FREE VIBRATIONS OF SANDWICH PANELS WITH A TRANSVERSELY FLEXIBLE CORE AND TEMPERATURE DEPENDENT CORE PROPERTIES

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ABSTRACT

The free vibration analysis of sandwich panels with a core that is flexible and compliant in the vertical direction and with temperature dependent mechanical properties is presented. The analysis is based on the high-order sandwich panel theory approach (HSAPT), and the equations of motions along with the appropriate boundary conditions are derived using the Hamilton principle. The study investigates the role of increasing temperature, through the degradation of the mechanical properties of the core, on the free vibration response of structural sandwich panels. The mathematical formulation includes two types of computational models. At first, following the HSAPT approach, the unknowns include the displacements of the face sheets as well as the shear stress in the core. This is referred to as the "mixed formulation" (MF). Secondly, it is assumed that the through-thickness distributions of the vertical and horizontal core displacements can be represented as polynomials, following the results of the HSAPT static case, and the effect of the variable mechanical properties are implemented directly. This is referred to as the "displacement formulation" (DF). The vibration response includes modes where the faces move laterally in-phase ("bending modes") and laterally out-of-phase ("pumping modes"), as well as in-plane modes where the faces move in-phase and out-of-phase. The results reveal a significant reduction of the eigenfrequencies as well as a shifting of the eigen-modes from higher to lower modes with increasing temperature.

1. INTRODUCTION

A structural sandwich panel can be considered as a special type of composite laminate where two thin, stiff, strong and relatively dense face sheets, which are often by themselves composite laminates, are separated by and bonded to a thick, lightweight and compliant core material. Such sandwich structures are being used as primary and secondary structural members due to their superior qualities in terms of: high strength and stiffness to weight ratios, ease of manufacturing, acoustic and thermal insulation, and flexibility in design.

Sandwich structures are often subjected to aggressive service conditions which may include elevated temperatures, which lead to thermally induced deformation loads and degradation of the mechanical properties.

The material properties of the constituents of a sandwich structure generally depend on the temperature field imposed. However, this dependency is usually ignored in the design process, even for applications where the induced temperature field causes the material properties to degrade significantly as the temperature increases. In many modern sandwich panel applications, the core material is made of polymer foam where significant changes in the properties may occur in the operating range of the temperatures. A PMI type of foam such as Rohacell® looses its heat distortion resistance at about 200^{o}C [1], while a PVC foam cores such as the various grades of Divinycell loose the strength at about 80-100^{o}C [2-4]. Hence, it becomes extremely important to understand how the degradation of the core properties affects the mechanical response including the load and the free vibration response.

Sandwich panels have been considered by many researchers. It is usually assumed that the core is “anti-plane” and incompressible, see the textbooks by Allen [5] and Zenkert [6]. Moreover, the models adopted for predicting the linear and non-linear load-response of sandwich panels are usually based on the “equivalent single layer” approach (ESL), where the layered panel is replaced by an equivalent single layer with homogenized (equivalent) mechanical properties, see Mindlin first-order theory [7], and Reddy's high-order theories [8]. The classical sandwich, ESL and high-order models usually disregard the changes in the height of the core (i.e. the compressibility) when the panel is deformed.

Only a few research works have addressed the response of sandwich panels with temperature-dependent mechanical properties, including Chen and Chen [9] and [10], Gu and Asaro [11], Birman *et al.* [12] and Liu *et al.* [13]. These works are based either on classical sandwich, ESL/FOSD or high-order models, and they all disregard the changes in the height of the core (compressibility) during the deformation of the sandwich panel. Accordingly, they yield inaccurate results when used for sandwich panels with a “soft/compliant” core, e.g. like a polymer foam

An approach that models the layered sandwich panel as made of two face sheets and a core layer that are interconnected through fulfilment of equilibrium and compatibility conditions, and thus incorporates the vertical flexibility of the core, offers significant advantages when considering sandwich structures with compliant/”soft” cores. This approach has been implemented through a variational principle into the High-Order Sandwich Panel Theory (HSAPT), and has been successfully used for the analysis of various linear and non-linear sandwich panel problems. A brief list of references include: Frostig [14], Frostig [15], Frostig *et a*l. [16].

Recently, Frostig and Thomsen [17] and [18] used the HSAPT approach to investigate the thermal buckling and non-linear thermo-mechanical response along with localized effects of sandwich panel with a compliant/“soft” core assuming temperature dependent mechanical properties.

However, the vibration response and its dependency of temperature has hitherto not been considered in open literature. The free vibration response of a sandwich structure with a compliant core is associated with both overall and through-thickness modes (the latter being denoted as local modes). In general the overall or global modes correspond to lower eigenfrequencies, and the local modes to higher eigenfrequencies. However, as the mechanical properties of the core degrade as a result of the elevated temperature, the rigidity of the core is reduced and the local modes may shift from higher frequencies into lower ones.

The objective of the investigation reported herein was to model the free vibration response of foam cored sandwich panels using the HSAPT approach including temperature dependent mechanical core properties. In particular, the investigation has studied how the eigenfrequencies and the corresponding eigenmodes change with increasing temperature which leads to degradation of the mechanical core properties. The results presented herein present a brief summary of the results presented in 2 recent papers by Frostig and Thomsen [19-20], to reference is made for further details.

2. MATHEMATICAL FORMULATION

The mathematical formulation includes two types of computational models. The first model follows the HSAPT approach in which the unknowns include the displacements of the face sheets as well as the core shear stress. This is referred to as the “mixed formulation” (MF). Secondly, it is assumed that the through-thickness distributions of the vertical and horizontal core displacements can be represented as polynomials which are specified a priori following the results of the HSAPT static case, and the effect of variable mechanical core properties are implemented directly. This is referred to as the "displacement formulation" (DF). It should be noticed that DF approach involves higher-order core stress resultants that have no physical interpretation, and the model yields higher-order modes that involve vibrations through the depth of the core that the MF (HSAPT) model cannot detect.

The mathematical formulation consists of a general section that presents the dynamic equations applicable to the two cmputational models, and which yields the governing equations of motions along with the boundary conditions for each model.

The equations of motions of the free vibration response are derived through the Hamilton principle which extremizes the Lagrangian that consists of the kinetic and the internal potential energy as follows:

$$\int_{t_1}^{t_2} \left(-\delta T + \delta U \right) \mathrm{d}t = 0 \qquad (1)$$

where T is the kinetic energy, t is the time coordinate that varies between the times t_1 and t_2; and U and V are the internal and external potential energies. The complete expressions for the first variations of T and U are given in [19].

The geometry, assumed temperature distributions and stress resultants are shown in Fig. 1. It is assumed that the displacements face sheets ($j=t,b$) obey the classical Bernoulli-Euler assumptions, and kinematic relations corresponding to small linear displacements of the face sheets and the core are adopted.

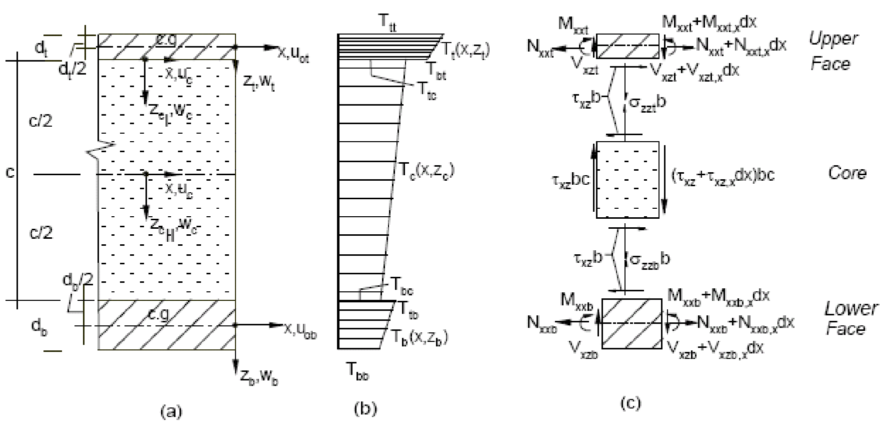

Figure 1: Geometry, temperature and stress resultants: (a) Geometry (MF and DF models); (b) Temperature Distribution; (c) Stress resultants (MF model).

The differences between the two sandwich models are a result of the description of the accelerations and the displacements in the core, as well as the solutions for the core stress and the displacements fields.

2.1 Mixed Formulation (MF)

The core, in this model, is regarded as a medium that transfers its inertia loads to the face sheets rather than resisting them by itself. Hence, the distributions of the accelerations through the depth of the core are assumed to follow the distributions of the static displacements under fully distributed loads. Thus, the dynamic stress and displacements fields of the core are assumed to be identical with the static stress and displacements fields without inertia loads terms. The distributions of the static displacements through the depth of the core are non-linear; quadratic for the vertical displacement, and cubic for the in-plane displacements when the mechanical properties of the core are uniform through its depth. These (non-linear) polynomial patterns (through the depth of the core) are usually a result of gradients of the vertical shear stresses, and they differ significantly from the linear distributions when localized loads or restrictive constraints are imposed on the sandwich structure. However, when fully distributed loads, such as inertia loads, are applied to the face sheets, the localized effects diminish leading to displacements distributions with small non-linearities. Hence, it can be rationally assumed that these distributions are linear rather then non-linear. Thus, the distributions of the accelerations, through the depth of the core, are assumed to be linear as follows:

$$u_{c,tt}(x,z_c,t) = u_{t,tt}(x,z_t = d_t/2,t)(1-\frac{z_c}{c}) + u_{b,tt}(x,z_b = -d_b/2,t)\frac{z_c}{c}$$
$$w_{c,tt}(x,z_c,t) = w_{t,tt}(x,z_t,t)(1-\frac{z_c}{c}) + w_{b,tt}(x,z_b,t)\frac{z_c}{c} \qquad (2)$$

This simplification is applied to the kinetic inertia terms only. The equations of motion are derived using the Hamilton's principle, Eq. (1); the expression for the kinetic energy along with the presumed accelerations with linear through-thickness distributions, Eqs. (2); the expression for the internal potential energy along with the kinematic relations of the faces and the core, and finally the compatibility conditions at the face-core interfaces. The resulting equations are given in [19].
In order to achieve an explicit description of the equations of motion of the face sheets in terms of the unknowns of the core, the displacement and thermal fields of the core must be determined first. The fields of the core are determined assuming that the core is orthotropic with the following constitutive relations:

$$\frac{\sigma_{zz}(x,z_c)}{E_{zc}(T_c(x,z_c))} = \frac{\partial}{\partial z_c} w_c(x,z_c) - \alpha_c T_c(x,z_c)$$
$$\frac{\tau_c(x,z_c)}{G_{xzc}(T_c(x,z_c))} = \frac{\partial}{\partial z_c} u_c(x,z_c) + \frac{\partial}{\partial x} w_c(x,z_c) \qquad (3)$$

where σ_{zz} and τ_c are the core transverse normal and shear stresses, w_c and u_c are the core transverse and in-plane displacements, T_c is the core temperature, x and z_c are the in-plane and transverse core coordinates, and $E_{zc}(T_c(x,z_c))$ and $G_{xzc}(T_c(x,z_c))$ are the vertical Young´s and shear moduli of the core that are known functions of the temperature distribution, respectively.
It is assumed that the temperature distribution through the depth of the core is linear with a gradient between the two face sheets.
The governing equations consist of a set of partial differential equations expressed in terms of space coordinates and time of the order of fourteen, see [19]. It should be noticed that the temperature induced deformation causes the sandwich panel to deform, and that the sandwich panel will oscillate with respect to the static deformed shape. The solution of the set of governing equations can be achieved numerically for general boundary conditions and external dynamic loads, or analytically in closed-form for the particular case of a simply-supported unidirectional panel [19].

2.2 Displacement Formulation (DF)

The DF model is used to investigate the accuracy of the results of the MF model, which uses the linear distributions of the accelerations through the depth of the core, see Eqns. (2). In the DF model quadratic and cubic polynomials are used to describe the variation

of the accelerations through the depth of the core. Hence, the DF formulation describes a sandwich panel that is more flexible than predicted using the MF model.
The advantage of the DF formulation is that the dynamic loads as well as its degrading mechanical properties are directly included in the equations of motion of the core, and that they are not a result of the interaction with the upper and the lower face sheets. This is achieved at the expense of predicting higher-order bending moments and shear stress resultants in the core that lack any physical interpretation. In addition, any constraint imposed on the core can be fulfilled only in the global sense, and not in the differential sense as is the case with the MF model. Thus, the MF is more physically consistent than the DF model in an overall sense.
The formulation of the DF model follows the same steps as the MF model, using the same basic equations, except that in this case the unknowns are the displacements of the face sheets and the core. In order to achieve this goal, the displacements fields of the core are assumed *a priori*, using the quadratic and cubic polynomial distribution of the static displacement fields of a core with uniform properties, see [14]. Here, the coefficients of these polynomials are the unknowns, and they are determined through the variational principle. The pre-assumed displacement fields of the core read:

$$\begin{aligned} u_c(x, z_c, t) &= u_o(x, t) + u_1(x, t)\, z_c + u_2(x, t)\, z_c^2 + u_3(x, t)\, z_c^3 \\ w_c(x, z_c, t) &= w_o(x, t) + w_1(x, t)\, z_c + w_2(x, t)\, z_c^2 \end{aligned} \qquad (4)$$

where u_k *(k*=0,1,2,3*)* are the unknowns of the in-plane displacements of the core, and w_l *(l*=0,1,2*)* are the unknowns of the core vertical displacements, respectively. The distributions of the accelerations and the velocities of the core are assumed to follow the static displacements, similar to the general approach that ignores the effect of the inertia terms, and they equal their second and first derivatives with respect to the time coordinate.
The governing equations of motion are formulated in terms of the fifteen unknowns: the longitudinal in-plane and vertical displacements of the face sheets, four Lagrange multipliers and the eleven polynomial coefficients of the core [19].
Also here the temperature induced deformation causes the sandwich panel to deform, and it oscillates with respect to the static deformed shape.
The set of governing equations does not have a general closed-form analytical solution. However, for the particular case of a simply-supported sandwich panel a closed-form solution exists.

3. NUMERICAL STUDY

The numerical study investigates the role of degrading core properties, as a result of elevated temperature, on the free vibration response of a unidirectional sandwich panel that is simply supported at the edges of the upper and the lower face sheets, and with the vertical displacements of the core prevented at the edges. The analyses have been conducted using the MF and DF models described in the preceding text. The considered sandwich panel configuration is shown in Fig. 2(a). For full details about the solutions of the MF and DF models as well as the numerical results, reference is made to [20].
A symmetric sandwich panel with a light PVC foam core of width 60 mm has been considered. The sandwich panel consists of two identical laminated face sheets and a PVC foam core. The face sheets are assumed to be glass/epoxy laminates with a thickness of 6 mm, an equivalent Young´s modulus of 18 GPa, and a density of 2000 kg/m^3. The foam core is assumed to be PVC foam, Divinycell HD100, with 60 mm thickness, Young´s and shear moduli of 85 MPa and 16 MPa, respectively, at room

temperature, and a density of 100 kg/m^3. The temperature field within the core is assumed to vary linearly across the height, see Fig. 2(a). The moduli of the core are temperature-dependent, and the $E_{cz}(T)$, $G_{xzc}(T)$ vs. temperature (T) plots appear in Fig. 2(b) in the temperature range of 20-80^{o}C based on the data [3-4].

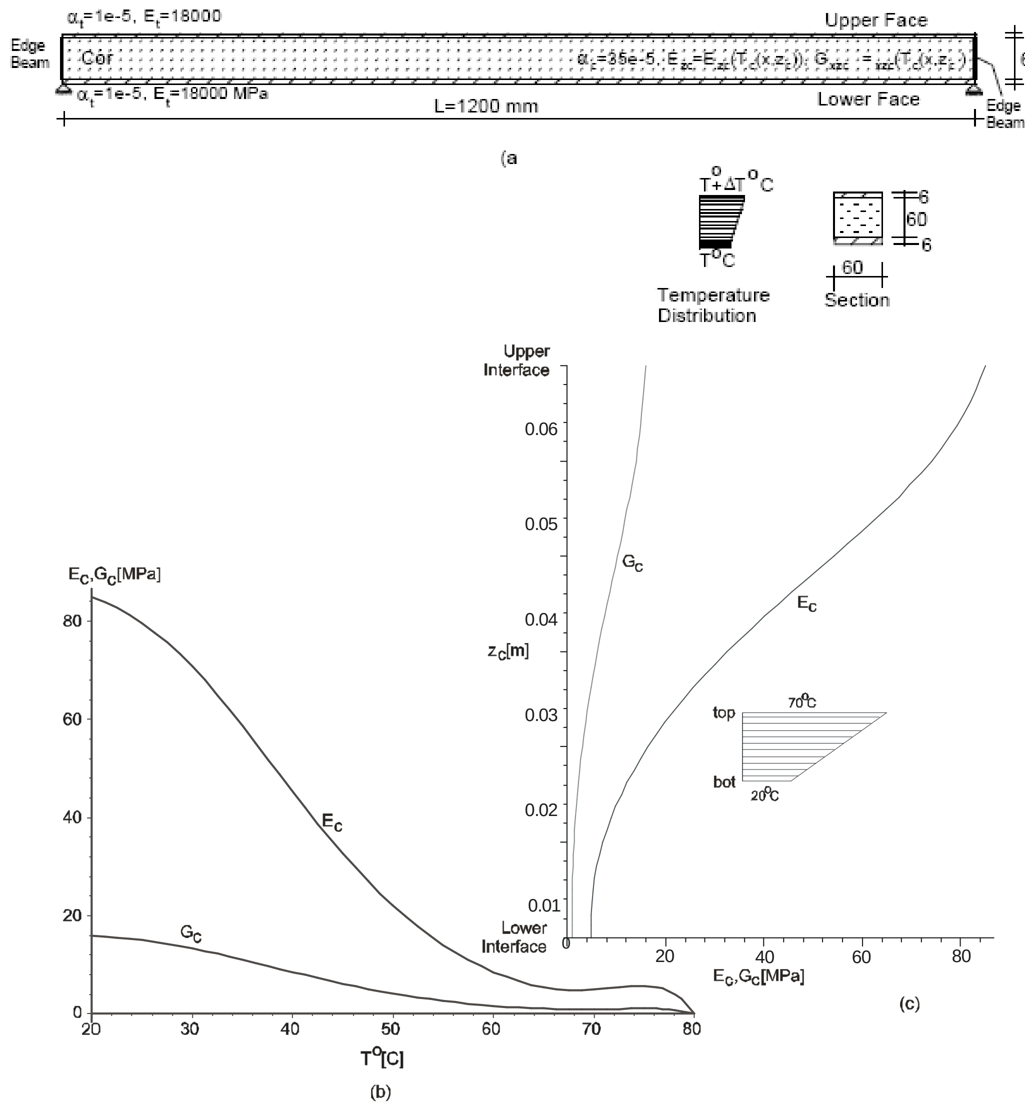

Figure 2: (a) Geometry of considered sandwich panel configuration and imposed temperature distribution; (b) Core moduli vs. versus temperature [3-4]; (c) Through-thickness variation of core moduli for a specific linear temperature distribution. (T=20 ^{o}C at lower face , and ΔT=50 ^{o}C across core height).

The through-thickness distributions of the core E and G moduli appear in Fig. 2(c) for the case of a thermal gradient of ΔT=50 ^{o}C across the core height. Two different cases of temperature distributions have been considered, see Fig. 2a. For the first case the the distribution is uniform and changing from 20 to 75 ^{o}C, while for the second case the temperature distribution is linear with a gradient ΔT across the core height, where the temperature at the lower face sheets remains at 20 ^{o}C and the temperature at the upper face sheet changes from 20 to 75 ^{o}C.

The non-dimensional eigenfrequencies that correspond to the first sinus half wave, m=1, at room temperature, T=20^{o}C and at T=75^{o}C, with a uniform and a linear temperature field, obtained using the MF and DF models appear in Table 1. The eigenfrequencies have been normalized with respect to the fundamental bending eigenfrequency of an equivalent single layer sandwich panel with a bending rigidity that equals the overall flexural rigidity of the considered sandwich panel.

Mode Number	Uniform Temp. Dist.		Linear Temp. Dist.	
	MF model	DF model	MF model	MF model
1	0.6479* *0.2202***	0.6479 *0.2202***	0.6337* *0.3246***	0.6377 *0.3288***
2	11.5589* *6.2785***	11.4430* *6.2764***	11.5589* *9.6875***	11.4320* *10.5967***
3	16.2939* *11.5590***	16.2701* *8.2739***	16.0428* *11.5590***	16.0221* *10.9097***
4	24.2916* *12.7115***	24.2872* *12.4318***	23.3905* *13.1102***	23.3846* *13.1313***
5		38.3922* *13.725***		36.8862* *21.7500***
6		73.1858* *19.3116***		70.4385* *52.0288***
7		87.6079* *22.6428***		84.2340* *61.5314***

Table 1: Non-dimensional eigenfrequencies, $\omega/\omega_{clas,}$ of the MF and DF models with m=1, at various temperatures and gradients for a uniform and a linear through-thickness distribution. Uniform temperature: *at T=20 °C, ** at T=75°C (in italic). Linear through-thickness distribution: *at ΔT=10 °C (T_t=30,T_b=20), **at ΔT=55°C (T_t=75, T_b=20).

The eigenfrequencies obtained for the first wave number, see Table 1, reveal that the values decrease significantly with increasing temperature, which causes degradation of the core properties. The results of the two models coincide at the lower eigenfrequencies. For the higher modes the DF results are always lower than the MF resultsl, which reflects that the DF model is more flexible than the MF model.

Generally, a sandwich panel with *uniform* mechanical properties displays four typical eigenmodes. Mode 1 corresponds to an overall vibration mode where the two face sheets undergo a vertical in-phase displacements (bending mode); Mode 2 corresponds to identical in-plane displacement of the two face sheets (in-phase axial mode); Mode 3 corresponds to in-plane displacements of the face sheets that are out of phase (out-of-phase axial mode); Mode 4 corresponds to vertical displacements of the face sheets that are in opposite directions (pumping mode). This corresponds to the eigenmodes given in Table 1 for a uniform temperature of T=20 °C.

However, when the uniform temperature increases to T=75 °C the first eigenmode coincides with the global mode bending mode, while the second eigenmode corresponds to the pumping mode rather then to the axial mode, which reflects a shift of the higher modes to lower modes as the temperature is increased, and the vertical rigidity of the core degrades. The third and the fourth modes correspond to the axial modes in phase and out of phase, respectively.

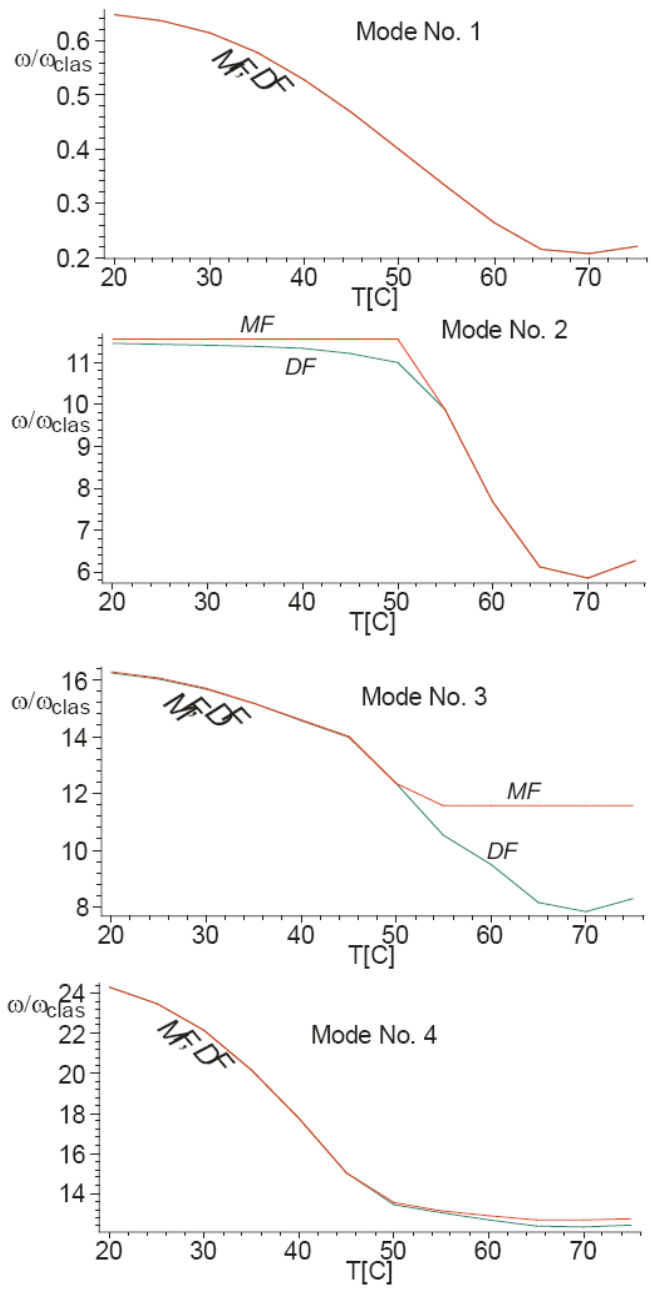

Figure 3: Eigenfrequencies vs. temperature assuming a uniform temperature distribution for the first four modes corresponding to the MF and DF models. MF model:————; DF model:————.

In Fig. 3, the eigenfrequency values vs. the uniform core temperature is presented for the first wave number. In the first mode (Mode no. 1) the results of the two computational models coincide. In the second mode (Mode no. 2) the eigenfrequency of the MF model remains almost unchanged up to 50°C after which it drops, while for the DF model the eigenfrequency reduces slightly up to 50°C. At temperatures above 55 °C the two models coincide. In the third mode (Mode 3) a large difference between the results of the two models above 50°C is observed. In the fourth mode the results almost coincide. The difference in the results is a result of the shifting of the higher eigenmodes to lower eigenmodes with increasing temperature.

CONCLUSIONS

The effect of degrading core material properties with increasing temperature has been investigated for the problem of free vibrations of sandwich panels with compliant and temperature dependent core properties (e.g. polymer foam cores). Two computational models for the analysis of sandwich structures have been discussed. The first model is a Mixed Formulation (MF) based on the HSAPT approach where the unknowns are the displacements of the face sheets and the core shear stress. In this model the stress resultants can be attributed a clear physical interpretation, and any conditions on the core are imposed in the strict differential sense. The second model is a Displacement Formulation (DF) which assumes that the through-thickness distributions of core vertical and horizontal displacements and accelerations are quadratic and cubic polynomials in terms of the the thickness coordinate, respectively. Hence, the unknowns in this model consist of the coefficients of these polynomials together with the face sheet displacements. This model implicates the existence of higher-order core stress resultants, which cannot be associated with any meaningful physical interpretation. Moreover, constraints that are imposed on the core may be defined in the overall sense only.

The two models have been used for conducting and extensive numerical study. This paper has presented results for the simple case of a simply supported unidirectional sandwich panel with composite face sheets and a Divinycell HD100 PVC core material. The obtained results reveal a significant reduction of the eigenfrequencies as well as a shifting of the eigenmodes from higher modes to lower ones with increasing temperature.

Sandwich structures often utilize compliant polymer foam core materials that display significant degradation of their mechanical properties with increasing temperatures. The degradation effects may be significant even within the working range of temperatures for realistic sandwich structures, and the results presented herein demonstrates that thermal fields imposed on such sandwich structures may influence the vibration response significantly.

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