SIMULTANEOUS STRUCTURAL AND MANUFACTURING OPTIMIZATION OF COMPOSITE PARTS USING NORMAL CONSTRAINT METHOD

Hossein Ghiasi, Damiano Pasini, Larry Lessard

Dept. of Mechanical Engineering, McGill University,
817 Sherbrooke W., Macdonald Eng. Building,
Montreal, QC, Canada H3A 2K6
Hossein.ghiasi@mcgill.ca

ABSTRACT

In composite materials, there is a strong interconnection between structure design and manufacturing design. The optimum design of a composite structure requires simultaneous optimization of structural performance and manufacturing process. Such a challenge calls for a Multi-Objective Optimization approach. A multi-objective optimization method aims at finding one or more non-dominated solutions in the criterion space, called Pareto optimal points. In this paper, a multi-objective optimization approach called normal constraint method is used to find a set of optimal designs for a Z-shaped composite bracket that simultaneously maximize structural and manufacturing performances. The set of solutions found by this approach is compared to the one obtained by a non-dominated sorting genetic algorithm. The result shows that the proposed method is able to find the Pareto frontier with the higher degree of accuracy than the evolutionary algorithm.

1. INTRODUCTION

Because of their high structural potential, laminated fibrous composite materials are successfully used in a wide range of structural applications. However, design with composite materials is more complex than design with metallic materials, because of the large number of design variables and the anisotropic properties of the fibrous materials. The design that objectives can be categorized as structural and manufacturing are more interconnected than in the design with isotropic materials. The strong interconnection is due to the fact that solidification and shaping of a composite part are performed at the same time.

It is observed that the approach to the design and optimization of composite materials is multidisciplinary and requires the simultaneous optimization of more than one objective [1-6]. The process of optimizing a set of objectives systematically and simultaneously is called multi-objective optimization. In contrast to single-objective optimization, a multi-objective problem does not have a unique solution, but a set of solutions that all fit a definition for an optimum, called Pareto optimality [7].

The Pareto optimal frontier consists of all non-dominated points in the criterion space. Multi-objective optimization aims at finding Pareto optimal points. Marler et al. [8] divided multi-objective optimization methods into three major categories. The first includes methods with a priori articulation of preferences, which leads to one point within the Pareto set according to the preference specified by the designer. The second groups methods with a posteriori articulation of preferences that provide a set of Pareto optimal solutions, among which a designer can select the final solution. In the third category, fall the methods with no articulation of preferences, which are similar to the methods in the first category where the user excludes at the onset selected parameters.

Because of their simplicity, methods with a priori articulation of preferences are of common interest in design and optimization of composite structures. However, these methods have the drawback of selecting the preference parameters before solving the problem. The user-defined parameters may include relative weight of objectives [9-12] or limits on the objectives [1, 3, 4, and 6], on which the final solution is strongly dependent.

There are a few works using methods with posteriori articulation of preferences. Among them, are attempts by Mohan Rao [12] and Pelletier [13]. Mohan Rao [12] used variable weighting in a weighted sum approach, and Pelletier [13] employed a non-dominated sorting genetic algorithm. The first method obtains Pareto optimal points, however it does not provide information about the distribution of points on the Pareto frontier. The second method finds a random distribution of Pareto optimal points, but only after a near infinite number of iterations.

In this paper, a multi-objective optimization strategy with a posteriori articulation of preferences called "Normal Constraint Method" [14] is adopted for simultaneous optimization of structural and manufacturing parameters of composite structures. The method is applied to the design of a Z-shaped composite bracket and the results are compared to the one obtained by a non-dominated sorting genetic algorithm.

The multi-objective optimization algorithm is introduced in the next section. The third section explains the composite design problem followed by the numerical results and validation in section four. The concluding remarks are presented in section five.

2. OPTIMIZATION PROCEDURE

The optimization procedure used in this research includes two parts. The first, called Normalized Normal Constraint Method (NNCM), is a multi-objective optimization method introduced by Messac [14]. This algorithm defines a set of optimization problems that must be solved to obtain a set of Pareto optimal points. The second, called Globalized Bounded Nelder-Mead (GBNM) method [15], is a local-global search based on several restarts of a simplex optimization. This algorithm solves the optimization problems defined by NNCM. Figure 1 shows the flowchart of the optimization procedure and the interaction among these two algorithms. Some details are not presented in this chart for the sake of clarity. The reader is referred to [15, 16] for more details.

2.1. Normal Constraint Method

Normal Constraint Method with normalized objective functions and with a Pareto filter, collectively abbreviated as NNCM [14], is an algorithm for generating a set of evenly spaced solutions on a Pareto frontier. It always yields Pareto optimal solutions, and the performance is independent of the scale of the objective function.

In this algorithm, first all the objectives are normalized to a scale of zero to one. Figure 2 shows a criterion space with two objectives, before (a) and after (b) normalization. The minimum value of each objective is found by performing a single-objective optimization. The solutions to these set of single objective optimizations are

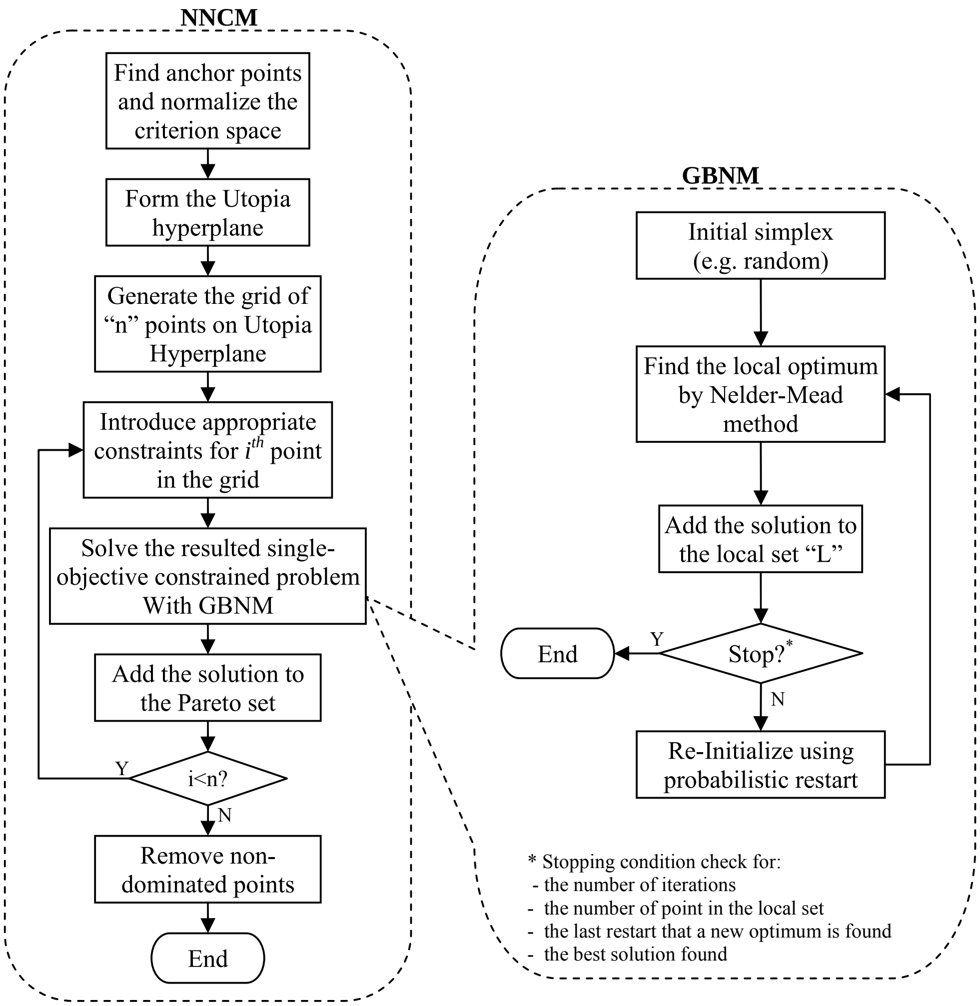


Figure 1. NNCM defines a set of single-objective optimization problems that must be solved by GBNM to provide the Pareto optimal points.

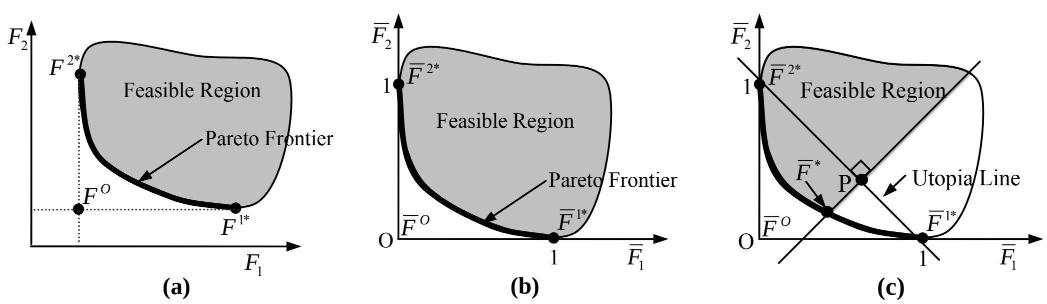


Figure 2. (a) A typical bi-criterion space (b) normalized criterion space (c) a single objective problem (i.e. $\min \overline{F}_2$) generated by normal constraint method.

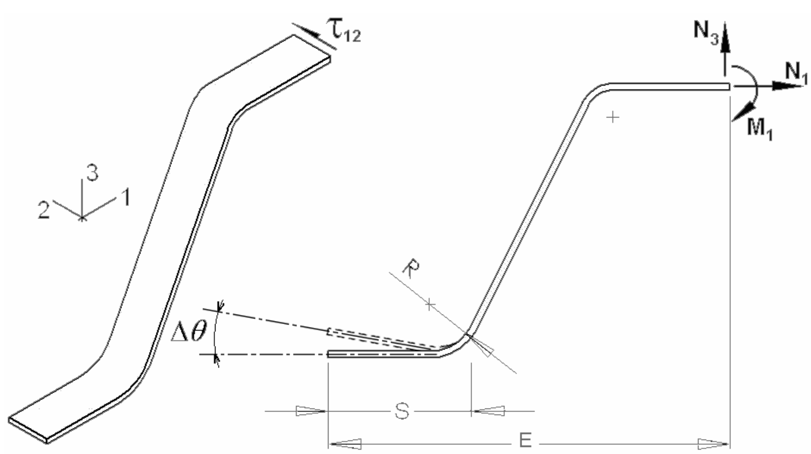
called "anchor points" (e.g. F_1^* and F_2^* in Figure 2). These are used to locate the vertices of the Utopia hyperplane (e.g. utopia line in Figure 2 (c)). Then, a grid of evenly distributed points is generated on the utopia hyperplane and each point is projected onto the Pareto optimal surface by solving a single-objective problem (Figure 2 (c)). Finally non-dominated points are removed by comparing each solution to all others.

2.2. Globalized Nelder-Mead method

To project a point from Utopia hyperplane to Pareto hyperplane, a single-objective optimization problem, shown in Figure 2(c), must be solved. For this purpose, the local-global search introduced by Luersen et al. [15] is used here. The method, called Globalized Bounded Nelder-Mead method (GBNM), finds the global optimum using a probability function that repeatedly restarts the Nelder-Mead from points different than those formerly used as an initial point or found to be a local optimum. In the next section, we apply the two described methods to the design of a composite bracket.

3. COMPOSITE DESIGN PROBLEM

A Z-shaped composite bracket, made of 16-ply balanced symmetric laminate of graphite/epoxy (AS4/8552) with fiber orientation of $[\pm\theta_1/\pm\theta_2/\pm\theta_3/\pm\theta_4]_s$ is considered as a test case (Figure 3). The goal is to find the optimum cross-sectional geometries and lamination sequences that minimize weight and spring-in and maximize strength. The composite bracket must not fail or delaminate under the given loading condition.



N_1=500 N/m
N_3=100 N/m
M_1=50 N.m/m
τ_{12}=300 N/m

Figure 3. Geometrical variables and applied loads on the composite bracket

Delamination is calculated in the curved regions where the angle shape causes high interlaminar normal stresses. The vertical deflection of less than 1*mm* and the spring-in of less than 0.5° are strictly required for an acceptable design. Semi-analytical models of first-ply-failure, delamination, deflection, and spring-in are generated in MATLAB and are used during the optimization process.

3.1 Optimization Set up

The multi-objective optimization method described in section three is used here to solve the composite design problem. The original objective functions are grouped into two sets of manufacturing and structural objectives through a weighted sum approach. The two objectives are then used to find the Pareto frontier.

The first objective deals with the structural performance and aims at minimizing the weight and maximizing the strength. The two objectives are collected into the following function:

$$\min\ F_s = \frac{W(gr)}{5gr} - \frac{R}{1.5} \tag{1}$$

where W is the weight and R is the load factor. The second objective deals with the manufacturing aspect and includes minimization of the spring-in. It is defined as:

$$\min\ F_m = |\Delta\theta| \tag{2}$$

After incorporating a set of inequality constraints described in the problem definition, the optimization problem can be stated as:

$$\begin{aligned} &\min\ \{F_s, F_m\} \\ &S.T.:\quad \{R \geq 1.5\ \wedge\ D \geq 2.0\ \wedge S_r \geq 10mm\ \wedge |\delta| \leq 1mm \wedge\ |\Delta\theta| \leq 0.5^{\circ}\} \end{aligned} \tag{3}$$

Where D is the delamination factor; S_r is the length of the flat area on the shoulder of the bracket after applying the fillet; δ is the maximum vertical deflection; and finally $\Delta\theta$ is the spring-in.

The first step in the optimization process (shown in Figure 1) is to find the anchor points. To this end, two single-objective optimization problems are solved; one optimizes only the structural objective while the manufacturing objective is set free to take any value. The other problem minimizes only the manufacturing objective and the structural one is ignored. We refer to the solution of the first problem as structural-only solution (F^{1*} in Figure 4), and the second problem as manufacturing-only solution (F^{2*} in Figure 4).

The values of the two objectives at the anchor points are used to normalize the objective functions. For the composite design problem in hand, the normalization is performed using the following transformation.

$$\overline{F}_s = \frac{F_s(x) + 0.037}{0.923}; \qquad \overline{F}_m = \frac{F_m(x) - 0.005}{0.144} \tag{4}$$

To find each point on the Pareto frontier, NNCM simplifies the mathematical problem into single-objective minimization problems with an additional nonlinear constraint on the objective functions. The additional nonlinear constraints make the feasible region tighter, and thus yield a solution of the single-objective minimization problem different than the anchor points. Here, the structural objective is minimized, while the manufacturing objective is constrained. The additional constraint restricts the difference between the normalized objectives. Thus the optimization problem is formulated as:

$$\begin{cases} \min\ \overline{F}_s \\ S.T.: \{R \geq 1.5 \wedge D \geq 2.0 \wedge S_r \geq 10mm \wedge |\delta| \leq 1mm \wedge |\Delta\theta| \leq 0.5^\circ \wedge \overline{F}_s - \overline{F}_m + a \geq 0\} \end{cases} \quad (5)$$

where the last term is the additional constraint applied by NNCM. The constant a is set to -0.5, 0, and 0.5 to obtain points A, B, and C in Figures 4 and 5.

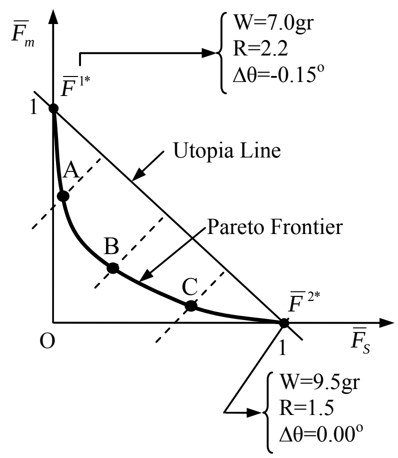


Figure 4. Two anchor points and utopia line for composite bracket design problem. NNCM tends to find points A, B, and C on the Pareto frontier.

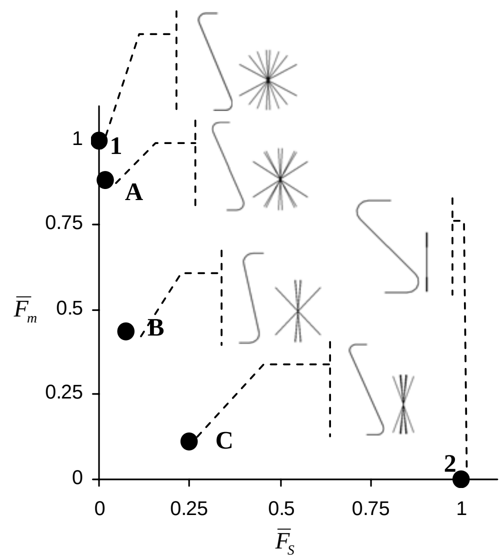


Figure 5. Graphical representation of 2D cross-section and lamination sequences of the Pareto optimal solutions. Crossed-line diagrams represent the fiber orientations.

4. NUMERICAL RESULTS AND DISCUSSION

Fig. 5 shows the results obtained in six trials, each consisting of 2000 function analyses. The corresponding lamination sequences and the 2D cross-section shapes are represented beside each point. Crossed-line diagrams represent the fiber orientations of the bracket. Vertical lines represent the fibers running along the length of the bracket, whereas horizontal ones show the fibers running along the width of the bracket (normal to the cross-section).

The solutions in this figure represent the Pareto frontier in the normalized criterion space (i.e. normalized structural objective $(\overline{F}_S)$ versus normalized manufacturing objective $(\overline{F}_m)$). The Pareto curve helps the designer to best understand the trade-off between the conflicting objectives. For instance, here the results show that starting from the best manufacturing design $(\overline{F}^{2*})$, penalizing only 10% the manufacturing objective, the structural objective is improved more than 75%.

This figure also shows that the design solution can move from the structural-only design (point 1) toward the manufacturing-only design (point 2). Moving from point 1 to 2 has a strong impact on the design: a) the geometry gradually changes from small brackets to elongated ones and b) the fiber orientations change from laminates that fibers are distributed in all directions to the laminates where fibers are mostly aligned at zero degrees. Table 1 summarizes the performance criteria of these solutions. It shows that the structural and manufacturing performances are also changed as the design point moves from best structural design toward the best manufacturing design. All these points are Pareto optimal and the designer can select among them based on a designer-selected criterion or on his experience.

Table 1. The performance criteria at five points located on Pareto frontier.

Point	1	A	B	C	2
Weight (gr)	**7.0**	**7.0**	**6.9**	**7.0**	**9.5**
Deflection (mm)	**0.046**	**0.073**	**0.099**	**0.166**	**0.133**
Load Factor	**2.2**	**2.1**	**2.0**	**1.8**	**1.5**
Spring-in (degree)	**-0.15**	**-0.14**	**-0.07**	**-0.02**	**-0.00**

To assess the efficiency of the approach, the results are compared with the one obtained by Non-dominated Sorting Genetic Algorithm (NSGA) [18]. NSGA was proposed by Srinivas et al. (1995) as a multi-objective optimization approach based on genetic algorithm. NSGA differs from simple genetic algorithm in term of ranking the solutions. In NSGA, individuals are ranked according to the level of non-domination and to each solution assigned a fitness equal to its non-domination level (i.e. 1^{st} level is considered as the best).

NSGA is conducted for the same total number of function analyses as used for NNCM. The optimal solutions found by using the two methods are illustrated in Figure 6. The Pareto frontier obtained by NNCM-GBNM clearly dominates the one obtained by NSGA. This means that the Z-shaped bracket is lighter, stronger, and with reduced spring-in.

One of the advantages of the improved GBNM is that not only returns a global solution, but also provides a set of local optima. Non-dominated points among these local optima can be used to gain an insight into the Pareto frontier. This is why the number of points on the Pareto frontier in Figure 6 is more than the one in Figure 5.

The results presented in this section shows that the proposed method is able to accurately and efficiently find the Pareto frontier for a composite design optimization problem. It confirms that the trade-off between the structural and manufacturing objectives exists and must be considered in the optimum design of composite parts.

5. CONCLUSION

This paper has tackled the multi-objective optimization of composite design to find the Pareto frontier. We have coupled the Normalized Normal Constraint Method (NNCM) to the local-global optimization method of Globalized Bounded Nelder-Mead (GBNM). The proposed approach is applied to the simultaneous structural and manufacturing optimization of a Z-shaped composite bracket. Comparison of the results shows that the

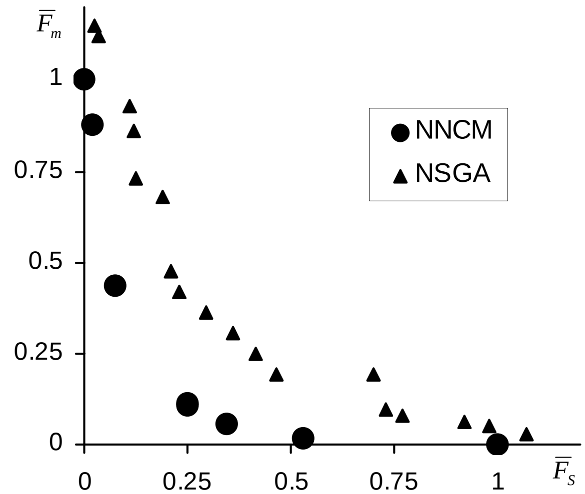


Figure 6 Pareto solutions obtained by NNCM clearly outperform the one obtained by NSGA.

Pareto frontier found by the proposed algorithm clearly dominates the one obtained by Non-dominated Sorting Genetic Algorithm (NSGA). The Pareto frontier helps the designer to find the best trade-off between conflicting objectives. The corresponding geometries, fiber orientations, and performance criteria of the optimal solutions of the composite bracket are plotted, showing the trade-off that exists between structural and manufacturing objectives.

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