicted schematically in Figure 13. Figure 13 illustrates the structural geometry, the finite element mesh, and the laminate configuration. Note that all dimensions are in inches. The results from the two types of analyses are shown in Figure 14. The deterministic results are in the left figure and the probabilistic in the right where the two CDF curves are shown, 1) just before fracture and 2) at fracture (dashed curve). What is interesting in this figure is that the two CDF curves are parallel and that the shape of the energy curve is also almost parallel to the probabilistic curves. The reason for this is that both curves are based on the same constitutive values. The probabilistic sensitivity factors before and after fracture are shown in Figure 15 for two probabilities: 0.001 and 0.999. The dominant sensitivities remain about the same. It is seen in the figure that the pressure dominates followed by the fiber volume ratio, the fiber longitudinal modulus and the ply thickness respectively. The reason that these two sets of probabilistic sensitivities are about the same is that they both represent the same physical system.
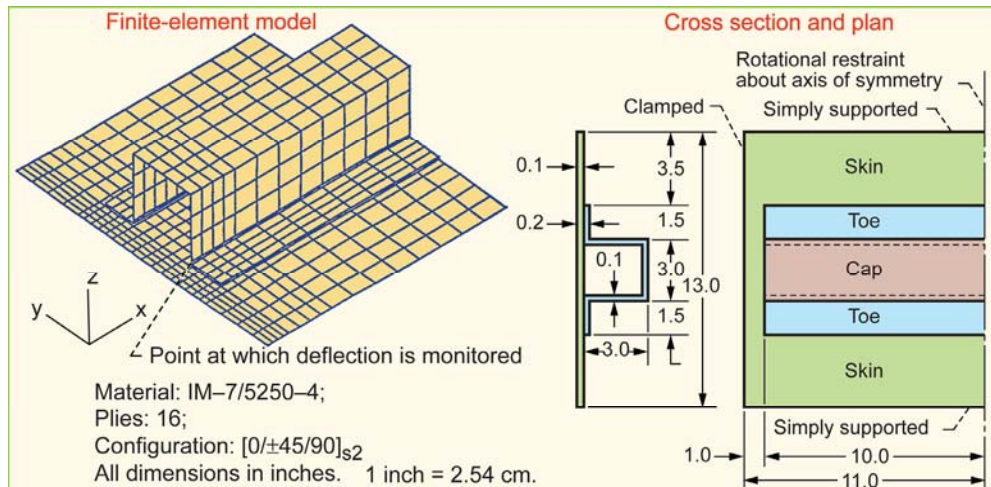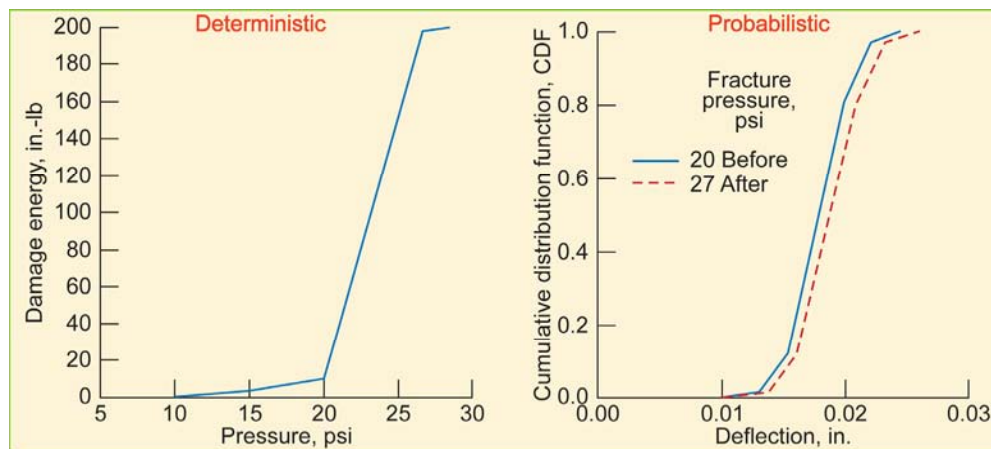
Figure 13.—Stiffened composite panel.

Figure 14.—Stiffened composite panel. Material, IM–7/5250–4; plies, 16; configuration, [0/±45/90]$_{s2}$.

CONCLUDING REMARKS

The results and discussions of probabilistic methods for structural reliability and risk lead to the following concluding remarks: The failure probability in metallic and composite structures can be computationally simulated. Probabilistic evaluation includes or can include some or all factors that influence component/system reliability. One important aspect is that the probability can be evaluated by the cumulative distribution function of the system response. Then the risk can be assessed by the remaining strength at some probability in the cumulative distribution function. Another important aspect of probabilistic evaluations is the probability sensitivities of all the variables that constitute the system design. The sensitivities can be used to fine-tune the design for more weight savings.

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