THE APPLICATION OF PARTICLE SWARM OPTIMISATION TO SANDWICH MATERIAL DESIGN

Craig W. Hudson, Joe J. Carruthers and A. Mark Robinson

NewRail
School of Mechanical & Systems Engineering, Newcastle University, NE1 7RU, UK
joe.carruthers@ncl.ac.uk

ABSTRACT

Sandwich materials, consisting of two thin, stiff facings separated by a low density core, can be used to produce structures that are both light and flexurally rigid. However, the optimisation of sandwich materials is not straightforward. This is because there are typically multiple design variables and multiple design objectives. Particle swarm optimisation is a stochastic method that is capable of finding optimal solutions within complex design spaces. It aims to mimic a flock of birds under the premise that an information-sharing group working towards common objectives is more likely to find good solutions than a sole agent acting independently. The application of particle swarm optimisation to multi-objective sandwich beam problems is described here. The free variables investigated include the facing thickness, and the facing and core materials. Furthermore, for the facings, multi-ply, oriented laminate constructions are considered. Based on these inputs, sandwich beams are optimised for stiffness, mass and cost. The results show that the particle swarm optimisation algorithm is effective at finding a range of optimal solutions for the given objectives.

1. INTRODUCTION

Sandwich materials typically consist of two relatively thin, stiff facings separated by a thicker, lower density core material or structure. Such assemblies have a number of characteristics that make them attractive for applications in transport and construction. Their high mass specific stiffness and strength make them a good enabling technology for lightweighting, leading to improved performance and / or lower life cycle costs. Sandwich materials also provide opportunities for design integration, i.e. the ability to combine different functionalities within a single material construction. For example, mechanical properties such as stiffness or strength can often be combined with thermal properties such as insulation. However, because sandwich materials are usually realised through an assembly of multiple parts and materials, and because functional integration usually means a given sandwich must satisfy multiple design objectives, their design and optimisation is rarely straightforward. Even for the simplest of constructions, a designer has the challenge of selecting the most suitable facing and core materials and determining their optimum thicknesses to meet the needs of the application. Furthermore, there will often be conflicting requirements (e.g. mass versus cost) that will need to be suitably reconciled. This paper describes the application of an optimisation algorithm known as “particle swarm optimisation” to sandwich material design. The intention is to assess the algorithm’s ability to identify optimum sandwich constructions subject to multiple design objectives.

2. PARTICLE SWARM OPTIMISATION (PSO)

2.1. Description of the Algorithm

Particle swarm optimisation (PSO) [1] is a stochastic method of optimisation that is capable of finding optimal solutions within complex design spaces. It aims to mimic a flock of birds under the premise that an information-sharing group working towards common objectives is more likely to find good solutions than a sole agent acting independently.

As described by Dong et al. [2], PSO is characterised by its simple implementation and excellent performance. It is particularly well suited to navigating poorly defined design spaces in order to identify global optimum solutions and has already been employed for the design of composite laminates. Suresh et al. [3] describe the optimisation of a laminated composite box-beam for a helicopter rotor blade. The objective was to meet a certain target stiffness. Design variables included the dimensions of the box-beam and the ply orientation angles of the laminate. A comparison of results obtained using PSO and an alternative genetic algorithm showed that PSO was always able to identify solutions that were closer to the target stiffness than the genetic algorithm. Also, in a separate performance evaluation, PSO was found to require less computational effort. Kathiravan and Ganguli [4] describe a similar analysis in which the optimum ply angles were sought for a composite beam in order to maximise strength. In this study, PSO was compared against a gradient-based optimisation technique. A number of different load cases were considered. For each load case the PSO algorithm identified material constructions that were at least as strong as, or stronger, than those identified by the gradient-based method.

The basic procedure for implementing a PSO algorithm is as follows:

1. A population, or “swarm”, of particles (candidate solutions) is first initialised in the design space with pre-assigned values for “position” and “velocity”.
2. The objective values of all these solutions are then calculated.
3. The objective values are compared with one another to identify the best solutions. This is done on the basis of *non-dominance*. A non-dominated solution is one which, when compared with another solution, has at least one objective value that is superior to the other solution, or is equal to the other solution across all objective values. The identified best solutions are held in a separate global best repository.
4. Each particle’s personal best ever position is monitored and continually updated.
5. Each particle’s velocity term is then calculated based upon the previous velocity, the global best solution, and the personal best solution of that particle.
6. Each particle’s position is updated based on the calculated velocity, thereby generating the next set of solutions for the subsequent iteration.
7. Steps 2 – 6 are repeated until a stopping criterion is satisfied, e.g. a fixed number of iterations or a defined convergence requirement.

Various permutations and modifications of the above process have been employed in optimisation analyses conducted by other researchers [5-7]. Some of these were implemented in the algorithm described in this paper to meet the specific needs of the investigation. They include a fast non-dominated sort procedure described by Deb et al. [5], a crowding distance operator to promote diversity of solutions by Reddy & Kumar [6], and a mutation operator similar to that of Coello Coello [7] to enrich the searching capability of the swarm. Additionally, a separate repository was employed to store and continually update the non-dominated solutions during the analysis to provide a straightforward means of accessing the resulting data.

Previous researchers have also investigated suitable swarm parameters for given applications [8-11]. Based on their recommendations, and unless otherwise stated, the PSO algorithm used in this study employed the following default values:

- Number of particles = 20.
- Number of iterations = 100.
- Size of best solutions repository = 50.
- Mutation probability = 0.1.
- $c_1 = 2$ (a parameter controlling the amount of influence a given particle's personal best has on its new position).
- $c_2 = 2$ (a parameter controlling information sharing within the swarm).
- $w = 0.01$ (a parameter controlling the influence of the particles' previous motions).

2.2. Multi-objective Handling

As a necessary complexity, the optimisations performed in this investigation required multiple objectives to be satisfied. Multi-objective handling in optimisation studies is not always straightforward and is really a topic in its own right [12]. However, a brief outline of the fundamental methodology for retrieving optimal solutions from multi-objective problems, based upon non-dominance, is described here.

Usually in multi-objective optimisations, there is no single optimal solution. Instead, a series of solutions exist that each contain an element of optimality. Consider, for example, an ordinary beam of fixed dimensions. Suppose also that there was a requirement to optimise the mass of this beam subject to a certain minimum stiffness. If the beam material was the only variable, the optimisation would be trivial. The material with the lowest density that still met the required stiffness would be selected. Similarly, if the sole objective was to minimise the cost of the beam, the optimal material would be the cheapest option.

However, if the objective was instead to optimise both the mass AND the cost of the beam subject to a certain minimum stiffness, the situation becomes less clear. This is because it is unlikely that the material that produces the lightest solution would also provide the cheapest solution. Instead, when both objectives are considered, a trade-off boundary between mass and cost is formed. The result is a set of solutions which, when all objectives are considered, show some degree of optimal quality. The optimal solutions within this set are non-dominated with respect to all the other known solutions and are called the *Pareto-optimal set*.

3. SANDWICH BEAM ANALYSIS

A typical sandwich construction is shown in Figure 1. It consists of two thin, stiff facings separated by a thicker, low density core. The effect of the core is to significantly increase the second moment of area of the section, and hence the flexural rigidity of the sandwich, with only a small increase in weight.

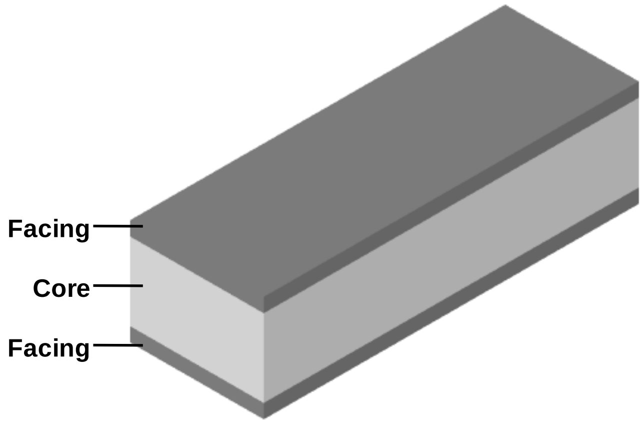

Figure 1: A typical sandwich construction.

The objective of the study was to apply PSO to identify optimal values for the facing thickness and optimal choices for the facing and the core materials of a sandwich beam subject to two design objectives. The first design objective was to maximise the mass specific flexural rigidity, D_m, as defined in Equation (1).

$$D_m = \frac{D}{m} \qquad (1)$$

where D is the flexural rigidity of the sandwich given by [13]:

$$D = E_f \frac{bt^3}{6} + E_f \frac{btd^2}{2} + E_c \frac{bc^3}{12} \qquad (2)$$

in which E_f and E_c are the Young's moduli of the facing and core materials respectively, b is the width of the sandwich beam, t is the thickness of one sandwich facing, c is the thickness of the sandwich core, and d is the distance between the centrelines of the opposing facings (= $t + c$). Equation (2) is applicable to sandwiches in which the two facings are (quasi-) isotropic and equal.

and m is the mass of the sandwich given by:

$$m = Lb(2\rho_f t + \rho_c c) \qquad (3)$$

in which ρ_f and ρ_c are the densities of the facing and core materials respectively, and L is the length of the sandwich beam.

The second design objective was to maximise the flexural rigidity per unit cost, D_c:

$$D_c = \frac{D}{C} \qquad (4)$$

where C is the total material cost of the sandwich.

Two sandwich beam optimisation problems were considered. Optimisation (1) was relatively straightforward, with the facing material restricted to aluminium and with a limited choice of isotropic core materials. For Optimisation (2), the choice of available materials (and hence the design space) was significantly expanded. Furthermore, Optimisation (2) allowed the use of oriented fibre reinforced polymer laminates for the facings. Table 1 summarises the two optimisation problems and Table 2 presents the material property data that was used as input to the analyses.

	Optimisation (1)	Optimisation (2)
Variables	• Core material • Facing thickness	• Core material • Facing material, including (if applicable): - Fibre material - Matrix material - Fibre volume fraction - Fibre orientation • Facing thickness
Objectives	Maximise flexural rigidity per unit mass, D_m, and flexural rigidity per unit cost, Dc	
Constraints	• Facing material = aluminium • Sandwich length, L, = 500 mm • Sandwich width, b, = 50 mm • Sandwich thickness = 50 mm	• Sandwich length, L, = 500 mm • Sandwich width, b, = 50 mm • Sandwich thickness = 50 mm

Table 1: Definition of the two sandwich optimisations investigated.

For the fibre reinforced polymer laminate facings, the following assumptions or limits were imposed upon the analysis:

- The thickness of a single lamina ply was taken to be 0.25 mm.
- Lamina orientations of 0^o, 90^o, $+45^o$ and -45^o were considered. Only balanced, symmetric, quasi-isotropic laminates were permitted.
- The number of plies within a facing laminate could vary as 4, 8, ..., 20, giving a range of discrete integer facing thicknesses of 1 – 5 mm. For the non-reinforced materials, facing thickness was treated as a continuous variable between the same limits.
- The fibre volume fraction, v_f, was treated as a discrete variable with values of 0.30, 0.35, ..., 0.70. The stiffness properties of each lamina were calculated from this fibre volume fraction and from the properties of the fibre and matrix materials [14].
- Classical laminate theory was used to derive the stiffness properties of the overall facing laminate.

	Material	Young's Modulus (GPa)	Shear Modulus (GPa)	Poisson's Ratio	Density (kg/m^3)	Cost (£/kg)
Core Materials	Balsa	4.7	-	-	190	7.72
	*Polystyrene foam: closed cell	0.028	-	-	50	1.55
	*Phenolic foam: closed cell	0.065	-	-	120	5.35
	Polymethacrylimide foam: rigid	0.088	-	-	75	45.75
	*Polyvinylchloride foam: rigid closed cell	0.020	-	-	30	10.70
Non-reinforced Facing Materials	*Aluminium: 5251, H4	72	-	-	2690	1.07
	Fir	13	-	-	435	0.89
	Hardboard: tempered	9	-	-	1130	0.42
	Low alloy steel: AISI 8650	211	-	-	7850	0.48
	Plywood: beech	5.5	-	-	750	0.89
Reinforced Facing Fibres	Carbon fibre: high modulus	380	170	0.11	1825	26.40
	Carbon fibre: high strength	235	105	0.11	1820	16.80
	Glass fibre: E grade	79	33	0.22	2575	1.50
Reinforced Facing Matrices	Epoxy resin	2.41	0.86	0.40	1255	1.34
	Phenol formaldehyde: casting resin	3.80	1.40	0.39	1280	1.01
	Polyester: cast, rigid	3.24	1.17	0.39	1220	1.10

Table 2: The material property data employed within the optimisation analyses. Those marked with an * were used in Optimisation (1). All the materials were considered in Optimisation (2). The figures are mean values taken from the CES Selector software [15].

4. RESULTS & DISCUSSION

4.1. Optimisation (1)

Figure 2 shows the non-dominated solutions identified by the PSO algorithm for the first, more straightforward, optimisation problem. The corresponding numerical results are shown in Table 3. It can be seen that two distinct optimal regions were found. Optimised constructions using a polystyrene foam core were identified for facing thicknesses of 3.1 – 3.7 mm. A broader range of optimised constructions were also identified using a PVC foam core in conjunction with 2.5 – 4.9 mm facings. The polystyrene-based constructions were generally more cost effective, whilst the PVC-based constructions offered lighter solutions. The phenolic foam core did not appear in the Pareto-optimal set at all. This leads to the conclusion that this material is not as well suited as the other two cores with respect to the given objectives.

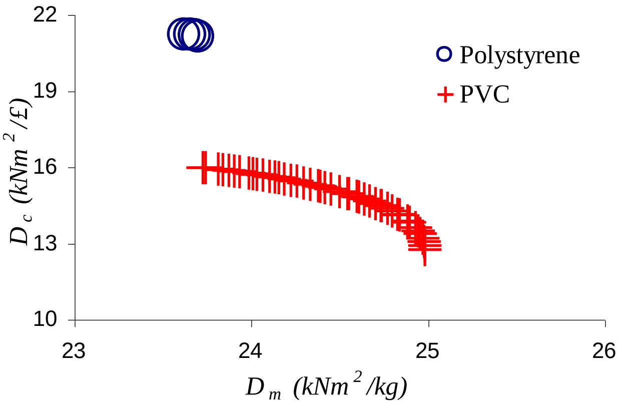

Figure 2: Identified solutions for Optimisation (1).

Core	Facing	t (mm)	m (kg)	Total cost (£)	D_m (kNm^2/kg)	D_c (kNm^2/£)
Polystyrene	Aluminium	3.1 – 3.7	0.52 – 0.60	0.59 – 0.67	24	21
PVC	Aluminium	2.5 – 4.9	0.40 – 0.76	0.79 – 1.14	24 – 25	13 – 16

Table 3: Optimised constructions and performance indicators for Optimisation (1).

4.2. Optimisation (2)

The results for the second, considerably more complex, sandwich optimisation are shown in Figures 3 – 4 and Table 4. Because of the increased complexity of the problem it was necessary to increase the number of iterations from 100 to 20,000 in order to obtain a complete set of non-dominated solutions. This very significant increase in the number of iterations was mainly because of the stipulation that only balanced, quasi-isotropic laminates would be permitted for compatibility with Equation (2). Any non-isotropic solutions generated by the algorithm, of which there were many, were automatically rejected. The inefficiency of this approach is acknowledged and will be addressed in future work.

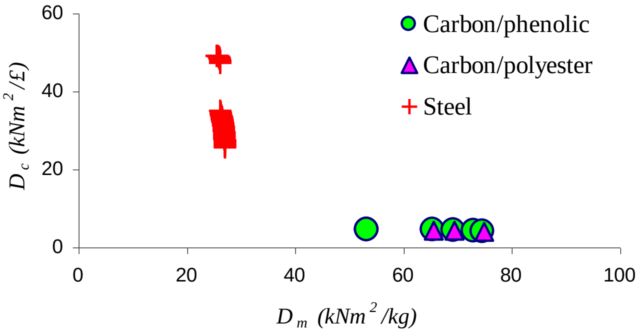

Figure 3: Identified facing solutions for Optimisation (2).

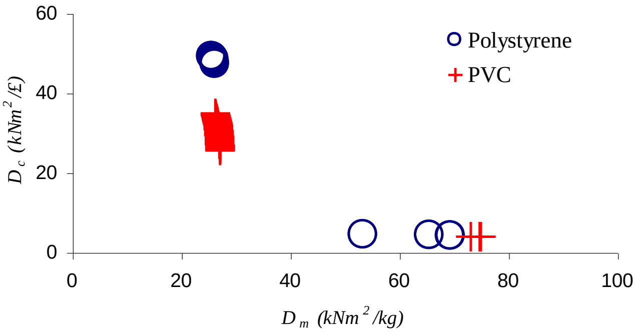

Figure 4: Identified core solutions for Optimisation (2).

Core	Facing	t (mm)	m (kg)	Total cost (£)	D_m (kNm^2/kg)	D_c (kNm^2/£)
Polystyrene	HM carbon / phenolic	1.0 – 3.0	0.16 – 0.33	1.98 – 5.72	53 – 69	4
Polystyrene	HM carbon / polyester	2.0 – 3.0	0.24 – 0.33	3.85 – 5.73	66 – 70	4
Polystyrene	Steel	1.9 – 3.3	0.88 – 1.48	0.49 – 0.77	26	47 – 49
PVC	HM carbon / phenolic	2.0 – 3.0	0.22 – 0.31	4.16 – 6.02	73 – 75	4
PVC	HM carbon / polyester	3.0	0.31	6.02	75	4
PVC	Steel	1.5 – 3.0	0.68 – 1.35	0.72 – 1.02	26 – 27	25 – 35

Table 4: Optimised constructions and performance indicators for Optimisation (2).

Firstly, it is clear that simply by expanding the range of material options available to the particles within the design space, a different set of solutions has been obtained compared to Optimisation (1). Polystyrene and PVC remain the core materials of choice, but aluminium has now been dropped as a facing material in favour of either steel (generally cheaper) or fibre reinforced polymers (generally lighter). Table 5 compares an optimised construction from Optimisation (1) with a number of optimised constructions from Optimisation (2). It can be seen that the expanded material database has yielded solutions with improved values for objective functions, D_m and D_c.

Core	Facing	t (mm)	m (kg)	Total cost (£)	D_m (kNm^2/kg)	D_c (kNm^2/£)
PVC	Aluminium	3.0	0.48	0.86	24.90	13.81
Polystyrene	HM carbon / polyester, v_f = 0.7, [0°/0°/90°/90°]s	2.0	0.24	3.85	65.68 (+164%)	4.16 (-70%)
PVC	HM carbon / phenolic, v_f = 0.7, [0°/0°/90°/90°]s	2.0	0.22	4.16	73.13 (+194%)	3.88 (-72%)
Polystyrene	Steel	3.1	1.39	0.73	25.78 (+4%)	48.96 (+255%)
PVC	Steel	2.3	1.05	0.88	26.79 (+8%)	31.69 (+130%)

Table 5: The benefits of expanding the design space – a comparison between an optimised construction obtained from Optimisation (1) (top, greyed) and optimised constructions based on the larger material database employed in Optimisation (2).

In terms of the fibre reinforced polymer facings, a number of distinct trends emerged:

- Phenolic and polyester matrix materials were favoured over epoxy. This appears to contradict normal practical experience of fibre reinforced polymers in which epoxy-based composites are generally expected to be stiffer than those based on polyesters or phenolics. However, it can be seen from Table 2 that the material property data [15] used as input to the optimisation contained higher stiffness and lower cost values for polyester and phenolic compared to epoxy, for similar densities. So whilst one might question the appropriateness of the input data used, the optimisation does appear to have identified the best performing matrix candidates from that input data.

- In terms of lay-up, [0°/90°] constructions were preferred to [+45°/-45°] constructions or combinations thereof. This is consistent with the predictions of classical laminate theory, in which [0°/90°] laminates are indeed stiffer when loaded along the principal material directions.

- For all optimal fibre reinforced polymer facing solutions, the PSO recommended a fibre volume fraction, v_f, of 0.7, the highest possible value. This is reasonable given the material input data employed and rule-of-mixtures approach to the calculation of lamina density, Young's modulus and cost. Considering the fibre and matrix material property data in Table 2, it can be shown that lamina Young's modulus increases with fibre volume fraction at a greater rate than either density or cost. Of course, this simple rule-of-mixtures approach neglects important considerations such as the fibre-matrix interface, but it does again illustrate that the PSO has produced sensible results within the scope of the model employed.

- With respect to facing thickness, t, the PSO has identified optimum values in the range of 2 – 3 mm, i.e. values that are intermediate within the permitted range of 1 – 5 mm.

5. CONCLUSIONS

The application of a particle swarm optimisation algorithm to sandwich material design has been described. It has been demonstrated that the algorithm was effective in identifying optimal solutions to the proposed problems, and that those solutions were sensible within the limitations of the material models and data employed. Having validated PSO for these simple sandwich cases, it is now in a position to be deployed for more sophisticated analyses.

REFERENCES

1- Kennedy, J., Eberhart, R., "Particle swarm optimization", *Proceedings of the IEEE Conference on Neural Networks*, Perth, Australia, 1995;4:1942-1948.

2- Dong, Y., Tang, J., Xu, B., Wang, D., "An application of swarm optimization to nonlinear programming", *Computers & Mathematics with Applications*, 2005;49:1655-1668.

3- Suresh, S., Sujit, P.B., Rao, A.K., "Particle swarm optimization approach for multi-objective composite box-beam design", *Composite Structures*, 2007;81:598-605.

4- Kathiravan, R., Ganguli, R., "Strength design of composite beam using gradient and particle swarm optimization. *Composite Structures*, 2007;81:471-479.

5- Deb, K., Pratap, A., Agarwal, S., Meyarivan, T., "A fast and elitist multiobjective genetic algorithm: NSGA-II", *IEEE Transactions on Evolutionary Computation*, 2002;6:182-197.

6- Reddy, M.J., Kumar, D.N., "An efficient multi-objective optimization algorithm based on swarm intelligence for engineering design", *Engineering Optimization*, 2007;39:49-68.

7- Coello Coello, C.A., Pulido, G.T., Lechuga, M.S., "Handling multiple objectives with particle swarm optimization", *IEEE Transactions on Evolutionary Computation*, 2004;8:256-279.

8- Zhang, L.P., Yu, H.J., Hu, S.X., "Optimal choice of parameters for particle swarm optimization", *Journal of Zhejiang University: Science*, 2005; 6A:528-534.

9- Coello Coello, C.A., Lechuga, M.S., "MOPSO: A proposal for multiple objective particle swarm optimization", *Proc. Congr. Evolutionary Computation (CEC'2002)*, 2002;1051-1056.

10- Perez, R.E., Behdinan, K., "Particle swarm approach for structural design optimization", *Computers & Structures*, 2007;85:1579-1588.

11- Eberhart, R.C., Shi, Y., "Comparing inertia weights and constriction factors in particle swarm optimization", *Proceedings of the IEEE Conference on Evolutionary Computation*, 2000;1:84-88.

12- Deb, K., *Multi-Objective Optimization using Evolutionary Algorithms*, Wiley, Chichester, 2001.

13- Zenkert, D., *An Introduction to Sandwich Construction*, EMAS Publishing, 1995.

14- Matthews, F.L., Rawlins, R.D., *Composite Materials: Engineering and Science*, Chapman and Hall, London, 1994.

15- *CES Selector Version 4.6*, Grata Design Limited, 2005.