A GENERALIZED DAMAGE MODEL FOR WOVEN PLY LAMINATES UNDER STATIC AND FATIGUE LOADS

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ABSTRACT

A generalized non-linear cumulative damage model for woven ply laminates subjected to static and fatigue loading is developed in this paper. The damage, consisting of small cracks running parallel to the fibers, leads to a loss of stiffness in the warp, weft and shear directions. The model presented here describes the evolution of the damage up to failure of the first ply. By replacing the woven ply by two stacked unidirectional plies corresponding to the warp and weft thicknesses, this general model is extended to cover a broad range of plies, from quasi-unidirectional to balanced woven plies. A continuum damage approach (CDM) is then used to define the behaviour of the two virtual unidirectional plies under static and fatigue loading conditions. The model is applied here to an unbalanced woven ply with glass reinforcement and the results of the simulations are compared with experimental data.

1. INTRODUCTION

Woven ply materials are used to produce helicopter blades because they are strong and easy to work. These structures are subjected in the air to both static and fatigue loads. The rupture/failure of composite materials can be due to many processes acting on various scales (Reifsnider[1], Wang[2]). Woven ply laminates show good resistance to delamination and are not sensitive to transverse rupture because of their woven structure (Hochard[3]). Experimental tests have shown that the rupture of the first ply leads quickly to the rupture of the whole laminate and the whole structure, In the case of structures consisting of laminates with woven plies, first ply failure models therefore generally suffice for modelling the rupture processes involved. A model based on a continuum damage approach (CDM) was developed to simulate balanced woven ply laminates [3] under static loading conditions. A preliminary non-linear cumulative damage model for unidirectional (UD) plies undergoing fatigue was presented by Payan[4]. This cumulative damage model was then extended to include balanced woven plies (Hochard[5]). In this model, the development of the damage depends on the maximum static load, the amplitude of the load during a cycle and the level of damage involved.

A generalized non-linear cumulative damage model for woven ply laminates undergoing static and fatigue loads is developed in this paper. The damage, consisting of small cracks running parallel to the fibers, leads to a loss of stiffness in the warp, weft and shear directions. The model presented here describes the evolution of the damage and the inelastic strain occurring in the shear direction up to failure of the first ply. By replacing the woven ply by two stacked unidirectional plies corresponding to the warp and weft thicknesses, this general model is extended to cover a broad range of plies, from quasi-unidirectional to balanced woven plies. A continuum damage approach (CDM) is then used to define the behaviour of the two virtual unidirectional plies under static and fatigue loading conditions. The model is applied here to an unbalanced woven ply with glass reinforcement and the results of the simulations are compared with experimental data.

2. DAMAGE BEHAVIOUR OF WOVEN PLIES

The model presented here is applied to an unbalanced woven ply with glass reinforcement. The unbalanced woven ply shows different behaviour in the warp and weft directions. The reinforcement is a glass five-harness satin in which 83% of the fibres runs in the warp direction and 17% in the weft direction. In order to study the behaviour of this material, tensile tests with discharge were performed on eight-ply laminates (Figure 1). Elastic damage behaviour was observed in the warp direction, and elasto-plastic damage behaviour in the weft and shear directions. The inelastic strains and the loading-unloading hysteresis observed (Figure 1) may be due to the slipping/friction processes occurring between the fibres and the matrix as the result of the damage.

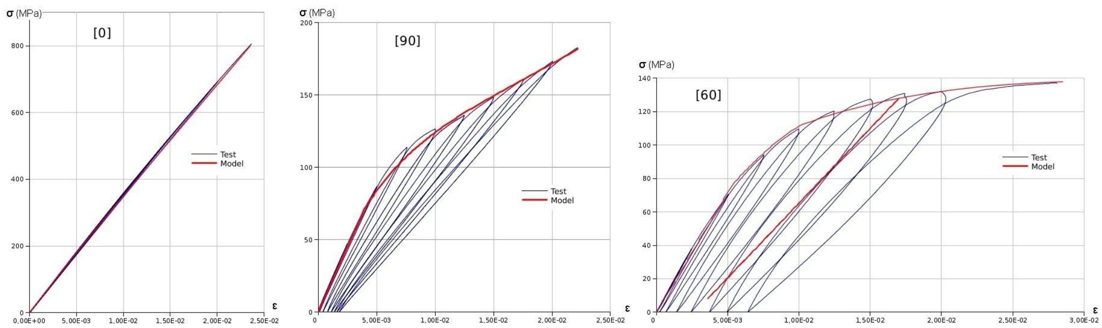

Figure 1: Evolution of the stress depending on the strain in [0°], [90°] and [±60°] laminates

Damage can be characterized by the loss of stiffness of the material. The processes that cause this damage are many and complex (Osada[6], Roy[7]). However, the main processes involved are fibre-matrix debonding, matrix microcracking on the scale of the fibres and small transverse cracks occurring on the scale of the yarns. These small cracks develop uniformly through the ply and are stopped by the woven texture. They do not evolve during the unloading, and during the loading, they continue to develop until saturation point is reached in the ply. The damage was measured on the tensile diagrams after unloading and expressed in terms of the loss of stiffness of the material resulting from the damage (Figure 1). The damage can be described by three variables d_1, d_2 and d_{12} corresponding to the loss of stiffness in the warp and weft directions and in the shear direction, respectively:

$$\begin{aligned} E_1 &= E_1^0\left(1-d_1\right) \\ E_2 &= E_2^0\left(1-d_2\right) \\ E_{12} &= E_{12}^0\left(1-d_{12}\right) \end{aligned} \qquad (1)$$

where E_1^0, E_2^0 and E_{12}^0 are the initial stiffnesses of the material.

Previous models describing the mechanical behaviour of laminated composites up to first failure have proved to be valid in the case of various types of materials such as unidirectional plies (Ladevèze[8]) and balanced woven plies (Hochard[4]). These models on the ply scale (the meso-scale) are based on thermodynamic expressions, where the internal variables, d_1, d_2 and d_{12} defined above are associated with the decrease in the stiffness. The plies are taken to be homogeneous and orthotropic and the damage to be constant throughout the thickness of the ply.

3. GENERALIZATION OF THE MODEL

In order to obtain a more general model which is applicable to a large class of plies ranging from quasi-unidirectional to balanced woven plies, we adopt the following assumptions: an unbalanced woven ply behaves like a $[0_\delta/90_{1-\delta}]$ laminate consisting of two virtual unidirectional plies, where the ratio between the warp and weft thicknesses is δ. The two virtual unidirectional plies constituting the $[0_\delta/90_{1-\delta}]$ laminate are defined as virtual (and they are denoted by UD*) because they do not really exist although they are similar. The classical laminate theory leads to:

$$\underline{\varepsilon} = \underline{\varepsilon}^{\text{virtual laminate}}$$
$$\underline{\underline{S}} = \underline{\underline{S}}^{\text{virtual laminate}} = \left(\delta\ \underline{\underline{Q}}_{0°}^{UD*} + (1-\delta)\ \underline{\underline{Q}}_{90°}^{UD*}\right)^{-1} \quad (2)$$

Assuming that no progressive damage occurs in the fibres direction in the UD* plies (brittle behaviour) and taking $\nu_{12}^{UD*}\nu_{21}^{UD*} << 1$, the equivalence between the unbalanced woven ply and UD* plies can be written as follows:

$$\begin{cases} d_1 = \dfrac{(1-\delta)\ d_2^{*90°} E_2^{0*}}{E_1^0} \\ d_2 = \dfrac{\delta\ d_2^{*0°} E_2^{0*}}{E_2^0} \\ d_{12} = \delta\ d_{12}^{*0°} + (1-\delta)\ d_{12}^{*90°} \end{cases} \quad (3)$$

where $d_2^{*0°}, d_{12}^{*0°}, d_2^{*90°}, d_{12}^{*90°}$ are the transverse damage and the shear damage in the unidirectional virtual plies oriented at angles of 0° and 90°, respectively, and E_2^{0*} is the initial transverse stiffness of the two UD* plies.

Assuming the existence of plane stresses and small perturbations, the strain energy of each UD* ply is written classically [8] in terms of the stresses as follows:

$$E_D^{ps} = \frac{1}{2}\left[\frac{\sigma_1^{\,2}}{E_1^0(1-d_1)} + \frac{\langle\sigma_2\rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle\sigma_2\rangle_-^2}{E_2^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\sigma_{12}^2}{E_{12}^0(1-d_{12})}\right] \quad (4)$$

where < . >$_+$ is the positive part and < . >$_-$ is the negative part. The tension energy and compression energy are split in order to describe the unilateral feature due to the opening and closing of the micro-defects in the transverse direction of the UD* plies. From this potential, thermodynamic forces associated with the tension and internal shear variables d_i (i=1, 2 and 12) are defined in each UD* ply:

$$Yd_1^{UD*} = \frac{\partial E_D^{ps}}{\partial d_1} = \frac{\langle\sigma_1\rangle_+^2}{2E_1^0\left(1-d_1\right)^2};\quad Yd_2^{UD*} = \frac{\partial E_D^{ps}}{\partial d_2} = \frac{\langle\sigma_2\rangle_+^2}{2E_2^0\left(1-d_2\right)^2};\quad Yd_{12}^{UD*} = \frac{\partial E_D^{ps}}{\partial d_{12}} = \frac{\sigma_{12}^{\,2}}{2E_{12}^0\left(1-d_{12}\right)^2} \quad (5)$$

In order to account for the coupling between the transverse traction and the shear during the development of the damage, we define an equivalent thermodynamic force and the maximum value of this force during the history of the loading in each UD* ply:

$$Y_{eq}(t) = \max_{\tau\le t}\left(a_s(Yd_2^{UD*})^l + b_s(Yd_{12}^{UD*})^m\right) \quad (6)$$

where a_s and b_s are the tension/shear coupling coefficients. Generally ([8], [3]), the power coefficients l and m are taken to be equal to 1/2, which gives an equivalent force which is proportional to the elastic strain. This value fits the experimental data satisfactorily (see next section).

The development of the internal variables depends on these equivalent thermodynamic forces. When traction is applied, the d_1 develops sharply because of the brittle behaviour of the fibres in the UD* plies. The evolutionary laws governing the internal variables d_2 and d_{12} in each UD* ply, which depend on the associated equivalent forces (6), still remain to be defined.

4. BEHAVIOUR UNDER STATIC LOADS

4.1 Damage evolution

The following classical law based on statistical considerations (Forquin[9]) describes the evolution of the damage (Figure 2):

$$\begin{cases} d_2^{ud} = \left\langle 1 - e^{-(Y_{eq} - Y_0)} \right\rangle_+ \\ d_{12}^{ud} = c\ d_2^{ud} \end{cases} \qquad (7)$$

where c and Y_0 are UD* ply coefficients and Y_0 corresponds to the threshold load. The shear damage is taken to be proportional to the transverse damage and c is generally equal to 1. The evolution of the transverse and shear damage are compared with the results of the identifications in Figure 2. Details of the identification procedure are given in the next section.

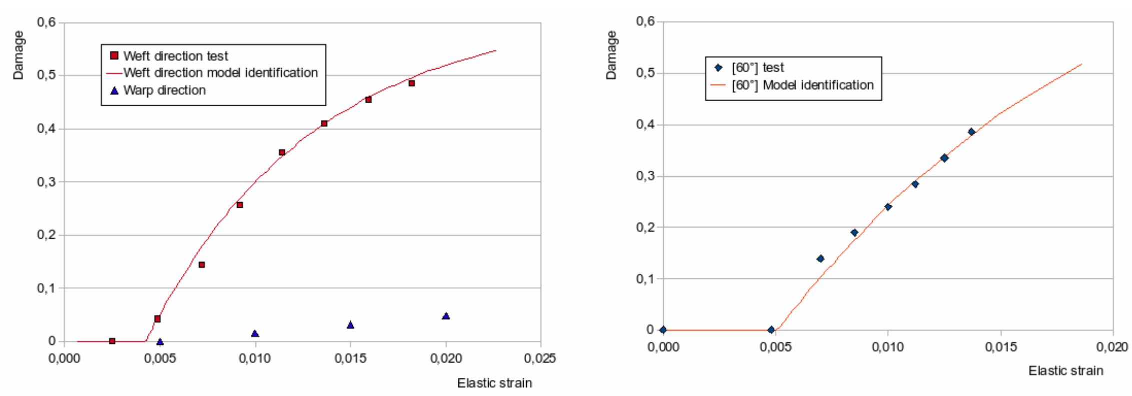

Figure 2: Evolution of the damage depending on the elastic strain.

4.2 Inelastic strain

Because of the fibres running in the warp and weft directions, only the inelastic strain exerted in the shear direction is significant. The inelastic strains occurring under shear loading conditions are described by a kinematic linear hardening law. The coupling between the damage and the plasticity is accounted for using the effective stress and the effective strain [8], which are defined as:

$$\tilde{\sigma}_{12}\dot{\tilde{\varepsilon}}_{12}^{p} = \sigma_{12}\dot{\varepsilon}_{12}^{p} \qquad where \quad \tilde{\sigma}_{12} = \frac{\sigma_{12}}{(1-d_{12})} \quad and \quad \dot{\tilde{\varepsilon}}_{12}^{p} = \dot{\varepsilon}_{12}^{p}(1-d_{12}) \qquad (8)$$

It is assumed that the stresses σ_1 and σ_2 do not influence the elastic field defined by:

$$f = \left|\tilde{\sigma}_{12} - C_0\tilde{\varepsilon}_{12}^{p}\right| - R_0 \qquad (9)$$

where R_0 denotes the initial threshold inelastic strain and C_0 denotes the linear law coefficient.

4.3 Types of rupture

During the tests, three types of rupture occur:

(i) Brittle rupture defined by a maximum thermodynamic force criterion: $Yd_1^{UD*} \leq Yd_1^{\max}$

This brittle rupture criterion resembles a maximum fibre strain criterion.

(ii) Rupture caused by instability due to the localisation of the damage. This instability corresponds to the maximum load observed on the curve describing the load with respect to the displacement.

(iii) Rupture associated with a maximum level of damage, depending on the material (damage equal to 1 in the UD*, in the case of the unbalanced glass/epoxy woven ply studied here).

5. IDENTIFICATION AND VALIDATION OF THE MODEL IN STATIC

The model presented here was implemented by MatLab in order to simulate the behaviour of the laminate. Various tests were carried out to assess and check the validity of the model. The coefficients used in the model were determined by performing standard industrial laboratory tests. Three tensile tests with discharge sufficed to identify these coefficients. The laminates used for this purpose were: [0], [90] and [60], which were all composed of 8 glass/epoxy unbalanced woven plies. These three tests were used to measure the elastic properties of the woven ply and eq. (2) was used to calculate the elastic properties of the virtual UD ply. The coefficients of the damage evolution law (eqs. (6) and (7)) governing the UD* plies were identified using the damage evolution measured in the [90°] and [60°] laminates and eq. (3). The coefficients of the inelastic strain law (eq. (9)) were determined using the [60°] laminate test.

The results of the identification tests are presented in Figure 1, which gives the stress/strain curves obtained in the three tests and Figure 2, which gives the evolution of the damage.

Experimental data obtained on out off-axis tests were compared with the results obtained with the model (Figure 3). The out off-axis tests gave rise to combined traction and shear loads. A good correlation was found to exist between the experimental data and the results of the simulations with various types of laminate, which confirmed the validity of the model. A good description of the non linear behaviour is necessary to be able to perform the finite element structure computations (Hochard[5,10]).

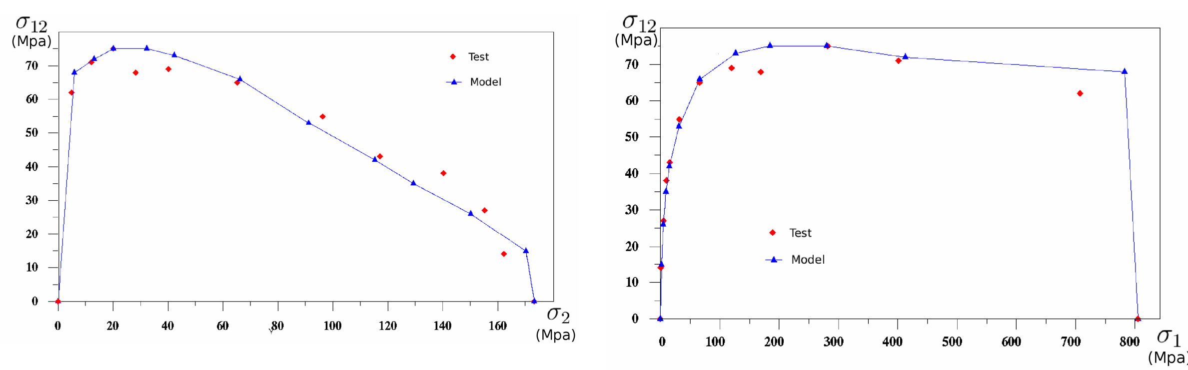

Figure 3. Validation of the model under combined static warp or weft traction/shear loads

6. BEHAVIOUR UNDER FATIGUE LOADS

The model for unidirectionnal plies previously presented in [4] is a non-linear cumulative damage model, which can be used to describe the development of damage under both static and fatigue loading conditions. With this law, the evolution of the damage depends only on the maximum load occurring during the cycle. This model was extended to balanced woven plies by taking into account the effects of the amplitude of the cyclic load on the evolution of the damage [5]. As in the static case, the validity of this model depends on the ‘diffuse damage’ phase (which is associated with small cracks), but only up to the first intra-laminar macro-crack (first ply failure model).

In the generalized damage model proposed here, as under static loads, the unbalanced woven ply is replaced by two stacked unidirectional plies corresponding to the warp and weft thicknesses. The behavior of the material subjected to fatigue loads is then defined, taking each of the UD virtual plies separately.

6.1. Damage evolution

The material is assumed to be brittle and non-sensitive to the cyclic loading in the direction of the fibres in the UD* plies. The in-plane transverse and shear moduli are modified under the assumption that a gradual damage process is involved. The development of the damage in the UD* plies depends on the maximum static and cyclic loads and their amplitude as well as on the level of damage involved. Furthermore, we define the cumulative damage as follows, where the damage variables (under shear and transverse tension loading) are obtained by adding two terms: the one part is due to the static loading and the other one is governed by fatigue loading:

$$d_2^{ud} = d_s + d_f \tag{10}$$

The static damage d_s evolution defined previously (equation (7)) depends on the maximum value of the equivalent force (6) reached during the history of the loading. Experiments have shown that the fatigue damage d_f evolution depends on both the maximum load Yd_i^{UD*} and the amplitude of the load $\Delta Y_{d_i}^{UD*}$ during a cycle. Accordingly, the following law was drawn up:

$$\frac{\partial d_f}{\partial N} = \left\langle a_f.(Yd_2^{UD*})^o.(\Delta Yd_2^{UD*})^p + b_f.(Yd_{12}^{UD*})^q.(\Delta d_{12}^{UD*})^r - Y_0^f \right\rangle_+$$

$$\text{where } \Delta Y_{d_i}^{UD*} = \frac{\left(\max_{\tau \in cycle(t)} (\sigma_i^{UD*}) - \min_{\tau \in cycle(t)} (\sigma_i^{UD*}) \right)^2}{2E_i^{ud}\left(1-d_i^{ud}\right)^2} \quad \text{for } i=2 \text{ and } 12 \tag{11}$$

where the parameter Y_o^f corresponds to the endurance threshold, i.e., the loading level below which no fatigue damage develops. An initial identification of the coefficients involved in the law is shown in the next section using [0°], [90°] and [45°] laminates. The shear damage is again assumed to be equal to the transverse damage (eq. (7)).

In this initial law, it is assumed that in the case of combined traction/shear loads, the contributions of each of the loads are added together (eq. 11). We are currently testing tubes in order to elucidate the influence of combined traction/shear loads on the evolution of damage under fatigue conditions.

6.2 Inelastic strain

During the fatigue loading process, residual strains can be observed, especially with shear loads. The inelastic evolution law defined previously for static loads (eqs. (8) and (9)) depends on the effective stresses. Since the fatigue damage increases during the cycles, the total damage increases, and so do the effective stresses. Using the same law (8) and (9) under fatigue loading conditions, the simulated inelastic strain evolution was in good agreement with the experiment evolution in the case of balanced carbon/epoxy woven plies (see [6]).

6.3 Types of rupture

The evolution of the damage occurring in [45°] laminates under various fatigue loads is shown in Figure 4. The first cycle of fatigue loading corresponds to static loading, and the damage evolution law (7) is used at this stage. The damage then increases slowly at each cycle until rupture occurs (Figure 4). As in the case of static loading, three types of rupture occur:

(i) Brittle rupture defined by a maximum thermodynamic force criterion: $Yd_1^{UD*} \leq Yd_1^{\max}$

(ii) Rupture by instability due to the localisation of the damage.

(iii) Rupture associated with a maximum level of damage, depending on the material.

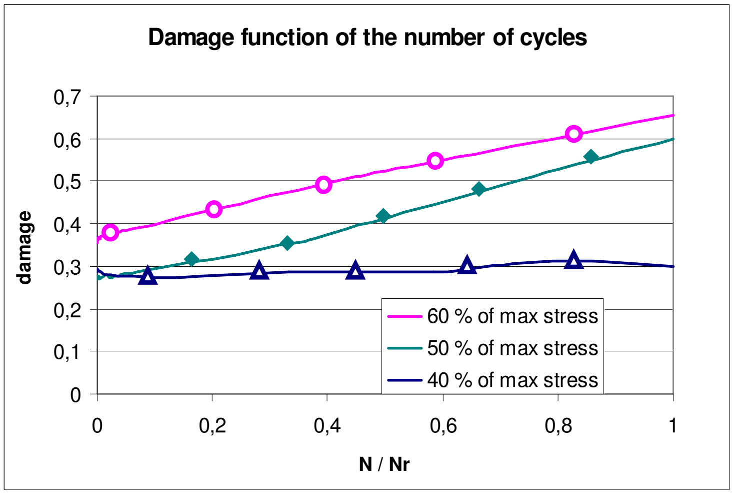

Figure 4. Damage depending on the number of cycles in [45°] unbalanced woven ply

The third type of rupture was confirmed by the experimental tests on [90°] laminates under various stress ratio fatigue loads (Figure 5). The level of damage was the same as before rupture, and corresponded to a very high level (amounting to almost 1) in the UD*.

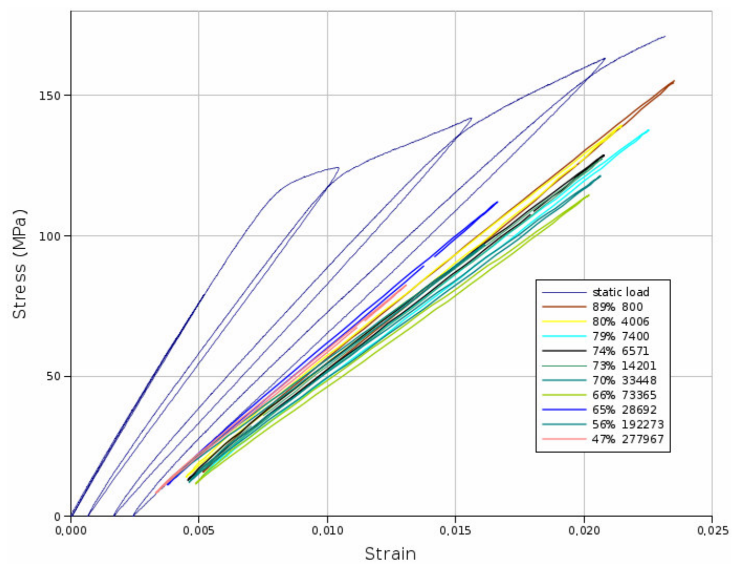

Figure 5. Decrease in stiffness occurring just before rupture in [90°] laminates

7. FIRST IDENTIFICATION OF THE MODEL FOR FATIGUE LOADING

The experimental fatigue loading tests on [0°] and [90°] laminates yielded similar results to those presented above (Figure 6). The S-N curves were similar and the influence of the amplitude of the load and the *R* ratio ($R = \sigma_{min} / \sigma_{max}$) was small. The endurance limit was found to be approximately 30% of the static load. The experimental and simulated results were compared after identifying the coefficients of the law (11) in the case of the [0°] and [90°] laminates. These results showed satisfactory agreement in view of the current state of development of the identification procedure (the *p* coefficient (eq. 11) is taken to be equal to 0, which leads to cancelling the influence of the load amplitude in the transverse direction of the UD* plies).

The results of the experimental fatigue load tests on [45°] are shown in Figure 7. The influence of the R ratio was found to be much greater with this laminate, where the level of shear stress is an important factor. As previously observed in the case of balanced carbon/epoxy plies [5], the influence of the R ratio is the predominant factor in shear loading contexts. The endurance limit amounts to around 40% of the static load with positive R ratios. The reason why this value is higher than that obtained with the [0°] and [90°] laminates, is that the slipping/friction processes occuring under shear loading conditions may can stop the propagation of small cracks. The experimental and simulated data were compared after identifying the coefficients of the law (11) in the case of the [45°] laminates (Figure 7). These data show satisfactory agreement and the influence of the R ratio was correctly taken into account.

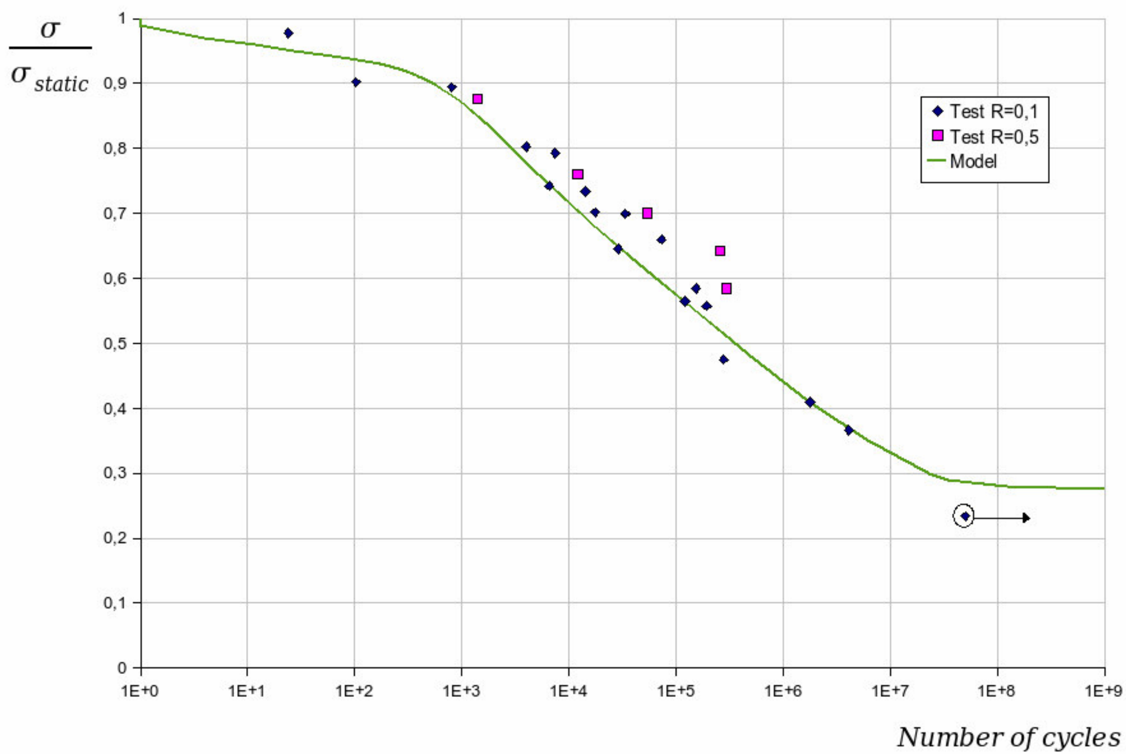

Figure 6. Normalized SN curve obtained on [90°] unbalanced glass/epoxy woven ply.

8. CONCLUSION

The cumulative damage model presented here accurately describes the loss of stiffness and the inelastic strain up to rupture of woven ply laminates subjected to static and fatigue loading. Thanks to the assumption consisting in replacing the woven ply by two stacked unidirectional virtual plies, this generalized model can be used to simulate the mechanical behaviour of various unbalanced woven plies, from quasi-unidirectional to balanced woven plies.

Rupture was defined here in terms of either the maximum force associated with damage (which is similar to a maximum strain criterion), the instability of the laminate response (especially in the case of shear loading) or the maximum level of damage (which was equal to 1 in the virtual UD plies in the case of the unbalanced glass/epoxy woven ply studied here). This model takes into account the contribution of the coupling between traction and shear stresses to the evolution of the damage.

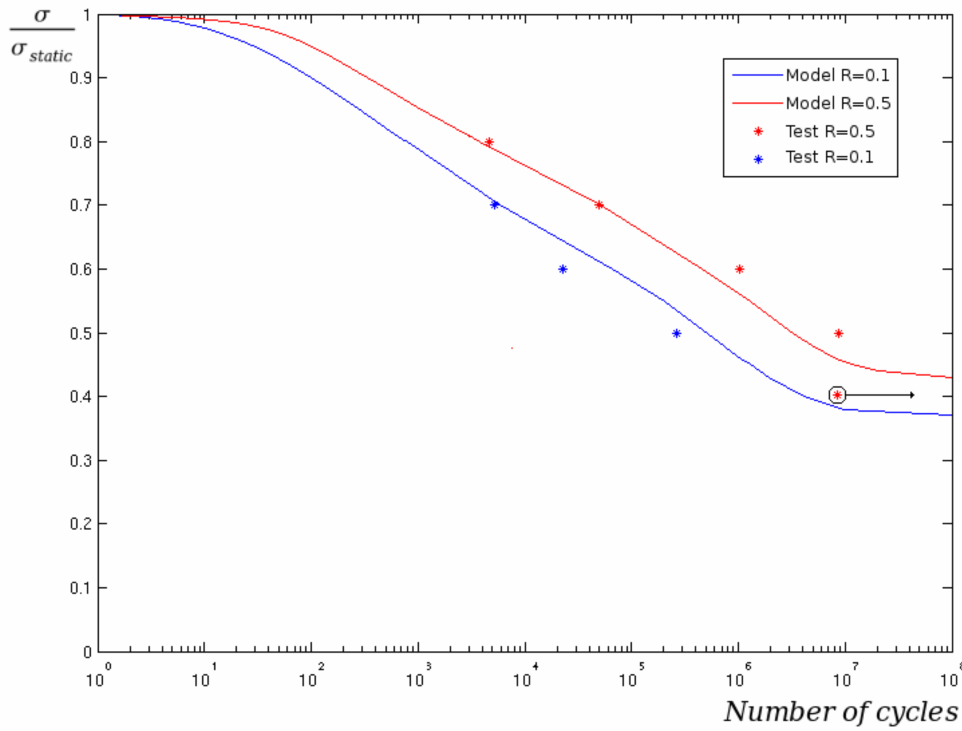

Figure 7. Normalized SN curve obtained on [45°] in the case of two *R* ratios

The cumulated damage evolution laws developed here make it possible to describe the evolution of the damage under static loading conditions (during the first cycle of the fatigue loading for example) as well as under fatigue loading conditions. In the latter case, the evolution depends on the maximum load exerted during the fatigue cycle and the amplitude of the fatigue load, especially in the case of shear stresses.

This approach, which lends itself to structural computations [5] and involves Characteristic Fracture Volumes (Hochard[10]), has been implemented in the Abaqus FEM code. Other extensions it is proposed to develop will involve the behaviour of these materials under compression loads and the influence of the temperature on the evolution of the damage.

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