ULTRASONIC WELDING OF THERMOPLASTIC COMPOSITES, MODELING AND SIMULATION OF THE PROCESS.

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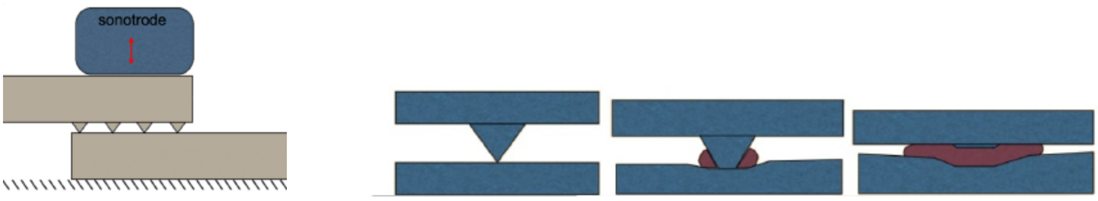
ABSTRACT

The process of ultrasonic welding is widely used in the industry. Nevertheless, its numerical modeling, essential for the aeronautic industry, is quite difficult because of the two time scales present in the process. After explaining the principle of the welding, a method of time homogenization is presented in order to write down three different thermal and mechanical systems of equations. Since one of those problems implies moving free surface, a numerical tool using the level-set method was used to solve them.

Keywords : Polymer, Welding, Time-homogenization, Visco-elasticity, Numerical simulation, Level-set.

1 INTRODUCTION

This work aims at modeling an original welding process for composite material with thermoplastic matrix. Triangular bulges, called "energy directors" are molded with matrix only, on a width of two centimeters on the border of one of the plate to be welded. The two plates are then positioned in order to cover each other on the width of the energy directors. The process then consists in applying an ultrasonic (20 KHz) sinusoidal compression stress between the two plates as shown in figure 1(a). The strain concentrates at the



(a) Assembly overview. (b) Melting and flowing of an energy director.

Figure 1: Ultrasonic welding of two plates

interface, in the energy directors, that melt because of the viscous dissipation [5]. The triangles then progressively flow on the whole interface and perform welding of the two composite planes (see figure 1(b)). Even if no fiber crosses the interface, the cover is wide enough to transmit the stress which allows the process to be used for large scale assemble.

The main problem when modeling and simulating such a process comes from the existence of two time scales, which would induce very fine time steps and therefore huge computation times. This process has been subject for few models. First we can mention Benatar [2, 5] who described the process as the succession of five phenomena physical phenomena. Some authors like Tolunay et al. [13] Suresh et al. [12] or Wang et al. [14] described the mechanical dissipation in a linear harmonic viscoelastic framework. Nevertheless, their models were limited to the thermal aspects only. We propose here a more general framework, which also enables to take into account the flowing of polymer.

2 MODELING OF THE PROCESS

2.1 General Thermo - Mechanical problem

Let us consider a single energy director as described in figure 2. The objective is to obtain a thermo-mechanical formulation able to describe the flow and heating of the polymer, due to the mechanical loading and the internal dissipation caused by vibrations.

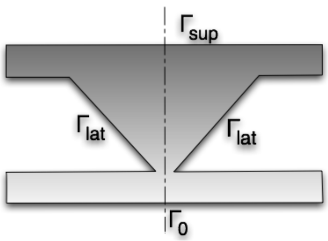


Figure 2: Geometry of the initial energy director.

Mechanical problem. Due to the high frequency of the sonotrode and the rather long time of the process, using a visco-elastic model for the polymer seems necessary. As a first approach, a Maxwell model in small displacement framework is used here :

$$\lambda \dot{\underline{\underline{\sigma}}} + \underline{\underline{\sigma}} = 2\eta \dot{\underline{\underline{\varepsilon}}} \tag{1}$$

where λ is the relaxation time, η the viscosity and ε the strain tensor. Neglecting the inertia terms and the volumic force, the equilibrium equation can be written as :

$$div(\underline{\underline{\sigma}} - p\underline{\underline{I}}) = \underline{0} \tag{2}$$

where σ is the extra stress tensor and p the pressure so that $\sigma - pI$ is the stress tensor.
The composites plates are supposed to be perfectly rigid compared to the energy director. The displacement of the tip of the director is then supposed to be null, whereas the displacement of the upper part can be split into a “micro- chronological” sinusoidal displacement $a.sin(\omega t)$ due to the short period of the ultrasound, and a “macro-chronological” displacement u_d due to the squeezing of the director during the process. The whole mechanical problem can finally be written as follow:

$$
\left\{\begin{array}{l}
\lambda \dot{\underline{\underline{\sigma}}} + \underline{\underline{\sigma}} = 2.\eta.\dot{\underline{\underline{\varepsilon}}} \quad on\,\Omega \\
div(\underline{\underline{\sigma}} - p\underline{\underline{I}}) = \underline{0} \quad on\,\Omega \\
div(\underline{u}) = 0 \quad on\,\Omega
\end{array}\right.
\quad \text{and} \quad
\left[\begin{array}{cc}
\underline{u} = \underline{u_d}(t) + \underline{a}sin(\omega t) & (\Gamma_{sup}) \\
\underline{u} = \underline{0} & (\Gamma_0) \\
(\underline{\underline{\sigma}} - p\underline{\underline{I}}).\underline{n} = 0 & (\Gamma_{lat})
\end{array}\right.
\tag{3}
$$

Thermal problem. The director is supposed to be insulated. A fraction α of the viscous part of the mechanical energy $\underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}}_{vis}$ is supposed to be dissipated in the director during the process. Considering the maxwell constitutive law (1), we can assume that $\dot{\underline{\underline{\varepsilon}}}_{vis} = \frac{1}{2\eta}\underline{\underline{\sigma}}$. The thermal problem can then be written as:

$$
\left\{\begin{array}{c}
\rho c\dot{\theta} = k\Delta\theta + \frac{\alpha}{2\eta}\underline{\underline{\sigma}} : \underline{\underline{\sigma}} \\
k.\underline{grad}(\theta).\underline{n} = 0\; on\, (\Gamma_{lat} \cup \Gamma_0 \cup \Gamma_{sup})
\end{array}\right.
\tag{4}
$$

where θ is the temperature.

2.1.1 Dimensionless quantities

In order to deal with comparable quantities, we will transform the initial problem into a dimensionless problem. We proceed by using characteristic magnitude of the process:

Viscosity : As set before, we choose a characteristic viscosity $\eta_0 = 10^7\, Pa.s$ that is equivalent to the newtonian viscosity at temperature $\theta = \theta_g$ (where $\theta_g = 143^\circ C$ is the glassy temperature).

Length : The characteristic length used in this work is $e = 300\ \mu m$ which is the initial height of the director.

Temperature : A temperature θ will be transformed into a dimensionless temperature θ^* by applying the following formula:

$$
\theta^* = \frac{\theta - \theta_{amb}}{\theta_{ref}}
\tag{5}
$$

where θ_{amb} is the ambient temperature and $\theta_{ref} = \theta_m - \theta_{amb}$ with θ_m the melting temperature. This allows to have an dimensionless temperature θ^* that varies around $[0,1]$.

Time : The time $\lambda_0 = 1\,s$ which is the time of the process is used as characteristic time.

Variables : We finally introduce dimensionless variables which are the following stared variables defined by:

$$
\left\{\begin{array}{c}
\underline{\underline{\sigma}} = \frac{\eta_0}{\lambda_0}\underline{\underline{\sigma}}^* \\
\underline{u} = e\underline{u}^* \\
\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^* \\
\underline{\underline{\varepsilon}} = grad_s^* u^* \\
\theta = \theta_{ref}\theta^* + \theta_{ref}
\end{array}\right.
\tag{6}
$$

Operators : In the same way, dimensionless space operators are introduced:

$$
grad \equiv \frac{1}{e}grad^*, \quad div \equiv \frac{1}{e}div^* \quad \text{and} \quad \Delta \equiv \frac{1}{e^2}\Delta^*
\tag{7}
$$

Dimensionless time derivation : One can notice that there are two time scales in the process: a short time scale of about 50 micro-seconds corresponding to the ultrasonic period that will be refered to as the micro-chronological time scale ; and a long time scale of about 1 second corresponding to the process time that will be refered to as the macro-chronological time scale.

Dimensionless time scales t^* and τ^* are then introduced such as $t^* = \frac{t}{\lambda_0}$ and $\tau^* = \omega t$. t^* being the macro-chronological time variable and τ^* the micro-chronological one. Then, one can define the time scale factor :

$$\xi = \frac{t^*}{\tau^*} = \frac{1}{\omega\lambda_0} \sim 10^{-5} \tag{8}$$

This good scales separation will reasonably allow us to use a technique of homogenization in time to model the process [6]. To proceed, each variable ϕ of the problem is written as a function of t^* and τ^*, the time derivative of ϕ being written as:

$$\dot{\phi} = \frac{d\phi}{dt} = \frac{1}{\lambda_0}\frac{\partial\phi}{\partial t^*} + \frac{1}{\lambda_0}\frac{1}{\xi}\frac{\partial\phi}{\partial\tau^*} \tag{9}$$

2.2 Homogenization.

2.2.1 Dimensionless resulting problems

Dimensionless equations corresponding to systems (3) and (4) can be deduced[1]:

$$\left\{\begin{array}{l}
\Lambda\xi^l\frac{\partial\underline{\underline{\sigma}}}{\partial t} + \Lambda\xi^l\frac{1}{\xi}\frac{\partial\underline{\underline{\sigma}}}{\partial\tau} + \underline{\underline{\sigma}} = 2.N\xi^n.\frac{\partial\underline{\underline{\varepsilon}}}{\partial t} + 2.N\xi^n.\frac{1}{\xi}\frac{\partial\underline{\underline{\varepsilon}}}{\partial\tau} \\
div(\underline{\underline{\sigma}} - p\underline{\underline{I}}) = \underline{0} \\
div(\underline{v}) = 0 \\
\left[\begin{array}{ll}
\underline{u} = \xi^p\underline{U_d}(t) + \underline{R}\xi^r sin(\tau) & (\Gamma_{sup}) \\
\underline{u} = \underline{0} & (\Gamma_0) \\
(\underline{\underline{\sigma}} - p\underline{\underline{I}}).\underline{n} = 0 & (\Gamma_{lat})
\end{array}\right.
\end{array}\right. \tag{10}$$

$$\left\{\begin{array}{l}
\frac{\partial\theta}{\partial t} + \frac{1}{\xi}\frac{\partial\theta}{\partial\tau} = A\xi^a\Delta\theta + B\xi^b\frac{1}{N\xi^n}\underline{\underline{\sigma}} : \underline{\underline{\sigma}} \\
\underline{grad}(\theta).\underline{n} = 0\ (\Gamma_{lat}\cup\Gamma_0\cup\Gamma_{sup})
\end{array}\right. \tag{11}$$

where, for the sake of legibility, the stared notation has immediately been dropped. All the capital letters introduced are of order 0 in ξ. Process and material order of magnitude found in the literature [2, 1, 7, 4, 9] allowed to determine the exponents of the equations :

$$\left\{\begin{array}{ccc}
|\underline{v_d}| & \sim & 300\,\mu m/s \\
k & \sim & 0.24\,W/m/K \\
\rho & \sim & 1300\,Kg/m^3 \\
c & \sim & 2000\,J/Kg/K \\
|\underline{a}| & \sim & 20\,\mu m
\end{array}\right.
\left\{\begin{array}{ll}
\underline{U_d}\xi^p = \frac{\underline{u_d}}{e} \sim \frac{2.10^{-4}}{3.10^{-4}} \sim 0,6 & \Rightarrow p = 0 \\
\underline{R}\xi^r = \frac{\underline{a}}{e} \sim \frac{2.10^{-5}}{3.10^{-4}} \sim 0,06 & \Rightarrow r = 0 \\
A\xi^a = \frac{k\lambda_0}{\rho c e^2} \sim \frac{0.8}{2.10^6(3.10^{-4})^2} \sim 3 & \Rightarrow a = 0 \\
B\xi^b = \frac{\alpha\eta_0}{\rho c\theta_{ref}\lambda_0} \sim \frac{10^7}{2.10^6.2.10^2.1} \sim 5.10^{-2} & \Rightarrow b = 0
\end{array}\right. \tag{12}$$

Concerning $\Lambda\xi^l$ and $N\xi^n$ we consider a temperature above the glassy temperature T_g. Using the work of Nicodeau [8], around $T = 410^\circ C$, which is widely above the melting temperature ($T_m = 334^\circ C$), $\lambda = 0.25\,s$, and around T_g extrapolating the arrhenius law,

[1]note that the constitutive law has been divided by $\frac{\eta_0}{\lambda_0}$ and the thermal equation by $\rho c\theta_{ref}$

$\lambda = 230\,s$. Therefore we can set $\frac{\lambda}{\lambda_0} \sim \xi^0$, ie $l = 0$. Proceeding the same way for the viscosity, we get $n = 0$.

Finally, the constitutive equation and boundary condition of the mechanical problem (10) become:

$$\left\{ \begin{array}{l} \Lambda \frac{\partial \underline{\underline{\sigma}}}{\partial t} + \Lambda \frac{1}{\xi} \frac{\partial \underline{\underline{\sigma}}}{\partial \tau} + \underline{\underline{\sigma}} = 2.N.\frac{\partial \underline{\underline{\varepsilon}}}{\partial t} + 2.N.\frac{1}{\xi} \frac{\partial \underline{\underline{\varepsilon}}}{\partial \tau} \\ \underline{u} = \underline{U_d}(t) + \underline{R} sin(\tau) \qquad (\Gamma_{sup}) \end{array} \right. \tag{13}$$

and the thermal constitutive equation:

$$\frac{\partial \theta}{\partial t} + \frac{1}{\xi} \frac{\partial \theta}{\partial \tau} = A\Delta\theta + \frac{B}{N} \underline{\underline{\sigma}} : \underline{\underline{\sigma}} \tag{14}$$

2.2.2 Asymptotic expansion

Asymptotic expansion in time consists in writing each unknown as an expansion in power of ξ :

$$\phi = \phi_0 + \phi_1 \xi + \phi_2 \xi^2 + ... \tag{15}$$

where ϕ stands for $\underline{\underline{\sigma}}$, p, $\underline{u}$, $\underline{\underline{\varepsilon}}$ or θ. Each ϕ_i is supposed to be periodic in τ as in Guennouni or Boutin & Wong [6, 3].

Later on, formulation of the mechanical problem in terms of velocity instead of displacement will be simpler. In our time derivation framework the velocity is defined as : $\underline{v} = \frac{\partial \underline{u}}{\partial t} + \frac{1}{\xi} \frac{\partial \underline{u}}{\partial \tau}$, which means that the velocity expansion starts at order -1 with:

$$\left\{ \begin{array}{c} \underline{v_{-1}} = \frac{\partial \underline{u_0}}{\partial \tau} \\ \underline{v_i} = \frac{\partial \underline{u_i}}{\partial t} + \frac{\partial \underline{u_{i+1}}}{\partial \tau} \qquad \forall i \geq 0 \end{array} \right. \tag{16}$$

We also expand the strain rate tensor as:

$$\left\{ \begin{array}{c} \underline{\underline{D_{-1}}} = \underline{\underline{grad_s v_{-1}}} = \frac{\partial \underline{\underline{\varepsilon_0}}}{\partial \tau} \\ \underline{\underline{D_i}} = \underline{\underline{grad_s v_{-1}}} = \frac{\partial \underline{\underline{\varepsilon_i}}}{\partial t} + \frac{\partial \underline{\underline{\varepsilon_{i+1}}}}{\partial \tau} \qquad \forall i \geq 0 \end{array} \right. \tag{17}$$

2.2.3 Identification on the mechanical problem.

Once injected in the previous problem, identification of different order of ξ leads to several sets of equations.

First, we can notice that the equilibrium equation, incompressibility constrain and two boundary conditions can be identified trivially at every order i of ξ. We get:

$$\left\{ \begin{array}{c} div(\underline{\underline{\sigma_i}} - p_i \underline{\underline{I}}) = \underline{0} \\ div(\underline{u_i}) = 0 \\ \left[\begin{array}{cc} \underline{u_i} = \underline{0} & (\Gamma_0) \\ (\underline{\underline{\sigma_i}} - p_i \underline{\underline{I}}).\underline{n} = 0 & (\Gamma_{lat}) \end{array} \right. \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{c} div(\underline{\underline{\sigma_i}} - p_i \underline{\underline{I}}) = \underline{0} \\ div(\underline{v_i}) = 0 \\ \left[\begin{array}{cc} \underline{v_i} = \underline{0} & (\Gamma_0) \\ (\underline{\underline{\sigma_i}} - p_i \underline{\underline{I}}).\underline{n} = 0 & (\Gamma_{lat}) \end{array} \right. \end{array} \right. \tag{18}$$

The displacement boundary condition on Γ_{sup} given in the system (13) only appears at order 0 as :

$$\left\{\begin{array}{ll} \underline{u_i} = 0 \quad \forall i > 0 & (\Gamma_{sup}) \\ \underline{u_0} = \underline{u_d}(t) + \underline{R}sin(\tau) & (\Gamma_{sup}) \end{array}\right. \quad \text{or} \quad \left\{\begin{array}{ll} \underline{v_{-1}} = \frac{\partial \underline{u_0}}{\partial \tau} = \underline{R}cos(\tau) & (\Gamma_{sup}) \\ \underline{v_0} = \frac{\partial \underline{u_1}}{\partial \tau} + \frac{\partial \underline{u_0}}{\partial t} = \frac{\partial \underline{u_d}}{\partial t} & (\Gamma_{sup}) \\ \underline{v_i} = 0 \qquad \forall i > 0 & (\Gamma_{sup}) \end{array}\right. \tag{19}$$

Then, the constitutive law (13) is identified at the first orders:

$$\text{order } \xi^{-1} : \qquad \Lambda \frac{\partial \underline{\underline{\sigma_0}}}{\partial \tau} + = 2N \frac{\partial \underline{\underline{\varepsilon_0}}}{\partial \tau} \tag{20}$$

$$\text{order } \xi^{0} : \qquad \Lambda \frac{\partial \underline{\underline{\sigma_0}}}{\partial t} + \Lambda \frac{\partial \underline{\underline{\sigma_1}}}{\partial \tau} + \underline{\underline{\sigma_0}} = 2N. \frac{\partial \underline{\underline{\varepsilon_0}}}{\partial t} + 2.N. \frac{\partial \underline{\underline{\varepsilon_1}}}{\partial \tau} \tag{21}$$

Introducing the short time average operator $< \cdot >= \frac{1}{|\kappa|}\int_{\kappa}(\cdot)d\tau$ where κ is an ultrasonic period, periodicity of $\underline{\underline{\sigma}}_i$ implies $< \frac{\partial \underline{\underline{\sigma}}_i}{\partial \tau} >= \underline{\underline{0}}$. We obtain from previous equation:

$$\Lambda \frac{\partial < \underline{\underline{\sigma_0}} >}{\partial t} + < \underline{\underline{\sigma_0}} >= 2N. \frac{\partial < \underline{\underline{\varepsilon_0}} >}{\partial t} \tag{22}$$

Finally, combining constitutive laws (20) and (22) , equilibrium, incompressibility and boundary conditions (18) and (19), we can write two mechanical problems.

Micro-mechanical problem : deals with short time τ:

$$\left\{\begin{array}{c} \Lambda \frac{\partial \underline{\underline{\sigma_0}}}{\partial \tau} = 2N \frac{\partial \underline{\underline{\varepsilon_0}}}{\partial \tau} = 2N\underline{\underline{D}}(\underline{v_{-1}}) \\ div(\underline{\underline{\sigma_0}} - p_0 \underline{\underline{I}}) = \underline{0} \\ div(\underline{v_{-1}}) = 0 \end{array}\right. \qquad \left[\begin{array}{ll} \underline{v_{-1}} = \underline{R}.sin(\tau) & (\Gamma_{sup}) \\ \underline{v_{-1}} = \underline{0} & (\Gamma_0) \\ \underline{\underline{\sigma_0}}.\underline{n} = 0 & (\Gamma_{lat}) \end{array}\right. \tag{23}$$

We get an hypo-elastic problem which is equivalent to an elastic problem in the small displacement framework. Indeed, for short time solicitation, the maxwell model is mainly elastic. This problem describes a stress fluctuation of order ξ^0 that is linked to a velocity of order ξ^{-1}. The boundary condition on Γ_{sup} is a micro-chronological harmonic condition only.

Macro-mechanical problem : deals with long time t:

$$\left\{\begin{array}{c} \Lambda \frac{\partial < \underline{\underline{\sigma_0}} >}{\partial t} + < \underline{\underline{\sigma_0}} >= 2N < \underline{\underline{D_0}} > \\ div(< \underline{\underline{\sigma_0}} > - < p_0 > \underline{\underline{I}}) = \underline{0} \\ div(< \underline{v_0} >) = 0 \end{array}\right. \qquad \left[\begin{array}{ll} < \underline{v_0} >= \underline{v_d}(t) & (\Gamma_{sup}) \\ < \underline{v_0} >= \underline{0} & (\Gamma_0) \\ < \underline{\underline{\sigma_0}} > .\underline{n} = 0 & (\Gamma_{lat}) \end{array}\right. \tag{24}$$

It describes the slow mechanical variation as a visco-elastic maxwell flow. The stress average is of order ξ^0 as well but is linked to a velocity of order ξ^0. The boundary condition on Γ_{sup} is the macro-chronological velocity condition only.

2.2.4 Identification of the thermal problem

Concerning the thermal problem, the boundary condition of isolation (11) is trivially identified at every order of ξ as:

$$\forall i \geq 0 \quad \underline{grad}(\theta_i).\underline{n} = 0 \quad (\Gamma_{lat} \cup \Gamma_0 \cup \Gamma_{sup}) \tag{25}$$

The constitutive equation (14) can be identified at order ξ^{-1} as $\frac{\partial \theta_0}{\partial \tau} = 0$, which implies that $< \theta_0 >= \theta_0(t)$, and at order ξ^0 as:

$$\frac{\partial \theta_0}{\partial t} + \frac{\partial \theta_1}{\partial \tau} = A\Delta\theta_0 + \frac{B}{N}\underline{\underline{\sigma_0}} : \underline{\underline{\sigma_0}} \tag{26}$$

This allows to define the temperature variation as the superposition of a short time fluctuation θ_1 one order smaller than the long time variation $\theta_0(t)$, which is completely defined by averaging the previous equation:

$$\frac{\partial \theta_0}{\partial t} = A\Delta\theta_0 + \frac{B}{N} < \underline{\underline{\sigma_0}} : \underline{\underline{\sigma_0}} > \tag{27}$$

Finally, the homogenization method allowed to split the initial multi time scale thermomechanical problem into three mono time scale coupled problems : the mechanical microchronological elastic problem that induces a source term in the thermal diffusion problem and a therm-dependent macro-chronological problem that gives the geometry changes.

3 NUMERICAL PROCEDURE

3.1 Simplification of the problem

Although the macro-chronological problem should strictly be visco-elastic, it was set as a purely viscous problem. Indeed, the macroscopic time of the process is rather long compared to the characteristic Maxwell time so that the steady state is suposed to be rapidly reached. Of course this assumption will be valid for rather high temperatures (close to θ_m). Nevertheless, for a better deformed shape representativeness, a strain rate dependent viscosity was used, therefore implying a non-linear flow problem.

Moreover, though rigorously valid above T_g the homogenized equation were used over the whole temperature range, but it was checked that th epresent formulations could be extended assuming that $n = -1$ and $l = -1$ in equation (10).

3.2 Resolution Scheme

The systems are solved using a classical Galerkin FEM. For each time step the three systems of equations are solved successively. Since coupling between the three systems is rather weak, they are solved iteratively until the residual of the three variational forms are simultaneously smaller than a prescribed tolerance. To solve each problem (which can be non-linear) a Newton-Raphson method was used.

The resolution is performed on a fixed mesh where the free surface is described by a levelset field. This is done using the C++ Library X-FEM. At each time step, once the three physical problems are solved, the levelset is propagated using a Hamilton-Jacobi method [10]. Then, using an operator splitting method [11], the temperature field is convected with the macro-chronological solution, using an SUPG method.

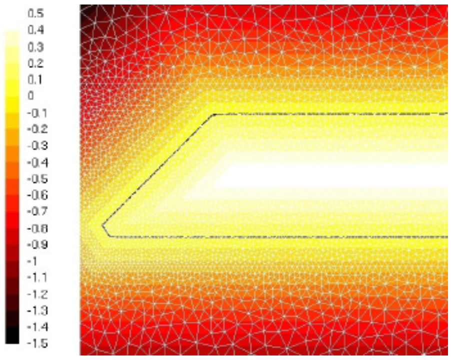


Figure 3: Initial Level Set. The iso-zero is in black.

The mesh is made of unstructured triangles over the whole domain, which is reduced to a half energy director, a part of the lower and upper plates and the void between directors. The initial levelset, that defines the free surface is shown on figure 3. Note that the singularity at the tip of the director has been smoothed by a bevel.

3.3 Material Data

The material parameters used in the simulations were chosen from the literature.

Elasticity : For the elastic micro-chronological system, the material was assumed to be isotropic with temperature dependent Young modulus E. In a first approximation based on [4, 8, 7]we propose a simple linear dependency:

$$E = 3.7e9 - 1e7.T(^{\circ}C)\,Pa \tag{28}$$

Viscosity : In order to take into account the shear-thining effect the viscosity was set as a Carreau law where the newtonian viscosity has an arrhenius type form [8] :

$$\begin{cases} \eta = \eta_0(T)\left[1 + (\lambda_{careau} D_{eq})^a\right]^{\frac{m-1}{a}} \\ \eta_0 = A.exp(\frac{E_a}{R.T}) \end{cases} \tag{29}$$

where $a = 0.7$, $m = 0.54$, $E_a = 74400$ and $A = 5,6e-3$

Thermal parameters : Using data provided by[1, 4, 7], we propose simple approximation for ρc and k :

$$\begin{array}{l} \rho c = 1.3\,10^6 + 5\,10^3 T(^{\circ}C)\,J/m^3/K \\ k = 0.24\,W/m/K \end{array} \tag{30}$$

4 RESULTS AND DISCUSSION

Despite the presentation of material data, we solved the dimensionless problems and therefore present dimensionless quantities in the following.

Figure 5 shows the evolution of the levelset negative part, which is the material domain. We clearly observe a flow of polymer that begins at the tip of the director. As the loading is pursued the flowing zone increases and fills the gap between the two plates. This is in agreement with the deformed shape of the micrography 4, made on a sample obtained with a stopped experiment. It would correspond to the simulated shape at time $t = 1.1\,s$. Next step would be to obtain better controlled interrupted experiments. We can also notice that a local refinement of the mesh around the levelset iso-zero would be necessary in order to describe the free surface more precisely.

This shape is explained by the temperature fields of figure 6 associated with the thermo-dependent mechanical parameters. It clearly shows that the tip of the director heats first as found in the literature [14, 2]. Despite those good first results, at this stage the temperature only reaches $\theta^* = 0.4$ which seems rather low according to the expected welding at $\theta^* = 1$.

This shows the need for finer instrumented experiments and material parameter identification. They are be the subject of ongoing works that will tend to validate and adjust the presented model.

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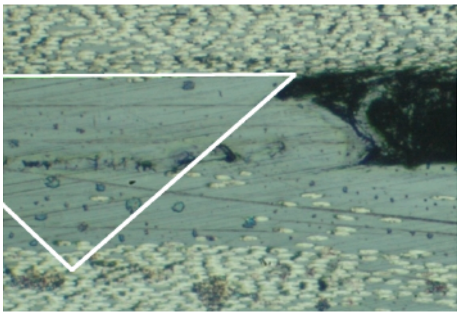
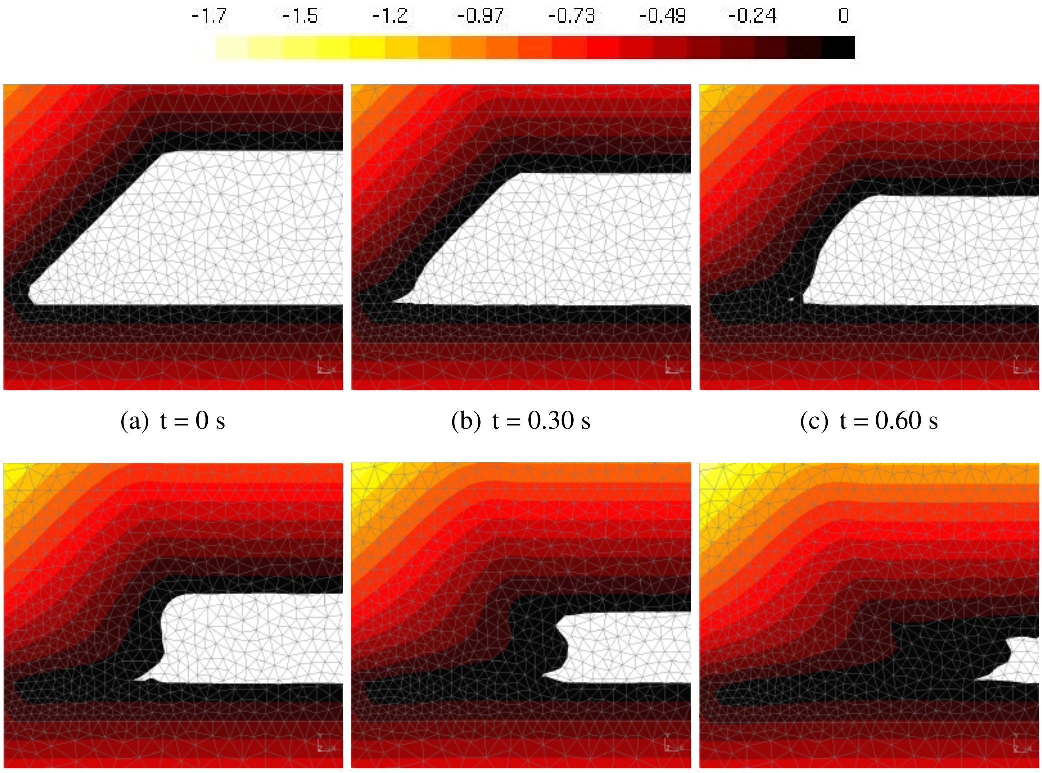


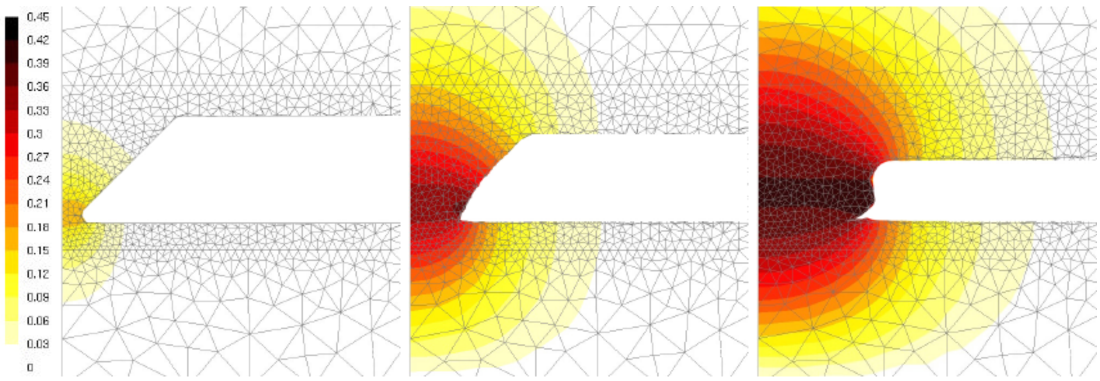
Figure 4: Micrography of a stopped experiment.



(a) t = 0 s (b) t = 0.30 s (c) t = 0.60 s

(d) t = 0.85 s (e) t = 1.10 s (f) t = 1.35 s

Figure 5: Evolution of the LevelSet. Negative part only.



(a) t=0.05 (b) t=0.40 s (c) t = 0.85 s

Figure 6: Temperature Field.