

# A SEMI-ANALYTICAL METHODOLOGY FOR THE RESIDUAL STRENGTH PREDICTION OF DAMAGED SANDWICH STRUCTURES

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## ABSTRACT

This paper presents experimental results for the residual compressive strength of impacted unsymmetrical sandwich specimens. The specimens were made of a honeycomb core of 28.4 mm thickness, which was embedded within CFRP face sheets of 0.63 mm and 2.7 mm thickness. Low velocity impacts from 1J up to 15J caused varying amounts of core and face sheet damage. Strain gauges near the impact damage area are used to record damage growth before the failure of the specimens.

Three simulation approaches are described for predicting the in-plane compression strength of impact damaged sandwich structures. While the first model uses finite volume elements for the core, the second FE model improves efficiency by using a 1D material law. Finally, a semi-analytical Ritz approach is developed in an attempt to further increase the computational efficiency.

The simulations show that the face sheet damage has to be included in the model for higher impact energies. Face sheet damage is included into the finite element models by applying a constant stiffness reduction factor for the damaged elements. Including face sheet damage into the semi-analytical model increases the computational time tremendously. Therefore, the semi-analytical approach should be used only for small impact energies with neglectable face sheet damage. The finite element model with the simplified core material law gives very accurate results for the residual strength and is computationally more efficient, especially when face sheet damage needs to be taken into account.

## 1. INTRODUCTION

In-plane compression is often the most dangerous load case for an impact damaged sandwich structure, since the reduced bending stiffness in the damage zone can lead to local stability failure. Typically this starts with compressive failure of the core material around the impact zone accompanied by growth of the dent in the impacted face sheet. This finally leads to global failure of the structural part [1].

The aim of this work is to provide an efficient simulation methodology for sandwich structures under in-plane compression after impact (CAI). Validation of the prediction is performed using test results for unsymmetrical sandwich structures with honeycomb core material and CFRP face sheets.

Several finite element (FE) models exist for the dent growth simulation. They include the material non-linearity of the crushed core as well as geometrical non-linearity. Davies et al. [2] use 8-node Mindlin composite plate elements for the face sheets and high-order 20-node brick elements capable of representing a bilinear strain distribution. Furthermore, Zenkert et al. [3] developed a three-dimensional FE model with about ten elements through the core thickness. Besides such time-consuming FE models a number of analytical and semi-analytical models have been developed by Xie [4], Minguet [5], Tsang [6], Tsang & Lagace [1] and Moody & Vizzini [7]. The applicability of these models to the structures under investigation is not completely clear:

- The authors use different definitions of the residual compressive strength.
- The authors use / modify model parameters such as core stiffness, ultimate strength and plateau strength in different ways.
- The authors include / neglect different parameters in their models.

Therefore it was necessary to identify the important parameters, which govern the behaviour of the structures at hand, in order to develop an efficient simulation methodology.

## 2. EXPERIMENTS

Impacted honeycomb sandwich specimens were loaded in uniaxial compression at the Department of Aerospace Technology at Dresden University (ILR Dresden), in order to study the failure mechanisms and to measure their residual stiffness and strength. The experimental results will be used for validation of the numerical model described in the following section.

### 2.1 Specimens

The experiments were performed on unsymmetrical honeycomb sandwich structures with CFRP face sheets (CYTEC 977-2/HTA). The inner skin carries most of the load and is therefore much thicker (2.7 mm) than the outer face sheet (0.63 mm), which mainly acts as an impact detector. The honeycomb core is made of resin-coated ARAMID fibre sheets (Hexcel HRH-10-3/16-3.0). The core thickness is 28.42 mm including the adhesive films for the attachment of the core material to the face sheets.

Five of these sandwich specimens were impacted with energies ranging from 1.0 J to 15.0 J, in order to introduce impact damage of varying severity. Details of the impact test programme have been published by Kärger et al. [9]. For reference, the compressive strength of a sixth, undamaged specimen was tested as well.

The impact damage was characterized with ultrasonic scanning. Table 1 shows the sizes of core and face sheet damage as well as the depth of the permanent indentation. Face sheet and core damage sizes are given in the two in-plane directions. The x-direction corresponds to the loading direction during the residual strength test. The 1 J impact caused significant core damage. The US-scan revealed that also the impacted face sheet is slightly affected, although the permanent indentation was hardly visible to the naked eye. The 6 J impact causes a clearly visible dent of almost 3 mm depth. The 15 J impact completely penetrated the face sheet, whereas the core damage size does not increase significantly compared to the 6 J impact.

Specimen No	Impact energy	Dent depth	Damage radius						Damage description	Residual strength
			Indentation		Face sheet		Core			
			x	y	x	y	x	y		
	$W_{imp}$	$Z_0$								$S_{res}$
	J	mm	mm	mm	mm	mm	mm	mm		kN
P00	0	---	0.0	0.0	0.0	0.0	0.0	0.0		78.1
P01	1	0.075	0.0	0.0	5.0	4.0	10.5	11.0	non-visible	65.1
P02	2	0.496	0.0	0.0	10.0	10.0	12.5	11.5	barely visible dent	62.4
P04	4	1.298	5.0	5.0	15.0	15.0	17.0	15.0	visible dent	53.2
P06	6	2.946	9.5	9.5	17.0	17.0	21.0	20.0	visible dent	39.3
P15	15	---	22.0	20.0	22.0	20.0	23.0	21.0	punched-out square	35.0

Table 1: Impact damage sizes and residual strengths for the six sandwich specimens.

## 2.2 Test programme

During compressive loading the cross head force and displacement were recorded. Strain gauges were applied to the outer and inner face sheet. The specimens with the smaller impact energies (1 J to 4 J) were equipped with several strain gauges on the impacted face sheet in increasing horizontal distance to the impact location in order to capture the expected growth of the impact dent (see right hand side of Figure 1).

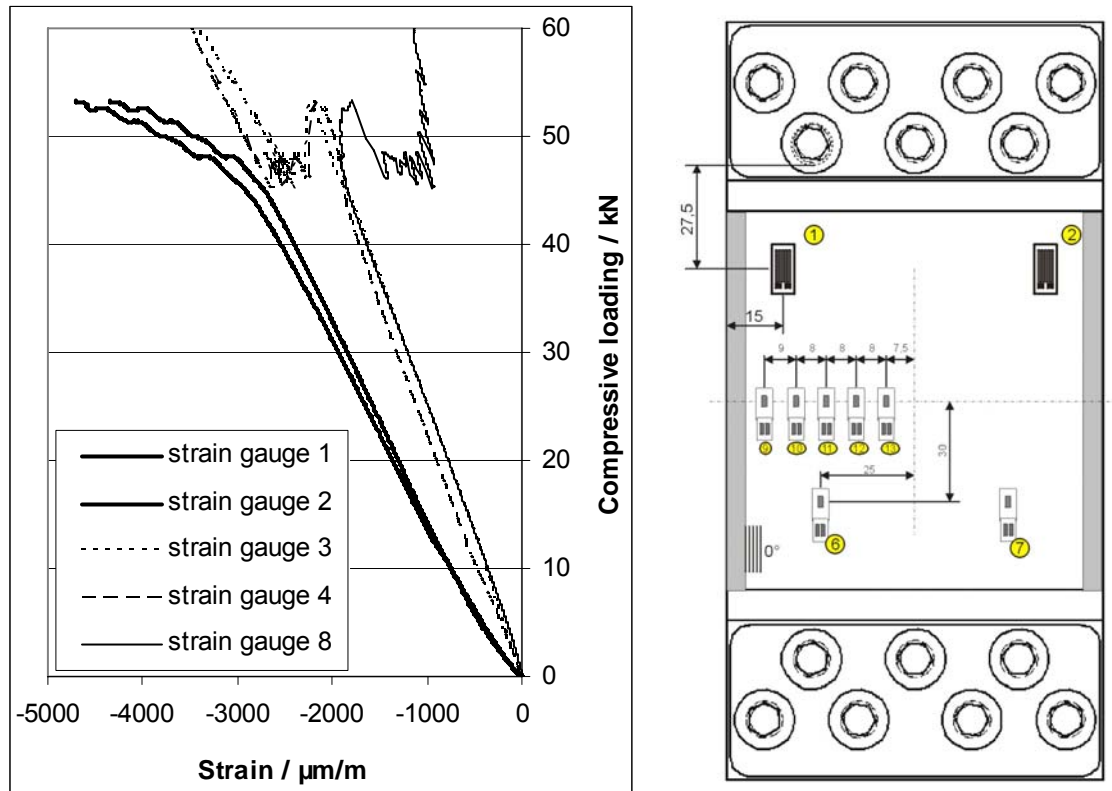


Figure 1: Strains in inner and outer face sheet vs. compressive loading of the sandwich panel P04 with a 4J impact damage. Strain gauges number 3 and 4 are located on the backside of strain gauges 1 and 2 (on the inner, undamaged face sheet). Strain gauge 8 is located in the centre of the inner, undamaged face sheet.

Figure 2 shows the compressive load over time for the specimen P04. During the loading of the panel the cross-head speed was kept constant at 0.5 mm/min, so time in Figure 2 is directly proportional to the displacement of the cross head.

After short initial non-linear behaviour the loading increases in a linear way up to about 42 kN. In the following non-linear region a growth of the impact dent was observed. This growth developed mainly transversely to the loading direction. Upon failure of the impacted face sheet the dent extended over the complete width of the specimen. Failure of the impacted face sheet at 53.2 kN results in an abrupt drop of the compressive load. After failure of the thin, impacted face sheet, the thicker, inner face sheet is able to sustain a much higher load. It collapses at 121.4 kN. The failure of the inner face sheet is not affected by the amount of BVID. The residual strength of these unsymmetrical sandwiches is defined as the maximum load that is reached before failure of the impacted face sheet.

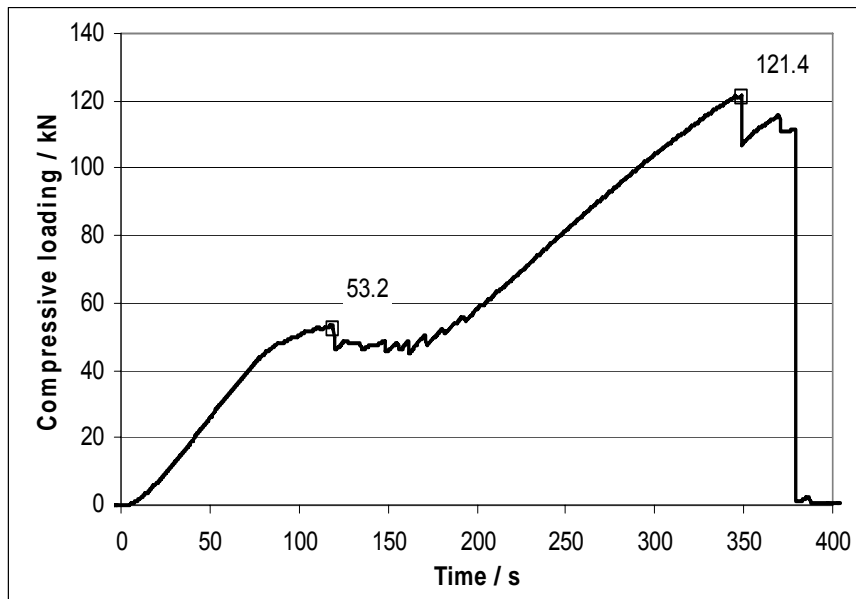


Figure 2: Load-time curve for specimen P04 with a 4J impact damage.

In the ideal case a pure compressive load would be introduced into the panel (no bending moment). However, Figure 1 exemplarily shows for the specimen P04 that bending is not negligible for the experiments described above. Between a loading of 10 kN up to 40 kN the compressive strain of all strain gauges increases in a linear way, with much larger strains in the outer face sheet (strain gauges 1 and 2) than in the inner face sheet (strain gauges 3, 4 and 8). After failure of the impacted face sheet at 53.2 kN recording of the strain gauges 1 and 2 stops. Destruction of the sandwich bond allows the thicker, inner face sheet to buckle. This is illustrated by the large discrepancy in strains of the inner face sheet near the load introduction (strain gauges 3, 4) and the centre (strain gauge 8) following the failure of the impacted face sheet.

In order to obtain the stress distribution between inner and outer face sheet, the strains measured at the strain gauges 1-4 are converted into stresses using stiffness parameters of the UD plies and 0/90 fabric layers as they were supplied by the panel manufacturer. A linear stress distribution across the width of the face sheets, constant strains in thickness direction of each face sheet and vanishing in-plane strains perpendicular to the loading direction are assumed. Cross checking the resulting compressive load of inner and outer face sheet with the measurement has shown that this method is sufficiently accurate.

### 3. SIMULATION METHODOLOGY

Three basic assumptions for all models are:

- The core material (honeycomb) can be homogenized. In-plane stiffness and strength are neglected.
- The inner, undamaged face sheet is not included in the model. The lower face sheet is assumed to undergo only in-plane deformations. This is justified by strain gauge measurements of the inner face sheet (Figure 1).
- Only the initial impact damage of the face sheets is modelled. Damage growth in face sheet during CAI failure process is not taken into account.

### 3.1 ABAQUS Model 1 (3D sandwich core)

A 3D model of the damaged sandwich structures has been built, in order to understand their behaviour under in-plane compressive loading. The core is modelled by six layers of 8-node volume elements, allowing for a piecewise linear, continuous deformation of the core in thickness direction and taking the transverse shear and normal stiffness into account. 4-node shell elements were used for the impacted CFRP face sheet.

The face sheet stiffness is obtained by laminate theory, assuming transversely isotropic material behaviour of UD and fabric layers. First order shear deformation theory is applied. The initial face sheet damage caused by the impact is modelled by reducing the stiffness in that region by a factor  $d$  according to Equation (1).

$$\begin{bmatrix} \sigma \\ \tau \end{bmatrix} = d \cdot \mathbf{C} \cdot \begin{bmatrix} \epsilon \\ \gamma \end{bmatrix} \quad (1)$$

For the impacts smaller than 4J no influence of the small delamination area on the stiffness properties of the face sheet is assumed ( $d = 1.0$ ). A degradation factor of 0.3 takes the substantial impact damage and resulting degradation of the compressive stiffness for the 4 J impact into account. Fibre breaks were visible on the panels impacted with energies of 6 J and 15 J, so for these specimens a face sheet degradation factor of 0.01 was used.

The elliptical impact dent in the impacted face sheet is defined by a double-cosine function. Its parameters are the measured dent depth and the indentation radii along the in-plane half axes.

The growth of core damage plays an important role during the failure process of the sandwich specimens. A failure criterion proposed by Besant et al. [8] was implemented in an ABAQUS-subroutine USDFLD, in order to take interaction of the transverse stresses for the 3D core model into account, cf. Equation (2).

$$\left( \frac{\sigma_z}{S_z} \right)^n + \left( \frac{\tau_{xz}}{S_{xz}} \right)^n + \left( \frac{\tau_{yz}}{S_{yz}} \right)^n \geq 1 \quad (2)$$

As recommended in [8] an exponent of  $n = 1.5$  is used.

Compression tests of the honeycomb sandwich material have shown that the compressive failure of the core material is followed by a sharp drop in the transverse normal stress, followed by further plastic compression of the crushed honeycomb cells. The compression process after failure takes place at more or less constant transverse normal stress of about 30% of the compressive strength. The material model proposed by Olsson [9] describes such behaviour. A somewhat simpler approach is a stiffness reduction analogous to Equation (1). In order to achieve an appropriate balance in the post-failure region, a stiffness reduction factor of  $d = 0.2$  has been used for the transverse normal core stiffness. The shear stiffness of the affected volume elements is set to 1% of the original stiffness. In the area of initial core damage (core damage due to impact) it is assumed, that the crushed sandwich core does not support the impacted face sheet.

The compressive loading is introduced by a stepwise increase ( $\Delta u$ ) of the prescribed displacement at one of the clamped sides of the panel. At the unloaded sides the out-of-plane displacement is set to zero, to account for anti-buckling guides used in the test.

### 3.2 ABAQUS Model 2 (1D sandwich core)

The analysis of the 3D sandwich core model requires a large amount of computational effort. Therefore a simpler, 1D model of the core is introduced by assuming the impacted face sheet to be supported by an elastic foundation of the Winkler type (Figure 3), with only one parameter describing the out-of-plane stiffness of the core. The smaller number of degrees of freedom will increase the stiffness of the model. In order to compensate for the increased stiffness, the out-of-plane shear stiffness of the sandwich core is neglected. The stiffness of the elastic foundation is determined by dividing the stabilized compression modulus provided by the manufacturer of the honeycomb core material by the height of the core.

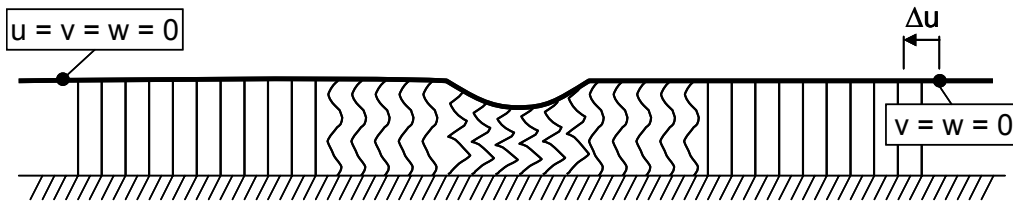


Figure 3: Cross section through the impacted sandwich panel in loading direction

The 3D core model core distributes stiffness and damage across the elements (Gauss-points). This is different for the 1D core model, where these parameters are associated to the nodes. This means that the core damage areas of the two models are slightly different for a given in-plane meshing of the sandwich specimen.

Instead of the simplified stiffness degradation approach (3D model) Olsson's model [9] is used. The effect of the different core material laws on the residual strength prediction is small, however. For example the 2J impact the 1D core model predicts a residual strength of 61.9 kN with the stiffness degradation model compared to 62.1 kN for Olsson's model.

### 3.3 Semi-Analytical Model

A semi-analytical model has been developed in order to investigate, whether the computational efficiency can be improved further.

The out-of-plane displacement of the impacted face sheet is approximated with double sinus functions through the whole panel area. Geometrical non-linearity is taken into account by von-Karman strain-displacement relations. The impact dent is modelled by prescribing a local, stress free initial deformation at the impact location.

As for the ABAQUS Model 2 the core is assumed as an elastic foundation of the Winkler type. For modelling the behaviour of the sandwich core Olsson's 1D model for brittle core materials is used [9]. In the area of the initial core damage resulting from the impact the impacted face sheet is assumed to be unsupported. Initially undamaged core material is assumed to fail at a critical transverse normal stress. Upon core failure the core stress is immediately reduced and kept constant under increasing compressive deformation. Since in Olsson's material model only one stiffness parameter governs transverse core deformation, an equivalent transverse stiffness  $k_c$  is determined. For the sandwich specimen presented in this paper it was found that  $k_c$  is equal to the transverse normal stiffness. Again, the transverse displacement is approximated as a

linear function over the core thickness. This leads to an increased transverse normal stiffness. As before, this is compensated by neglecting the transverse shear stiffness of the core.

Face sheet damage can be also included in the model. However, this increases the computational effort tremendously. The compressive loading is applied by a far-field stress. At each load step, stationary values of the potential energy are sought. The non-linear set of equations is solved by Newton's method or a modified arc-length method.

#### 4. SIMULATION RESULTS AND VALIDATION

As observed in the experiment, all simulations show the growth of the impact dent transverse to the loading direction. This is shown exemplarily for the specimen P04 and the ABAQUS model 2 in Figure 4.

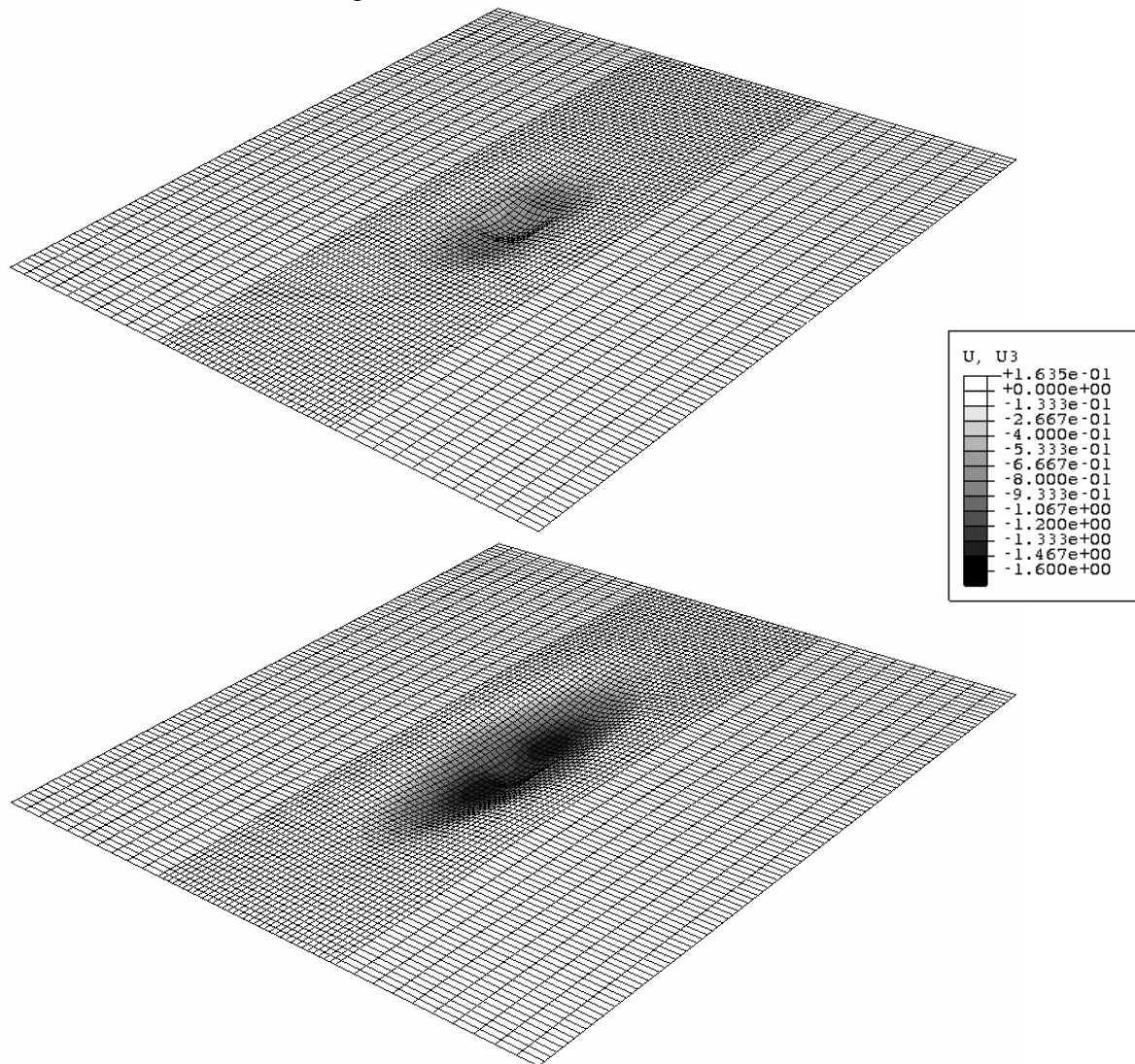


Figure 4: Transverse displacement of the impacted face sheet for the ABAQUS model 2 and the 4J impact damage, upper figure: prescribed in-plane displacement  $\Delta u = 0.251$  mm, lower figure:  $\Delta u = 0.411$  mm.

The residual strengths shown in Table 2 are the loads of the complete structure at the moment of failure of the impacted face sheet. Since the simulation only provides failure

loads of the damaged face sheet, these loads are transformed into loads of the complete structure.

Specimen No	Impact energy J	Face sheet Degr. factor	Residual strength			
			ABAQUS 1 kN	ABAQUS 2 kN	semi-anal. M. kN	Exp. kN
P01	1.0	1.00	-	64.9	64.5	65.1
P02	2.0	1.00	62.0	62.1	62.3	62.4
P04	4.0	1.00	-	56.3	56.5	53.2
		0.70	-	55.2	53.0	
		0.30	-	53.0	-	
P06	6.0	0.01	-	41.2	-	39.3
P15	15.0	0.01	-	42.7	-	35.0

Table 2: Residual strengths from simulation and experiment.

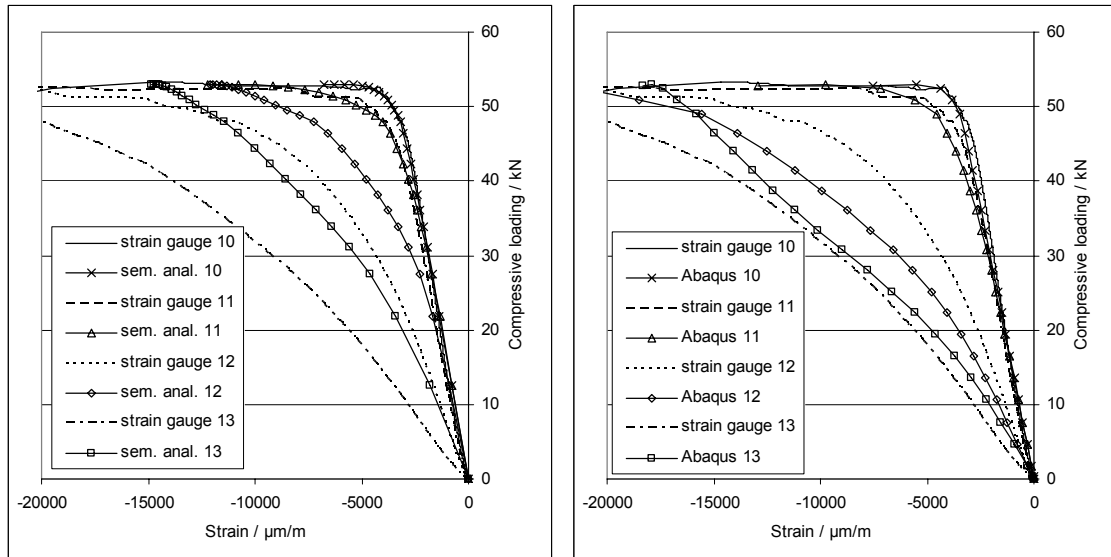
The ABAQUS model 1 (3D core model) is applied only for the 2 J impact damage. It is used as reference for the more efficient ABAQUS model 2 (1D core model). The ABAQUS model 1 predicts a residual strength of 62.0 kN for the specimen with the 2 J impact, which is in excellent agreement with the prediction of the simpler 1D core model (62.1 kN) and the residual strength of 62.4 kN measured in the experiment. Furthermore, it was found that the deformation states of 1D and 3D core model agree very well before damage growth. During damage propagation the 1D model underestimates the face sheet deformations by 6% in the center of the damage.

The ABAQUS model 2 (1D core model) gives very good results for the residual strength of specimen with impacts of up to 6 J. The simulation overestimates the failure load for the 15 J impact. The far field stress in the upper face sheet is slightly lower than for the P06 specimen at collapse of the upper face sheet. However, the predicted residual strength is slightly larger compared to the specimen with the 6 J impact, because evaluation of the strain gauges of both face sheets yielded a smaller bending moment being introduced into the P15 specimen.

For impacts with less severe face sheet damages (impacts of 1 J and 2 J) the semi-analytical model predicts the residual strength very well. In these cases it is possible to neglect the face sheet damage. Including the face sheet damage in the model increases the computational effort tremendously. Therefore, only one simulation is performed including the face sheet damage in the semi-analytical model. Table 2 shows that the stiffness reduction factor of 0.7 leads to a residual strength of 53.0 kN very close to the experimental result of 53.2 kN.

Figure 5 (a) shows that strain gauge measurements outside of the damage area (strain gauges 10 and 11) correspond well with the simulation results of the semi-analytical model. The stiffness reduction factor of 0.7 for the face sheet damage captures the softening effect at the border of the face sheet damage (strain gauge 12) quite well. The semi-analytical model with the stiffness reduction factor of 0.7 underestimates the compressive strains at strain gauge 13, which is situated inside the impact damage area (cf. Table 1). For the stiffness reduction factor of 0.7 the compressive strains of the ABAQUS model 2 are very close to the ones of the semi-analytical model.





(a) semi-analyt. model with degr. factor of 0.7

(b) ABAQUS model 2 with degr. factor of 0.3

Figure 5: Strains in loading direction in the vicinity of the 4J impact vs. compressive loading of the panel. For strain gauge positions see Figure 1.

A smaller stiffness reduction factor of 0.3 results in higher compressive strains at the four strain gauge locations (Figure 5 (b)). Inside the damage area (strain gauge 13) the strains are predicted quite accurately up to a compressive load of about 40 kN. The stiffness reduction factor of 0.3 gives better results for the strain gauges close to the centre of the face sheet damage. Close to the border of the damage the stiffness reduction is less severe than in the centre of the damage area, where a larger amount of fibre and matrix fracture is expected. This might be the reason, why the stiffness reduction factor of 0.3 overestimates compressive strains at the border of the face sheet damage (strain gauge 12). This leads to the conclusion that the stiffness reduction factor should be varied from the border of the impact damage toward the damage center.

## 5. DISCUSSION AND CONCLUSIONS

The 3D FE model (ABAQUS model 1) was able to accurately account for the residual compressive strength behaviour of the sandwich structures under investigation. The non-linear distribution of through-thickness deformations in the core and transverse shear effects in the damage area affect the structural behaviour substantially.

With the simplified FE-model (ABAQUS model 2) and the semi-analytical model it was possible to increase the computational efficiency at the cost of approximating the transverse displacement as a linear function over the core thickness. This was compensated by neglecting the transverse shear stiffness of the core.

A stiffness reduction factor accounts for the face sheet damage. The ABAQUS model 2 gives very good results for the residual strength of specimens with impacts of up to 6 J. For the massive face sheet damage produced by the 15 J impact, the model overestimates the residual strength.

The semi-analytical model predicts the residual strength well for impact energies of up to 4 J, without needing to take face sheet damage into account. Including face sheet damage into the semi-analytical model improves its results for the 4 J impact, but the

computation time increases tremendously. Therefore, the ABAQUS model 2 should be applied for impact damages with substantial face sheet damage.

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## REFERENCES

- 1 Tsang, P.H.W. and Lagace, P.A. "Failure mechanisms of impact-damaged sandwich panels under uniaxial compression". *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 35nd Structures, Structural Dynamics, and Materials Conference*, Hilton Head, pp. 745–754, 1994.
- 2 Davies GAO, Hitchings D, Besant T, Clarke A, Morgan C. Compression after impact strength of composite sandwich panels. *Composite Structures* 2004; 63:1-9.
- 3 Zenkert D, Shipsha A, Bull P, Haymann B. Damage tolerance assessment of composite sandwich panels with localised damage. *Composite Science and Technology* 2005; 65:2597-2611
- 4 Xie Z. Damage Tolerance of Low Velocity Impacted Composite Sandwich Structures. Dissertation, University of Maryland, College Park, 2003.
- 5 Minguet PJ. A model for predicting the behavior of impact – damaged minimum gage sandwich panels under compression. *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics, and Materials Conference*, Baltimore. 1112–1122, 1991.
- 6 Tsang PHW. Impact resistance and damage tolerance of composite sandwich panels. Dissertation, Massachusetts Institute of Technology, Cambridge, 1994.
- 7 Moody RC, Vizzini AJ. Incorporation of a compliance change due to impact in the prediction of damage growth in sandwich panels. *Proceedings of the 13<sup>th</sup> International Conference on Composite Materials*, Beijing, 2001.
- 8 Besant T, Davies GAO, Hitchings D. Finite element modelling of low velocity impact of composite sandwich panels. *Composites: Part A* 2001;32:1189–1196.
- 9 Olsson., R. "Methodology for predicting the residual strength of impacted sandwich panels". *Report FFA TN 1997-09*, The Aeronautical Research Institute of Sweden, 1997.
- 10 Kärger L, Baaran J, Tessmer J. Rapid simulation of impacts on composite sandwich panels inducing barely visible damage. *Composite Structures* 2007;79:527-534.