BUCKLING OF COMPOSITE PLATES SUBJECTED TO SHEAR AND LINERLY VARRYING LOADS

Gabriella Tarján and László P. Kollár

Dept. of Mechanics, Materials and Structures,
Budapest University of Technology and Economics,
H-1521 Budapest, Műegyetem rpt. 3. Hungary
lkollar@eik.bme.hu

ABSTRACT

In this paper explicit expressions are developed for the calculation of the buckling loads of long, rectangular composite plates with orthotropic layup. These results can be used for the local buckling analysis of thin-walled fiber reinforced plastic beams. For *linearly varying axial load* the known results for hinged supports are corrected and new results are presented for built-in and constrained edges; for *shear load* new results are presented for constrained edges. The results are based on the Rayleigh-Ritz method.

1. INTRODUCTION

Thin-walled beams and columns may fail in three ways: (i) the stress reaches the strength of the material, (ii) global buckling, or (iii) local buckling of the walls.

When the beam is made of composite materials local buckling is a mayor consideration in design, because its stiffness/strength ratio is low, compared to conventional materials, such as steel or alumina.

Critical load due to local buckling of thin walled beams can be calculated by modeling the wall segments as long plates and by assuming that the edges common to two or more plates remain straight [1-8]. A conservative estimate of the local buckling load can be obtained if the restraining effect is neglected and the long edges (common to two or more plates) are assumed to be simply supported. Buckling loads of simply supported long plates subjected to axial load, linearly varying load (bending) and shear load (Figure 1) are available in the literature [9, 3, 4] and are given in Table 1 (rows: 1, 5, 6 and 8).

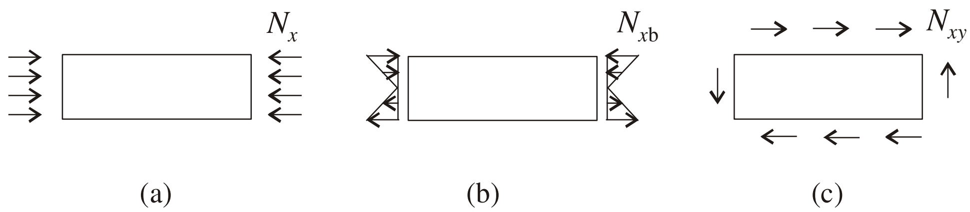


Figure 1. Uniform compression (a), linearly varying compression (b) and shear (c) acting on a plate

Note, however that the rotation of the long edges is restrained by the adjacent wall segments, and it has a significant effect on the buckling load. Buckling load of plates with restrained edges are available in the literature only for a few cases (see the next section), and as a consequence it cannot be applied for the local buckling analysis of thin walled beams subjected to transverse loads.

In this paper those cases of plates with restrained edges will be treated, which are necessary for the analysis of local buckling.

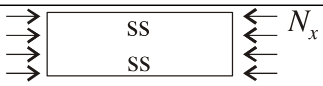
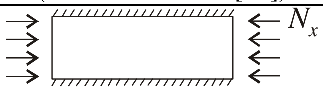
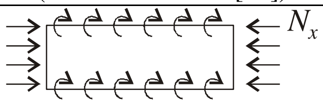
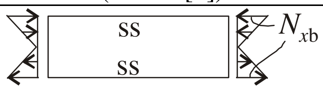
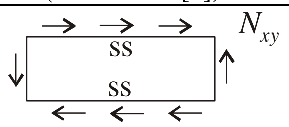
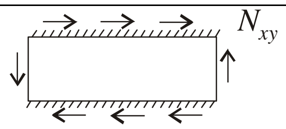
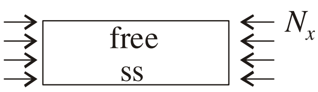
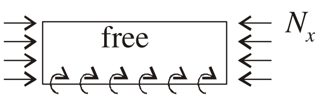
	Supports and loading	Buckling load	
1	 (Lekhnitskii [9])	$N_{x,\mathrm{cr}} = \frac{\pi^2}{L_y^2}\left[2\sqrt{D_{11}D_{22}} + 2(D_{12}+2D_{66})\right]$	
2	 (Veres and Kollár [10])	$N_{x,\mathrm{cr}} = \frac{\pi^2}{L_y^2}\left[3.125\sqrt{D_{11}D_{22}} + 2.33(D_{12}+2D_{66})\right]$	
3	 (Veres and Kollár [10])	$N_{x,\mathrm{cr}} = \frac{\pi^2}{L_y^2}\left[4.53\sqrt{D_{11}D_{22}} + 2.62(D_{12}+2D_{66})\right]$	
4	 (Kollár [3])	$N_{x,\mathrm{cr}} = \frac{\pi^2}{L_y^2}\left[2\sqrt{1+4.139\xi}\sqrt{D_{11}D_{22}} + \left(2+0.62\xi^2\right)(D_{12}+2D_{66})\right]$	
5	 (Lekhnitskii [9])	$N_{xb,\mathrm{cr}} = \frac{\pi^2}{L_y^2}\left[13.4\sqrt{D_{11}D_{22}} + 10.4(D_{12}+2D_{66})\right]$	
6	 (Seydel [11], explicit expression in [4])	$N_{xy,\mathrm{cr}} = \frac{4}{L_y^2}\sqrt[4]{D_{11}D_{22}^3}\,(8.125+5.045K)$	$K \le 1$
		$N_{xy,\mathrm{cr}} = \frac{4}{L_y^2}\sqrt{D_{22}(D_{12}+2D_{66})}\left(11.7+\frac{1.46}{K^2}\right)$	$1 \le K$
7	 (Seydel [11], explicit expression in [4])	$N_{xy,\mathrm{cr}} = \frac{4}{L_y^2}\sqrt[4]{D_{11}D_{22}^3}\,(15.07+7.08K)$	$K \le 1$
		$N_{xy,\mathrm{cr}} = \frac{4}{L_y^2}\sqrt{D_{22}(D_{12}+2D_{66})}\left(18.59+\frac{3.56}{K^2}\right)$	$1 \le K$
8	 (Barbero [12])	$N_{x,\mathrm{cr}} = 12\frac{D_{66}}{L_y^2}$	
9	 (Kollár [13])	$N_{x,\mathrm{cr}} = \frac{7}{\sqrt{1+4.12\zeta}}\frac{\sqrt{D_{11}D_{22}}}{L_y^2} + 12\frac{D_{66}}{L_y^2}$	

Table 1. Buckling loads of long plates available in the literature. $\xi = 1/(1+10\zeta)$ and $\zeta = D_{22}/(kL_y)$ for plates rotationally restrained by springs, $\xi = 1/(1+0.61\zeta^{1.2})$ and $\zeta = D_{22}L_y/GI_{12}$ for plates rotationally restrained by stiffeners, $K = (2D_{66}+D_{12})/\sqrt{D_{11}D_{22}}$. (Note that for case 5 Lekhnitskii derived the constants: 13. 9 and 11.1)

2. LONG PLATES WITH RESTRAINED EDGES

Depending on the configuration of the restraining wall, two kinds of restraining effects must be considered [3]: either both edges are attached to adjacent walls or one of the long edges is free (Figure 2). In the first case the restraining effect is equivalent to that of a rotational spring:

$$M_y = (\pm) k \frac{\partial w}{\partial y}, \tag{1}$$

while in the second case it is equivalent to the effect of a torsional stiffener [14]:

$$M_y = (\pm) GI_t \frac{\partial^3 w}{\partial x \partial y^2}, \tag{2}$$

where k is the rotational spring stiffness, GI_t is the rotational stiffness of the stiffener, M_y is the restraining moment, w is the deflection, x is the longitudinal coordinate, and y is the coordinate perpendicular to the long edges.

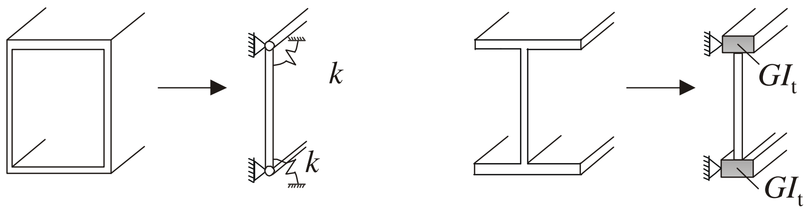


Figure 2. Web with restrained edges. The restraining wall segment (a) may have two edges attached to adjacent walls or (b) has one free edge

Buckling loads of long plates with restrained edges subjected to *axial load*, are also available in the literature [3], and summarized in the fourth and ninth row of Table 1. (These were applied for the local buckling analysis of columns subjected to axial load [1, 5, 6]). Note, however that no solutions are available for long plates subjected to bending or shear. These cases are important for calculating the web buckling of beams subjected to bending or transverse loads, and hence it will be treated in this paper.

3. PROBLEM STATEMENT

We consider rectangular composite plates (Figure 3), where one of the sizes is significantly longer than the other ($L_x >> L_y$). The layup of each wall segment is orthotropic and symmetrical with respect to its midplane.

The rotation of the two long edges are restrained by either rotational springs or stiffeners (see Section 2).

The plate may be subjected to axial compression, linearly varying load or shear as shown in Figure 1. We wish to determine the buckling load of the composite plates.

It is assumed that the material behaves in a linearly elastic manner and the deformations are small.

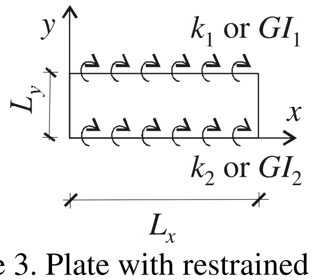


Figure 3. Plate with restrained edges

4. SOLUTION BY THE RAYLEIGH-RITZ METHOD

Buckling load of orthotropic plates can be calculated e.g. by the Rayleigh-Ritz method. For *simply supported* plates solution is given by Whitney [15], which is also presented in [4]. The displacements are assumed in the form of a trigonometrical series:

$$w = \sum_{i=1}^{I} \sum_{j=1}^{J} w_{ij} \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y}, \tag{3}$$

which satisfies the boundary conditions. Here L_x and L_y are the sizes of the plate and I and J are the number of terms. The details of the solution – for simply supported edges, uniform compression and uniform shear – are given in [4], pages 112-114.

This procedure was applied for the presented problem, with the following modifications: The strain energy contains terms due to the rotation of the long edges taking into account either the k spring constant or the GI_t torsional stiffness. In calculating the work of the external loads in addition to the shear and uniform compression a linearly varying axial force was taken into account (Figure 1b).

The linearly varying axial load is given by the following expression:

$$N_{xb} = -1 + \frac{2}{L_y} y\,. \tag{4}$$

The displacement is assumed to be in the form of Eq.(3). The w_{ij} constants are calculated by the Rayleigh-Ritz method from the principle of stationary potential energy:

$$\frac{\partial \pi_{\mathrm{P}}}{\partial w_{ij}} = \frac{\partial (U + \Omega)}{\partial w_{ij}} = 0\,, \tag{5}$$

where the strain energy is [4]:

$$U = \frac{1}{2} \int_0^{L_x} \int_0^{L_y} \left[D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + D_{66} \left(\frac{2\partial^2 w}{\partial x \partial y} \right)^2 + D_{12}\, 2 \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] dy dx + U_R \,. \tag{6}$$

When the long edges of the plate are rotationally restrained by springs U_R is:

$$U_R = \frac{1}{2} \int_0^{L_x} \left[k_1 \left(\left. \frac{\partial w}{\partial y} \right|_{y=0} \right)^2 + k_2 \left(\left. \frac{\partial w}{\partial y} \right|_{y=L_y} \right)^2 \right] dx\,, \tag{7}$$

while in the case of torsional stiffeners:

$$U_R = \frac{1}{2} \int_0^{L_x} \left[GI_{t1} \left(\left. \frac{\partial^3 w}{\partial x \partial y^2} \right|_{y=0} \right) \left(\left. \frac{\partial w}{\partial x} \right|_{y=0} \right) + GI_{t2} \left(\left. \frac{\partial^3 w}{\partial x \partial y^2} \right|_{y=L_y} \right) \left(\left. \frac{\partial w}{\partial x} \right|_{y=L_y} \right) \right] dx\,. \tag{8}$$

The potential of the external forces λN_x and λN_{xy} are given in [4], while for λN_{xb} it is:

$$\Omega = \frac{1}{2}\int_0^{L_x}\int_0^{L_y}\left[-\lambda N_{xb}\left(\frac{\partial w}{\partial y}\right)^2\right]dydx\,. \tag{9}$$

Substituting the displacement function, w (Eq.3) and the external loads, N_{xb}, N_x, N_{xy}, into the expression of the potential energy (Eqs. 5-8) the stationary condition (Eq.5) results in the following system of equations:

$$\sum_{k=1}^{I\times J} G_{kl}w_k = \lambda\sum_{k=1}^{I\times J} b_{kl}w_k\,, \qquad l = 1,2,3...I\times J\,, \tag{10}$$

where

$$\begin{aligned} k &= (i-1)J + j, && \begin{Bmatrix} i = 1,2,3...I \\ j = 1,2,3...J \end{Bmatrix}, \\ l &= (m-1)J + n, && \begin{Bmatrix} m = 1,2,3...I \\ n = 1,2,3...J \end{Bmatrix}. \end{aligned} \tag{11}$$

and G_{kl} and b_{kl} are given below:

$$\begin{aligned} G_{kl} &= \frac{1}{4}L_xL_y\pi^4\left[D_{11}\left(\frac{i}{L_x}\right)^4 + 2(D_{12}+D_{66})\left(\frac{i}{L_x}\right)^2\left(\frac{i}{L_x}\right)^2 + D_{22}\left(\frac{i}{L_y}\right)^4\right]\delta_{lk} + \\ &+ \frac{1}{2}L_x\pi^2\left[k_1\frac{jn}{L_y^2} + k_2(-1)^j(-1)^n\frac{jn}{L_y^2}\right]\delta_{mi} + \\ &\frac{1}{2}L_x\pi^4\left[GI_1\frac{ijmn}{L_x^2L_y^2} + GI_2(-1)^j(-1)^n\frac{ijmn}{L_x^2L_y^2}\right]\delta_{mi} \end{aligned} \tag{12}$$

$$b_{kl} = \frac{1}{4}L_xL_y\pi^2\left[\left(\frac{i}{L_x}\right)^2 + \frac{im}{L_x^2}\right]\delta_{lk} + \frac{1}{2}L_xL_y\frac{im}{L_x^2}f_{jn}\,, \tag{13}$$

with

$$\delta_{lk} = \begin{Bmatrix} 1 & \text{if} & k = l \\ 0 & \text{if} & k \neq l \end{Bmatrix}, \qquad \delta_{mi} = \begin{Bmatrix} 1 & \text{if} & m = i \\ 0 & \text{if} & m \neq i \end{Bmatrix}, \tag{14}$$

and

$$f_{jn} = \frac{(-1)^{(j-n)}-1}{(j-n)^2} - \frac{(-1)^{(j+n)}-1}{(j+n)^2}. \tag{15}$$

The lowest eigenvalue, λ of Eq.(10) results in the critical load parameter, and the eigenvector, w_k gives the bucking shape of the plate (Eq. 3).

Note that the above displacement function satisfies the geometrical boundary conditions, and hence it can be applied, but a high number of terms must be taken into account to reach proper accuracy. In calculating the buckling loads 71 terms were taken into account in the y, and 41 in the x direction.

The results calculated by the Rayleigh-Ritz method will be considered as the "accurate" buckling loads. In the following sections we will fit curves to these data to arrive at explicit expressions for the calculation of the buckling load.

5. EXPLICIT EXPRESSIONS

5.1 Linearly varying compression (bending)

For linearly varying load first we verified Lekhnitskii's result (5^{th} row, Table 1) and it was found that the constants, proposed by Lekhnitskii, are a little bit high. According to our calculation instead of 13. 9 and 11.1 one must use 13.4 and 10.4. (Lekhnitskii used a two term approximation, which gives the results within 7 percent). Lekhnitskii's expression, with the modified constants gives the accurate results for $0 \le K \le \infty$ within 0.4 percent, and the results are always on the safe side.

Then the plate with built-in edges was investigated. For this case, to obtain proper accuracy, we suggest to use different expressions for two intervals of parameter K. The expressions are given in Table 2 (first row). Both expressions are on the safe side, the maximum error of the first one is 1.42 %, while that of the second one is 2.17%.

We recall Bank's statement [1] that in most practical cases $K \le 1$. (For an isotropic plate $K=1$. When $K \ll 1$ the bending stiffness, while for $K \gg 1$ the torsional stiffness dominates.) We agree with this statement, however if one applies a layup in which the ± 45 layers dominate, K might be higher than 1. This is the reason why we included also these results in the paper.

When the edges are elastically restrained the constants must vary between the constants for the case of simply supported edge (13.4 and 10.4), and those of the case of built-in edges (26.8 and 12.9 or 30.1 and 11.5). Approximate expressions for plates elastically restrained by springs are given in the 2^{nd} row of Table 2, the maximum error in the buckling load is 2.73%.

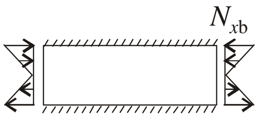
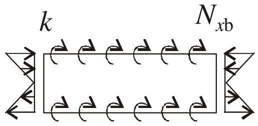
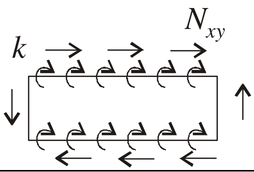
Supports and loading	Buckling load	
1 	$N_{xb,cr} = \frac{\pi^2}{L_y^2}\left[26.8\sqrt{D_{11}D_{22}} + 12.9(D_{12} + 2D_{66})\right]$	$K \le 3$
	$N_{xb,cr} = \frac{\pi^2}{L_y^2}\left[30.1\sqrt{D_{11}D_{22}} + 11.5(D_{12} + 2D_{66})\right]$	$3 \le K$
2  (k, hinged or built-in)	$N_{xb,cr} = \frac{\pi^2}{L_y^2}\left[(13.4 + 13.4\xi)\sqrt{D_{11}D_{22}} + (10.4 + 2.5\xi^4)(D_{12} + 2D_{66})\right]$	$K \le 3$
	$N_{xb,cr} = \frac{\pi^2}{L_y^2}\left[(13.4 + 16.7\xi^{1.4})\sqrt{D_{11}D_{22}} + (10.4 + 1.1\xi^{\delta})(D_{12} + 2D_{66})\right],$ $\delta = \min\{K, \ 8\}$	$3 \le K$
3 	$N_{xy,cr} = \frac{4}{L_y^2}\sqrt[4]{D_{11}D_{22}^3}\left(8.125\sqrt{1+2.44\xi} + (5.045 + 2.035\xi^2)K\right)$	$K \le 1$
	$N_{xy,cr} = \frac{4}{L_y^2}\sqrt{D_{22}(D_{12} + 2D_{66})}\left(11.71 + 6.88\xi + \frac{1.46 + 2.1\xi}{K^2}\right)$	$1 \le K$

Table 2. Buckling loads of long plates with restrained edges. The expressions given in the second row can also be applied when the tensile side is hinged or built-in. (ξ and K are given in Table 1.)

We may observe that the tensile side of the plate is almost flat (Figure 3), and hence the boundary condition on the tensile side does not influence the buckling load, only the restraining effect of the compressive side must be considered.

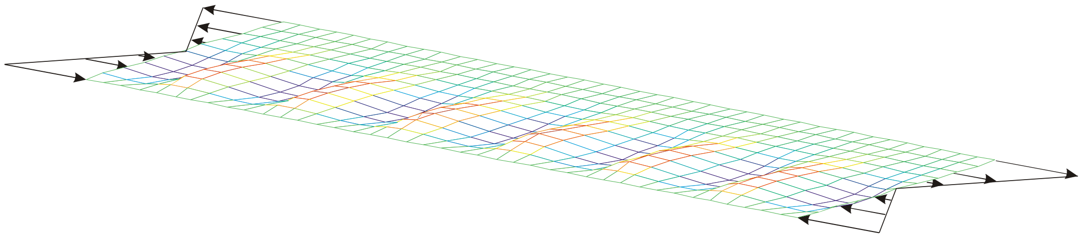


Figure 3. Buckling shape of a plate with restrained edges subjected to a linearly varying axial load

5.2 Shear load

Next, plates subjected to shear load (Figure 1c) were considered. We verified Seydel's result [11] (see 6th and 7th rows, Table 1). We obtained his data within 2%. Then we calculated several cases of plates with restrained edges (Figure 4), and then curves were fitted on the results.

To obtain proper accuracy we suggest different expressions for $K \leq 1$ and for $1 \leq K$. These are given in the 3rd row of Table 2 for plates elastically restrained by springs. The results are on the safe side and the maximum error of the expressions is 3.05%.

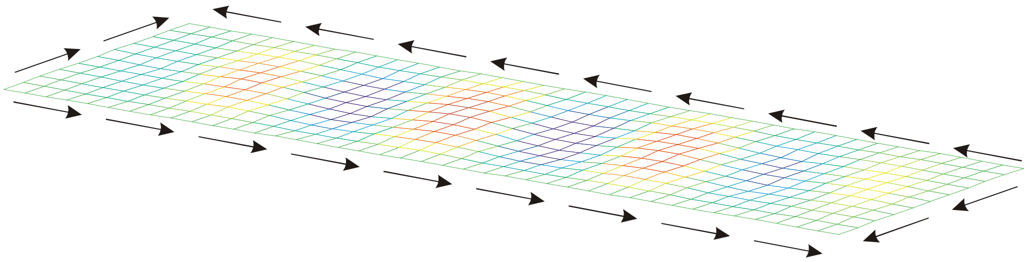


Figure 4. Buckling shape of a plate with restrained edges subjected to shear load

6. NUMERICAL EXAMPLES

To illustrate the usage and accuracy of the above method two numerical examples are presented. In each case we use an E-glass/vinylester plate, the material of which was also used by Bank ([1], p. 421). The thickness of the plate is t=3 mm, while its width is $L_y = 225\,\text{mm}$. The other dimension of the plate is much higher than L_y, it is $L_x = 2000\,\text{mm}$. The material properties are: E_1=17.927 GPa, E_2=6.895 GPa, G_{12}=2.93 GPa, ν_{12}=0.33, and the flexural rigidities are calculated as:

$$D_{11} = \frac{E_1 t^3}{12\left(1-\nu_{12}^2 \dfrac{E_2}{E_1}\right)} = 42.1\,\text{Nm}, \quad D_{22} = \frac{E_2 t^3}{12\left(1-\nu_{12}^2 \dfrac{E_2}{E_1}\right)} = 16.2\,\text{Nm}$$

$$D_{12} = \nu_{12} D_{22} = 5.34\,\text{Nm}, \quad D_{66} = \frac{G_{12} t^3}{12} = 6.59\,\text{Nm}, \quad K = \frac{2D_{66} + D_{12}}{\sqrt{D_{11} D_{22}}} = 0.71$$

First we consider a plate, which is subjected to a *linearly varying end load* (Figure 1b). When the edges are simply supported the critical load is (Table 1, 5th row):

$$\left(N_{x\mathrm{b,cr}}\right)^{\mathrm{ss}}=\frac{\pi^2}{L_\mathrm{y}^2}\left[13.4\sqrt{D_{11}D_{22}}+10.4\left(D_{12}+2D_{66}\right)\right]=105.77\frac{\mathrm{N}}{\mathrm{mm}}. \tag{16}$$

The Rayleigh-Ritz solution results in 106.22 N/mm.

We assume that the rotation of both edges is constrained by rotational springs with the spring constant $k=800$ Nm/m .

In this case the buckling load is calculated as (Table 2, 2nd row)

$$\zeta=\frac{D_{22}^{\mathrm{w}}}{k_1 b_\mathrm{w}}=0.090, \qquad \xi=\frac{1}{1+10\zeta}=0.5264, \tag{17}$$

$$N_{x\mathrm{b,cr}}=\frac{\pi^2}{L_\mathrm{y}^2}\left[(13.4+13.4\xi)\sqrt{D_{11}D_{22}}+\left(10.4+2.5\xi^4\right)\left(D_{12}+2D_{66}\right)\right]=142.37\frac{\mathrm{N}}{\mathrm{mm}}. \tag{18}$$

The Rayleigh-Ritz solution results in 141.10 N/mm. The buckled shape is shown in Figure 3.

The buckling load was also calculated as a function of the plate length, L_x (Figure 5). The number of waves depends on the length of the plate, which is clearly seen on the shape of the results of the Rayleigh-Ritz calculation (Figure 5): when the length is under 0.2 m, there is only one (half) wave, while if it is between about 0.2 and 0.4 m, there are two (half) waves, etc. For longer plates the "long plate approximation" (given in Table 2) is reasonable. In [4] it is suggested that the long plate approximation can be used, if the following condition holds:

$$L_x \ge 3L_\mathrm{y}\sqrt[4]{\frac{D_{11}}{D_{22}}} \tag{19}$$

We may observe that the long plate approximation has a very good accuracy even for much shorter plates.

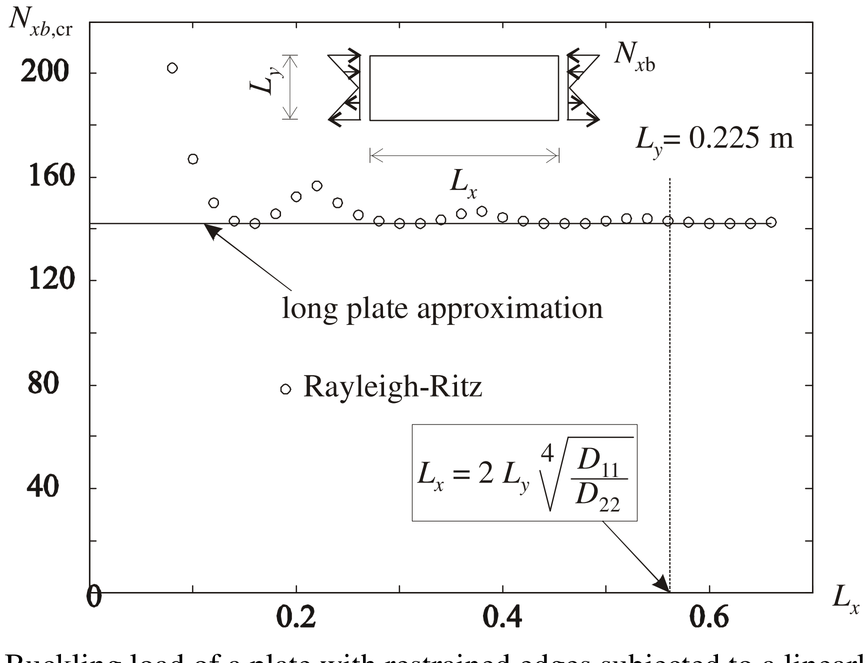


Figure 5. Buckling load of a plate with restrained edges subjected to a linearly varying axial load as a function of the plate length

Consider a plate, which is subjected to *shear load* (Figure 1c). When the plate is simply supported, the critical load is calculated as (Table 1, 6th row, $K = 0.71$)

$$\left(N_{xy,\mathrm{cr}}\right)^{\mathrm{ss}} = \frac{4}{L_{\mathrm{y}}^{2}}\sqrt[4]{D_{11}\left(D_{22}\right)^{3}}\left(8.125 + 5.045K\right) = 19.016\frac{\mathrm{N}}{\mathrm{mm}}. \qquad (20)$$

The Rayleigh-Ritz solution results in 19.56 N/mm.

We assume that the rotation of both edges is constrained by rotational springs, with the spring constant $k = 800$ Nm/m. In this case, the buckling load is (Table 2, 4th row, with $\xi = 0.5264$ given by Eq.17):

$$N_{xy,\mathrm{cr}} = \frac{4}{L_{\mathrm{y}}^{2}}\sqrt[4]{D_{11}\left(D_{22}\right)^{3}}\left(8.125\sqrt{1+2.44\xi} + \left(5.045 + 2.035\xi^{2}\right)K\right) = 26.42\frac{\mathrm{N}}{\mathrm{mm}}, \qquad (21)$$

The Rayleigh-Ritz solution results in 27.26 N/mm. The buckled shape is shown in Figure 4.

The buckling loads were calculated also by a finite element code, which gave the results of the Rayleigh-Ritz solution within 1%.

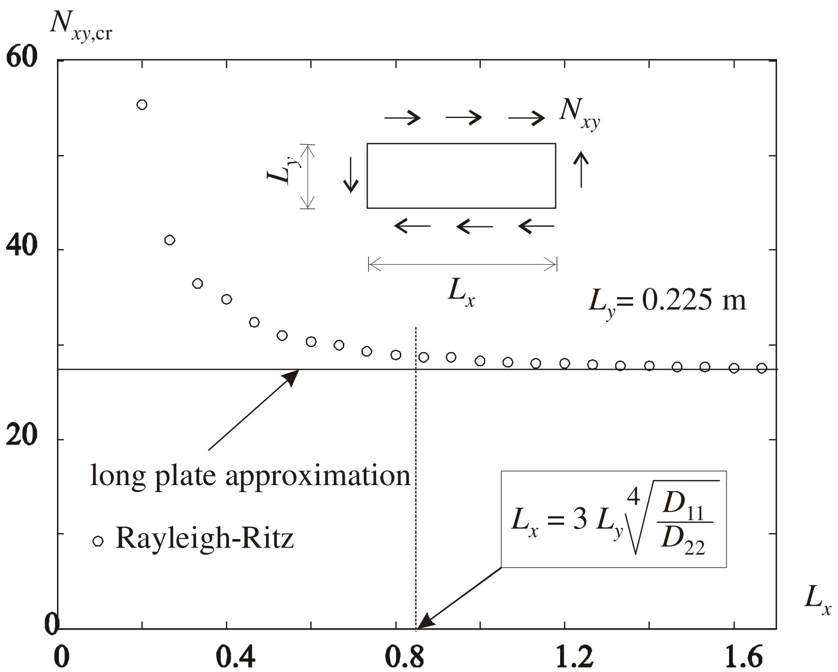


Figure 6. Buckling load of a plate with restrained edges subjected to shear load as a function of the plate length

The buckling load was also calculated as a function of the plate length, L_x (Figure 6). Note that the waves are skewed (Figure 4), and hence the short edges has a more significant effect, than in the case of linearly varying axial load. As a consequence the buckling load is almost monotonically decreasing with the length. (The number of waves, naturally, depends on the length of the plate.) For longer plates the “long plate approximation” (given in Table 2) is reasonable. The length, when the long plate approximation is suggested (Eq.19) is shown by dotted line in Figure 6. The expression can be used also for shorter plates, however the accuracy is smaller.

7. DISCUSSION

Approximate expressions were presented for the calculation of the buckling load of long orthotropic composite plates with restrained edges. The plates were subjected to linearly varying compression or shear load.

Using the above expressions the local buckling analysis of thin walled composite beams given in the literature can be extended to all practical cases.

The analysis was developed for thin-walled members where the layup of each wall is symmetrical. For thin-walled members with unsymmetrical walls the presented formulas can be applied, as an approximation. In this case the reduced bending stiffnesses must be used [8]. The matrix of these reduced stiffnesses is calculated by Eq.(29) of [3].

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REFERENCES

1. Bank, L. C. "*Composites for construction"*, J. Wiley & Sons. Hobken, 2006.
2. Bleich, F. "*Buckling of metal structures"*, McGraw-Hill, New York, 1952.
3. Kollár, L. P. "Local buckling of fiber reinforced plastic composite members with open and closed cross sections." *J. Struct. Eng.,* 129(11), 1503-1513. 2003.
4. Kollár, L. P., and Springer, G. "*Mechanics of composite structures",* Cambridge University Press, Cambridge, 2003.
5. Mottram, J. T. Discussion of „Local Buckling of FRP Composite Structural Members with Open and C55losed Cross Sections," by László P. Kollár. *J. of Structural Engineering.* 2004.
6. Mottram, J. T. „Determination of critical load for flange buckling in concentrically loaded pultruded columns," *Composites Part B: Engineering,* 35(1), pp. 35-47. 2004.
7. Qiao, P., Davalos, J. F., and Wang, J. "Local buckling of composite FRP shapes by discrete plate analysis." *J. Struct. Eng.,* 127(3), 245-255. 2001.
8. Webber, J.P.H, Holt, P.T, and Lee, D.A. „Instability of Carbon Fibre Reinforced Flanges of I section Beams and Columns." *Composite Structures*. Vol. 4, 245-26. 1985
9. Lekhnitskii, S. G. "*Anisotropic plates",* Gordon and Breach science Publishers, New York, 1968.
10. Veres, I. A., and Kollár, L. P. "Buckling of orthotropic Plates with different edge supports" *Journal of Composite Materials,* Vol.35, 625-635. 2001.
11. Seydel, E. "Über das Ausbeulen von rechteckigen isotropen oder orthogonal isotropen Platten bei Schubbeanspruchung.“ *Ingenieur-Archiv,* 4, 169, 1933.
12. Barbero, E. J. "*Introduction to composite materials design",* Taylor &Francis, Philadelphia, 1999.
13. Kollár, L. P. "Buckling of unidirectionally loaded Composite Plates with one free and one rotationally restrained unloaded edge." *J. Struct. Eng.,* 128(9), 1202-1211, 2002.
14. Timoshenko, S. P., and Gere, J. M. "*Theory of elastic stability,*" 2nd Ed., McGraw-Hill, New York, 1961.
15. Withney, J. M. "*Structural analysis of laminated anisotropic plates",* Technomic, Lancester, 1987.