TRANSIENT RESPONSE OF A THREE LAYER SANDWICH PLATE WITH ELECTRORHEOLOGICAL CORE AND ORTHOTROPIC FACES

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ABSTRACT

The present work is as investigation of the transient response of a rectangular three layer sandwich plate with Electro-rheological (ER) core and orthotropic face sheets. ER fluids which are a class of smart materials exhibit reversible changes in their mechanical properties when they are subjected to an electric field. By applying an electric field to the core layer is changed the dynamic characteristics of the structure, the ER layer can be used for vibrations suppressing and reducing the settling time of the sandwich plate subjected to a transient excitation.

In this study, an FE model of the structure is developed and a modified direct integration algorithm is used to simulate the impulse response of the proposed sandwich plate. The derived finite element model and the solution procedure are programmed as a code. Since the behavior of electro-rheological materials are different in the pre-yield and post-yield regions, it is necessary to identify the state of the ER material throughout the solution time. According to numerical results which are compatible with previous experimental observations, the produced shear stains in the core is greater than the required strain for exiting the core material from its pre-yield regime. Hence, the core is modeled as a Bingham plastic material. Based on the proposed model, effects of changes in the applied electric filed on the structure settling time and its natural frequencies are represented for different core thicknesses.

1. INTRODUCTION

Electrorheological (ER) fluids which are a kind of smart materials with controllable rheological properties. When an electric field is applied to these fluids, they respond by forming chain-like structures, which results in an enhancement of their rheological properties, including viscosity, yield stress, and shear modulus, by several orders of magnitude. ER clutches [1, 2] and journal bearings [3] are some examples for extensive applications of ER fluids. But the exceptional characteristic of ER fluids is their typical response time, which is of the order of a few milliseconds. In fact, this inherent feature of these fluids was the main reason for tremendous research activities in development of their extensive applications in systems with controllable damping [4, 5].

Various mathematical models have been introduced by researchers to present the dynamic behavior of ER fluids in the pre-yield and the post-yield stages [6-8]. Experimental investigations imply that if a small excitation gain applied to an ER fluid, the fluid does not pass its pre-yield stage and traditional linear vibration theories can be modified to describe the vibration characteristics of the system. On the other hand, higher excitation gains would be able to cause the ER fluid entering its yield region and consequently make the system response nonlinear. The Bingham model has been dominantly used by researchers for describing the post-yield behavior of ER fluids [9, 10]. According to this model, the ER treated system yields a non-harmonic displacement response under a sinusoidal excitation force.

Having extended the applications of ER fluids, the vibration [11] and the dynamic stability [12] problems for beams and plates with ER core were investigated. In such structures, applying an electric field to elastic layers changes the mechanical properties

of the core layer and consequently influences the dynamic properties of the structure.
Linear viscoelastic models which have been widely used by previous researchers in modeling of ER fluid-based structures can only describe the pre-yield behavior of these materials [9, 13]. Using these theories for expressing the shear modulus and the loss factor of ER fluids, a growth of some hundred percents in natural frequencies of a sandwich structure are predicted by increasing the strength of the applied electric filed [14]. However, some experimental evaluations exhibit no significant change in resonant frequencies of such a system [13]. The origin of these discrepancies can be the much lower shear modulus and yield stress of available ER fluids, which can easily cause the material reaches its yield point in comparison the other viscoelastic damping materials such as rubber. Therefore, viscoelastic modeling of ER fluids in damping layer treatment applications with relatively high excitation gains does not seem to be appropriate due to the existence of the great inconsistency with experimental results.
In contrast to most of the earliest studies on harmonic analysis of sandwich structures with ER core layer for which Kevin viscoelastic models were used, Rezaeepazhand and Pahlavan [15] proposed a finite element model of a three layer beam structure based on the Bingham plastic model which was capable of simulating the post-yield behavior of ER fluids in a sandwich beam. In their dynamic model, a direct time-integration-based algorithm had been applied to model the transient response of a cantilever sandwich beam.
In the present work and in continue of our previous study [15], the transient response of a cantilever sandwich shell structure with an ER core and orthotropic face sheets subjected to an impulse load as an arbitrary excitation is investigated. The modified direct integration method has been used in evaluation of the influence of the applied electric field on the structure settling time and the natural frequencies of the structure for different thicknesses of the ER core.

2. BINGHAM MODEL FOR THE POST-YIELD BEHAVIOR OF ER FLUIDS

Typical ER fluids are colloidal fluids of the particle dispersion type, exhibiting the characteristics of Bingham fluids in their post-yield stage. An ER fluid is changed from a near Newtonian fluid in which its particles are free to move in the absence of electric field to the Bingham plastic behavior in which its particles are aligned in a chain by applying the electric field. The yield stress of these materials varies with the application of an electric field according to the following equation [16] :

$$\tau_p(E) = \eta_p \dot{\gamma} + \tau_E(E) \qquad (1)$$

τ_p (Pa) and η_p (Pa.s) are the shear stress and the basic viscous coefficient of this fluid. γ (1/s), E (V/mm) and τ_E (Pa) are the shear rate, the electric field and the fluctuating stress, respectively. Consequently, when these fluids are used in an ER device, it can be configured as a mechanical impedance having a variable friction damper.

3. DESCRIPTION OF THE PROBLEM

Consider a rectangular three-layer sandwich shell as shown in Figure 1 in which an ER layer is restricted by two orthotropic layers, a constraining layer at top and a base layer at bottom. Mathematical model of this structure is constructed based on the following assumptions:

1- The ER core behaves as a Bingham plastic material.
2- Rotary inertia and shear deformation of the elastic layers are negligible.
3- No normal stress can be produced in the core layer.
4- No slipping occurs between the neighbor layers.

5- The transverse displacements of all points on a cross-section are equal.
6-The material line in the core layer would be straight after deformation (as shown by lines B-C and B$^{/}$- C$^{/}$ in Figure 2 and similarly for y-z plane).

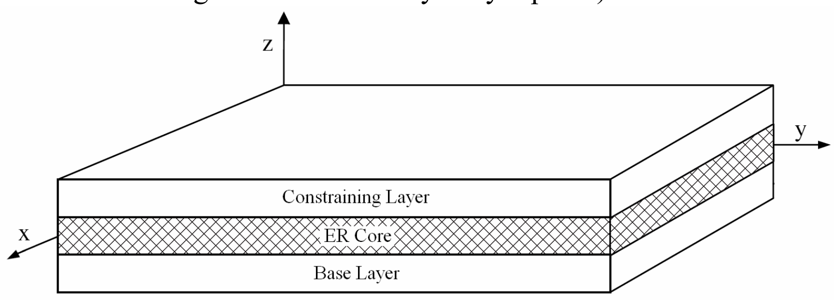

Figure 1. The rectangular sandwich shell with ER core and orthotropic face sheets.

According to the strain-displacement relations of the orthotropic layers and the geometrical characteristics and properties of the layers, the elastic strains of the constraining and the base layers (indexed by 1 and 3, respectively) can be expressed as follows.

$$\{\varepsilon_1\}=\begin{Bmatrix}\varepsilon_{x1}\\ \varepsilon_{y1}\\ \gamma_{xy1}\end{Bmatrix}=\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0 & 0 & -z_1\frac{\partial^2}{\partial x^2}\\ 0 & 0 & \frac{\partial}{\partial y} & 0 & -z_1\frac{\partial^2}{\partial y^2}\\ \frac{\partial}{\partial y} & 0 & 0 & \frac{\partial}{\partial x} & 0\end{bmatrix}\begin{Bmatrix}u_1\\ u_3\\ v_1\\ v_3\\ w\end{Bmatrix}=[L_1]\{u\} \tag{2}$$

$$\{\varepsilon_3\}=\begin{Bmatrix}\varepsilon_{x3}\\ \varepsilon_{y3}\\ \gamma_{xy3}\end{Bmatrix}=\begin{bmatrix}0 & \frac{\partial}{\partial x} & 0 & 0 & -z_3\frac{\partial^2}{\partial x^2}\\ 0 & 0 & 0 & \frac{\partial}{\partial y} & -z_3\frac{\partial^2}{\partial y^2}\\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0\end{bmatrix}\begin{Bmatrix}u_1\\ u_3\\ v_1\\ v_3\\ w\end{Bmatrix}=[L_2]\{u\} \tag{3}$$

In these relations, u_i and v_i are the displacements of the mid-plane of the i^{th} layer in the x and y directions respectively and z_i is the distance of the mid-height of layer i. Also w denotes the transverse displacement of the layers. Considering the geometry, the following relationship for the shear strains of the core layer can be obtained.

$$\begin{Bmatrix}\gamma_{x2}\\ \gamma_{y2}\end{Bmatrix}=\begin{bmatrix}\frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x}\\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y}\end{bmatrix}\begin{Bmatrix}u_2\\ v_2\\ w\end{Bmatrix} \tag{4}$$

According to Figure 2, the displacement gradients in Equation (4) can be replaced by the following equivalent expressions for the core shear stains in terms of the displacements of the elastic layers.

$$\frac{\partial u_2}{\partial z}=\frac{h_1+h_3}{2h_2}\frac{\partial w}{\partial x}+\frac{u_1-u_3}{h_2} \tag{5}$$

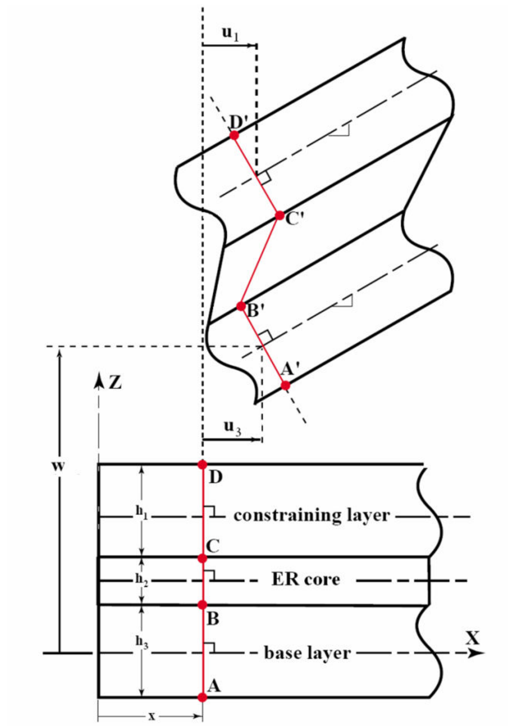

Figure 2. Configuration of the layers before and after deformation.

$$\frac{\partial v_2}{\partial z} = \frac{h_1 + h_3}{2h_2}\frac{\partial w}{\partial y} + \frac{v_1 - v_3}{h_2} \tag{6}$$

Substituting the above equation into Equation (4), the shear strains of the ER layer can be written as:

$$\{\gamma_2\} = \begin{Bmatrix} \gamma_{x2} \\ \gamma_{y2} \end{Bmatrix} = \begin{bmatrix} \frac{1}{h_2} & \frac{-1}{h_2} & 0 & 0 & \frac{d}{h_2}\frac{\partial}{\partial x} \\ 0 & 0 & \frac{1}{h_2} & \frac{-1}{h_2} & \frac{d}{h_2}\frac{\partial}{\partial y} \end{bmatrix} \{u\} = [Q]\{u\} \tag{7}$$

where $d = h_2 + \frac{1}{2}(h_1 + h_3)$. (8)

4. FE MODELING OF THE PROBLEM

Consider a four-node multi-layer shell element as shown in Figure 3 in which the transverse displacements of all layers are equal. Each node has 7 degrees of freedom $u_{1i}, u_{3i}, v_{1i}, v_{3i}, w_i, w_{i,x}, w_{i,y}$.

Accordingly, the in-plane and the transverse displacements of the sandwich plate can be written in terms of a nodal displacement vector and a shape function matrix where its components are common shell shape functions at its corners.

$$\{u(x,t)\} = [N(x,y)]\{q(t)\} \tag{9}$$

The total energy of the element π can be written:

$$\pi = T - V + U \tag{10}$$

where T and V are the kinetic and the potential energies of the element, respectively. Also U denotes the work done by external forces. While the face sheets are assumed to have no shear deformation, the strain energy of the elastic layers is as follows.

$$V_s = \frac{1}{2}\int_{V_1} \{\varepsilon_1\}^T [E_1]\{\varepsilon_1\}\, dv + \frac{1}{2}\int_{V_3} \{\varepsilon_3\}^T [E_3]\{\varepsilon_3\}\, dv \tag{11}$$

The total strain energy of each layer can be written as:

$$\{\varepsilon_1(x,y,t)\} = [D_1(x,y)]\{q(t)\} \tag{12}$$

$$\{\varepsilon_3(x,y,t)\} = [D_3(x,y)]\{q(t)\} \tag{13}$$

Where $[D_1(x,y)] = [L_1][N(x,y)]$ and $[D_2(x,y)] = [L_2][N(x,y)]$. Therefore,

$$V_s = \frac{1}{2}\int_{V_1} \{q\}^T [D_1]^T [E_1][D_1]\{q\}\, dv + \frac{1}{2}\int_{V_3} \{q\}^T [D_3]^T [E_3][D_3]\{q\}\, dv \tag{14}$$

in which

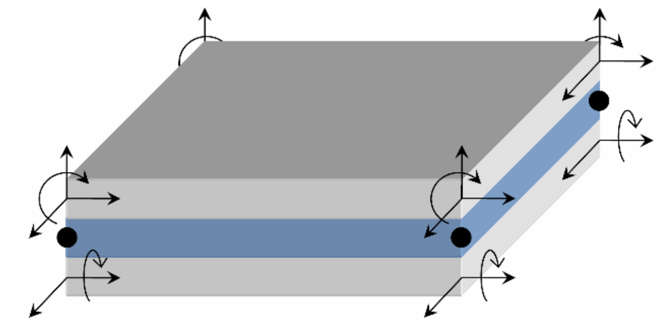

Figure 3. A three layer shell element with 4 nodes and 7-DOF per node.

$$[E_i] = \begin{bmatrix} \dfrac{E_{xi}}{1-\upsilon_{xi}\upsilon_{yi}} & \dfrac{\upsilon_{xi}E_{yi}}{1-\upsilon_{xi}\upsilon_{yi}} & 0 \\ \dfrac{\upsilon_{yi}E_{xi}}{1-\upsilon_{xi}\upsilon_{yi}} & \dfrac{E_{yi}}{1-\upsilon_{xi}\upsilon_{yi}} & 0 \\ 0 & 0 & G_i \end{bmatrix}, \quad i = 1, 3 \tag{15}$$

E_{xi} and E_{yi} are the orthotropic elasticity modulus, υ_{xi} and υ_{yi} are the orthotropic Poisson's ratios and G_i is the shear modulus.

As described before, according to Bingham plastic model, the shear stress of a particle type ER fluid could be expressed as Equation (1). Therefore, the total energy lost by the ER core layer can be written as:

$$E_{er} = \int_{V_2} \{\gamma_2\}^T \{\tau_E(E)\}\, dv + \int_{V_2} \{\gamma_2\}^T \eta_p \{\dot{\gamma}_2\}\, dv \tag{16}$$

Substitution of Equation (10) into Equation (7) results in:

$$\{\gamma_2(x,t)\} = [Q][N]\{q\} = [R]\{q\} \tag{17}$$

$$\{\dot{\gamma}_2(x,t)\} = [R]\{\dot{q}\} \tag{18}$$

and thus,

$$E_{er} = \int_{V_2} \{q\}^T [R]^T \{\tau_E(E)\}\, dv + \int_{V_2} \{q\}^T [R]^T \eta_p [R]\{\dot{q}\}\, dv \tag{19}$$

Neglecting the rotary inertia of the elastic layers, their kinetic energy due to their in-plane and their transverse displacements can be written as:

$$T_e = \int_{V_1} \{u\}^T \rho_1 \{\ddot{u}\}\, dv + \int_{V_3} \{u\}^T \rho_3 \{\ddot{u}\}\, dv \tag{20}$$

$$T_e = \int_{V_1} \{q\}^T [N]^T \rho_1 [N] \{\ddot{q}\} dv + \int_{V_3} \{q\}^T [N]^T \rho_3 [N] \{\ddot{q}\} dv \quad (21)$$

On the other hand, the kinetic energy of the core layer includes an energy term associated with transverse displacement and another energy term associated with rotation of elements.

$$T_c = \int_{V_2} \{u_2\}^T \rho_2 \{\ddot{u}_2\} dv + \int_{V_2} \{\gamma\}^T [J_2] \{\ddot{\gamma}\} dv \quad (22)$$

where $[J_2]$ is the mass moment of inertia matrix for a definite volume of ER core. Its dimensions are defined by the element length and width and the core thickness. Therefore,

$$T_c = \int_{V_2} \{q\}^T [N]^T [H]^T \rho_2 [H][N] \{\ddot{q}\} dv + \int_{V_2} \{q\}^T [R]^T [J_2][R] \{\ddot{q}\} dv \quad (23)$$

in which $H = [0 \quad 0 \quad 0 \quad 0 \quad 1]$.

Finally, the work done by external forces f_p can be written as follows:

$$W = \sum \{u\}^T \{f_p\} = \sum \{q\}^T [N]^T \{f_p\} \quad (24)$$

According to the Hamilton's principle for $\delta\pi = 0$, the governing equation of motion for a damped sandwich shell element can be expressed in the following form,

$$[M^e]\{\ddot{q}\} + [C^e]\{\dot{q}\} + [K^e]\{q\} + \{F_{ER}{}^e\} = \{F_{ext.}{}^e\} \quad (25)$$

in which,

$$\begin{aligned}[M^e] = &\int_{V_1} [N]^T \rho_1 [N] dv + \int_{V_2} [N]^T [H]^T \rho_2 [H][N] dv + \\ &\int_{V_2} [R]^T [J_2][R] dv + \int_{V_3} [N]^T \rho_3 [N] dv\end{aligned} \quad (26)$$

$$[C^e] = \int_{V_2} [R]^T \eta_p [R] dv \quad (27)$$

$$[K^e] = \int_{V_1} [D_1]^T [E_1][D_1] dv + \int_{V_3} [D_3]^T [E_3][D_3] dv \quad (28)$$

$$\{F_{ER}{}^e\} = \int_{V_2} [R]^T \tau_E(E) \{\mathrm{sgn}(\dot{\gamma})\} dv \quad (29)$$

$$\{F_{ext.}{}^e\} = \sum [N]^T \{f_p\} \quad (30)$$

It should be noted that $\{\mathrm{sgn}(\dot{\gamma})\}$ in Equation (29) is appeared because of the nature of ER core layer which produces only a resistant force against the motion.

5. DIRECT INTEGRATION METHOD

Direct integration or explicit integration method is an effective general algorithm for solving dynamic problems. In this method the total solution time is divided into several intervals. In each interval, the procedure performs a complete FE analysis. Substitution of equivalent difference formulas for velocity and acceleration vectors into the equation of motion results in a recursive formula that specifies the displacement vector at time $t+\Delta t$ with respect to displacement values at time t and time $t-\Delta t$ [17].

$$\begin{aligned}&\left(\frac{1}{(\Delta t)^2}[M] + \frac{1}{2\Delta t}[C]\right)\{U_{t+\Delta t}\} = \\ &\{f(t)\} - \left([K] - \frac{2}{(\Delta t)^2}[M]\right)\{U_t\} - \left(\frac{1}{(\Delta t)^2}[M] - \frac{1}{2\Delta t}[C]\right)\{U_{t-\Delta t}\}\end{aligned} \quad (31)$$

Details of the modified procedure can be found in [15, 17]. The derived finite element model and the solution method are programmed as a code. In the next section, effects of some parameters in vibration response of this sandwich shell are studied.

6. EVALUATION OF THE TRANSIENT METHOD

Owing the lack of the published works on dynamic response of sandwich structures with ER core based on Bingham model, the finite element model and corresponding explicit solution algorithm were evaluated distinctly. Gaining the confidence of the correctness of the procedure and the computer program by evaluating various cases including sandwich shells and beams, the transient behavior of a model sandwich shell with ER core is obtained and the results are represented in the next section.

Table 1. Material properties of orthotropic face sheets.

E_x	307	*Gpa*
E_y	358	*Gpa*
υ_x	0.2	-
υ_y	0.2	-
ρ	3750	kg/m^3
G	126.9	*Gpa*

7. RESULTS AND DISCUSSION

For numerical analysis, a three layer sandwich cantilever plate with orthotropic faces is considered. The face sheets are made up of Aluminium Oxide with mechanical properties listed in Table 1. Table 2 indicates the stucture geometries. As an arbitrary excitation, a unit impulse load applied at a free corner of the plate is considered. The particle type ER fluid used for this study consists of chemical starch as particles and silicone oil as base liquid. $\tau_E(E)$ is the field-dependent yield stress of the ER fluid which is given by αE^{β}. α and β are constants of the ER fluid. The values of $\alpha = 427$ and $\beta = 1.2$ are determined experimentally by Jung et al. [18] .

Table 2. Geometries and materials properties.

Plate length	500	*mm*
Plate width	300	*mm*
Thickness of the constraining layer	1	*mm*
Thickness of the base layer	1	*mm*
Thickness of the ER layer	2	*mm*

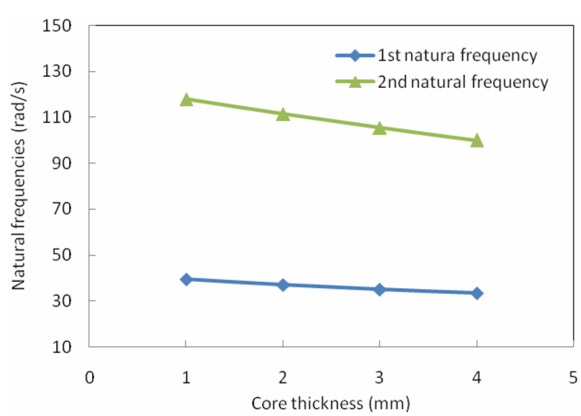

Figure 4. Dependence of the structure natural frequencies on the core thickness regardless of the applied electric field when h_1=h_3=1mm.

7.1 EFFECTS OF THE CORE THICKNESS AND THE APPLIED ELECTRIC FIELD ON NATURAL FREQUENCIES OF THE SYSTEM

Dependence of the first two natural frequencies of the system on thickness of the core layer is illustrated in Figure 4. Applying various levels of electric field into the ER layer which behaves as a Bingham fluid found to have no effect on natural frequencies of the structure. In fact, applying an electric field can only improve the damping treatment of the system, not its stiffness. Since the equivalent mass of the system grows up by increasing the thickness of the core layer, the natural frequencies of the structure are expected to be decreased.

7.2 EFFECTS OF THE ELECTRIC FIELD STRENGTH ON DYNAMIC BEHAVIOR OF THE STRUCTURE

Influence of applying electric field on dynamic response of the proposed structure is presented in Figure 5. In this case, the geometries are according to Table 2. In order to study the effects of applied electric field, various voltages are applied to the faces. The graphs illustrate the displacement of the plate corner on which the external force has been exerted.

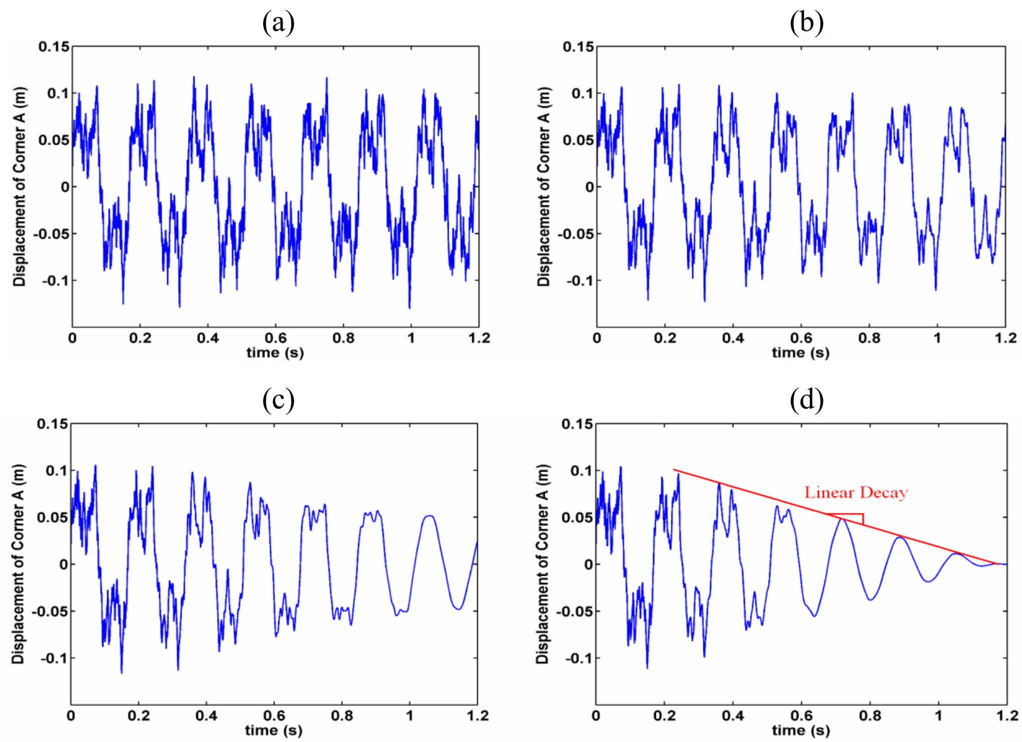

Figure 5. Impulse response of the structure for different applied electric fields: (a) zero, (b) 1, (c) 2 and (d) 3 KV/mm.

According to Figure 5-a, applying no electric field does not have any considerable influence on vibration suppression time. As shown in Figure 5, the damping behavior of the system improves by applying higher levels of electric field and thus, a shorter settling time is achieved. It can be concluded that the viscous damping component of the core has negligible effect (Figure 5-a) in comparison with the damping component associated with applying electric field (Figure 5-d).

The viscous damping force of ER fluids acts similar to a viscous damper in a vibrating system. However, the other term of the damping force produced in the result of applying electric field can be configured as a coulomb damper, according to Equation

(1). Since the electric field-caused damping force is dominating in the system, a linear decay of vibration amplitude exists instead of a combination of a linear and an exponential decay. Figures 5-c and 5-d clearly show the coulomb damping behavior of the ER core layer.

7.3 EFFECT OF THE ELECTRIC FIELD STRENGTH AND THE CORE THICKNESS ON THE SYSTEM SETTLING TIME

The energy lost in the core layer increases by raising the thickness of the core layer. Consequently, the equivalent damping of the structure is expected to be improved. As Figure 6 illustrates, enhancement of the damping properties of the structure can be achieved by increasing the thickness ratio of the core layer. However, due to the growth of the mass and the mass moment of inertia of the system and also the reduction of the shear damping forces by increasing the ER layer thickness, stronger electric fields has less effect in higher values of the core thickness.

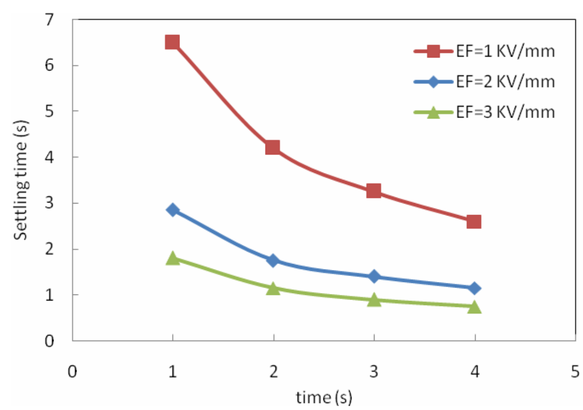

Figure 6. Dependence of the system settling time on the thickness of the core layer (h_1=h_3=1 mm).

8. CONCLUSION

Transient response of a three layer cantilever sandwich plate with an ER core subjected to an impulse excitation was studied. A finite element model for the beam has been constructed and direct integration algorithm has been used to simulate the time-dependent behavior of the structure. Due to the considerably low yield stress of ER fluids and their characteristics in the post-yield region, the Bingham plastic model was used in derivation of FE model of sandwich beam with ER core. The results presented herein, indicated that the applied electric field and the thickness of the core layer have considerable influence on damping treatment of the structure. While the electric field values were found to have no considerable effect on the system natural frequencies at a specified core thickness, it was observed that the settling time of the structure decreases by increasing the thickness of the core layer. This can be explained by the growth of the energy lost by the ER layer.

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