FREE RADIAL VIBRATIONS IN SPHERICAL SANDWICH SHELL

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ABSTRACT

Free vibrations across the thickness of a closed spherical shell are studied. A finite solution to the one-dimensional (along a radius) wave problem was derived for aclosed spherical layer, and used in constructing a model for joint vibrations of a sandwich shell. The boundary-value problem of joint free vibrations of the compound body was complected with contact requirements on the interfaces. Mathematical technique showed compliance with a searching operation for the general solution of the homogeneous equations that met the requirement of the six-order main determinant zeroing. The transcendental equation for the defined problem on free vibrations was derived. A correlation between wave numbers and thicknesses as well as physical properties of the spherical layers was ascertained. The curves were constructed for the main eigenfrequency versus stiffness of face layers and geometric parameters characteristic of a sandwich shell.

1. INTRODUCTION

Among many problems of structural dynamics the transverse waves acting normally to sandwich plates and shells should be of interest using thick and essentially dissimilar layers. The well-developed approaches based on variational principles mainly refer to the evaluation of vibration process for multilayer composite plates and shells within the framework of the classical Kirchhoff-Love kinematics for rigid layers and with due account of transverse shears in a soft interlayer, [1,2,3]. The wave propagation direction is usually assumed parallel to the face plane of a layered structure. A scrutiny of the wave process in a layered structure still invites further refinement of solution using a layerwise decomposable model, Hohe [4], Sorokin [5]. The rejection of the hypothesis of incompressible normal in all layers of a structure with arbitrary elastic and geometric parameters, at a joint dynamic behavior of layers, requires the elaboration of special solution technique for boundary-value problem for wave equations. The compatibility of free wave fields requires a common characteristic such as an eigenfrequency. The most simplified alternate solution of a dynamic boundary-value problem for a closed spherical sandwich shell which is methodically important for determining the eigenfrequency spectrum, is presented in this study.

2. RADIAL VIBRATIONS OF A HOLLOW ELASTIC BALL

At first we will obtain the final solution of the homogeneous equation for radial vibrations of an elastic sphere with a central spherical cavity. Assuming that the displacement depends only on a radial coordinate r and two its components are equal to zero, i.e., $u_\theta = u_\varphi = 0$, the basic equation of motion is reduced to the scalar equation of only one nonzero component $u_r \equiv u$ in the form

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\left(\frac{\partial u}{\partial r} - \frac{u}{r}\right) - \rho\frac{(1-2\nu)(1+\nu)}{E(1-\nu)}\frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

In this one-dimensional wave equation (1) the coefficient dependant on specific density ρ, Young's module E and Poisson ratio ν of the homogenous material is inverse value of its squared acoustic velocity which is characterized by only a volume dilatation. Thus the volumetric stresses in the material and the dependence of the solution on the surface curvature are taken into account.

For solving eqn (1) one uses the method of separation of variables. The final form of the $k-\mathrm{th}$ solution entering the complete function system of eqn (1) can be represented as

$$u_k(r,t) = U_k(r)T_k(t) \tag{2}$$

$$T_k(t) = a_k \sin(\omega_k t) + \mathrm{b_k}\cos(\omega_k t) \tag{3}$$

$$U_k(r) = C_{1k}\left[\frac{\sin\left({r}/{\Lambda_k}\right)}{r^2} - \frac{\cos\left({r}/{\Lambda_k}\right)}{\Lambda_k r}\right] + C_{2k}\left[\frac{\sin\left({r}/{\Lambda_k}\right)}{\Lambda_k r} + \frac{\cos\left({r}/{\Lambda_k}\right)}{r^2}\right] \tag{4}$$

The eigenfrequency of vibrations in eqn (3) according to the number Λ_k is

$$\omega_k = \frac{1}{\Lambda_k}\left[\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}\right]^{1/2}, \quad k = 1,2 \ldots \tag{5}$$

The natural vibrations of a homogeneous sphere with a central cavity depend on the boundary conditions for fastening the external and internal spherical surfaces. From these conditions and eqn (4) one can find the eigenvalues Λ_k.

3. EIGENVALUE PROBLEM OF THREE-LAYER SPHERICAL SHELL

Free radial vibrations of three rigidly connected (in the sense of equal contact displacements and stresses) spherical shells are characterized by a frequency spectrum common for all spherical regions. Since the eqn (5) contains the physical parameters and the wave number $1/\Lambda_k^{(i)}$ of each layer $i = 1,2,3$ at the constant eigenfrequency ω_k for all layers the eigenvalue problem consists in determination of a layerwise infinite spectrum of $\Lambda_k^{(i)}$, $k = 1,2 \ldots$ from the boundary conditions including its formulation on the interfaces.

3.1 Boundary conditions and the requirements on the interface

In the case of isotropic materials, the radial stress of the dissimilar layers can be expressed by using the Lamé constants λ_i and μ_i:

$$\sigma_r^{(i)} = (\lambda_i + 2\mu_i)\frac{\partial u^{(i)}}{\partial r} + 2\lambda_i\frac{u^{(i)}}{r}, \quad i = 1,2,3 \tag{6}$$

In eqn (6) index i denotes the number of a layer but index k is omitted for simplicity. Substituting eqn (4) and its derivative found at $r = R_1$, $i = 1$, $C_1 = C_1^{(1)}$, $C_2 = C_2^{(1)}$ and accordingly at $r = R_4$, $i = 3$, $C_1 = C_1^{(3)}$, $C_2 = C_2^{(3)}$ into the right-hand side of eqn (6) and equating the results to zero, we obtain the boundary conditions for free surfaces.

By analogy, we obtain the conditions for clamped surfaces equating the only $u^{(1)}$ and $u^{(3)}$ to zero accordingly at $r = R_1$ and at $r = R_4$, see Fig. 1.

In order to meet the requirement for continuity on the interfaces it is necessary to write the displacements and stresses with the indexes $i = 1,2$ at $r = R_2$ and $i = 2,3$ at $r = R_3$, and thereafter the eqn (4) and eqn (6) set conditions for the equality of two terms according to the pair of indexes defined.

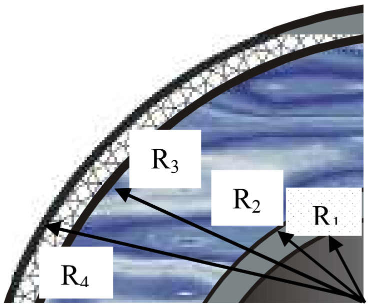


Figure 1: The quarter of a normal section of a layered sphere with a central cavity.

3.2 Matrix notation of the homogeneous set of equations

The system of boundary conditions for three rigidly bound layers under study brings to the canonical form of a matrix product

$$\boldsymbol{AC} = \boldsymbol{0}, \quad \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} C_1^{(1)} \\ C_2^{(1)} \\ C_1^{(2)} \\ C_2^{(2)} \\ C_1^{(3)} \\ C_2^{(3)} \end{bmatrix} \tag{7}$$

The components of the matrix $\boldsymbol{A}$ can easily be determined after elementary operations associated mainly with transformation of the trigonometric functions entering into expressions for the six above-mentioned boundary conditions. In the case of free outer surfaces twenty components are of the following form

$$a_{11} = k_1 R_1 \nu_1^0 + (k_1^2 R_1^2 - \nu_1^0)\tan(k_1 R_1), \quad a_{22} = 1 + k_1 R_2 \tan(k_1 R_2),$$

$$a_{12} = k_1^2 R_1^2 - \nu_1^0[1 + k_1 R_1 \tan(k_1 R_1)], \quad a_{21} = \tan(k_1 R_2) - k_1 R_2,$$

$$a_{23} = [k_2 R_2 - \tan(k_2 R_2)]\frac{\cos(k_2 R_2)}{\cos(k_1 R_2)},$$

$$a_{24} = -[1 + k_2 \mathrm{R}_2 \tan(k_2 R_2)]\frac{\cos(k_2 R_2)}{\cos(k_1 R_2)},$$

$$a_{33} = [(\nu_2^0 - k_2^2 R_2^2)\tan(k_2 R_2) - k_2 R_2 \nu_2^0]\frac{\mu_2 \nu_1^0 \cos(k_2 R_2)}{\mu_1 \nu_2^0 \cos(k_1 R_2)},$$

$$a_{31} = k_1 R_2 \nu_1^0 + (k_1^2 R_2^2 - \nu_1^0)\tan(k_1 R_2),$$

$$a_{32} = k_1^2 R_2^2 - \nu_1^0[1 + k_1 R_2 \tan(k_1 R_2)],$$

$$a_{34} = [\nu_2^0(1 + k_2 R_2 \tan(k_2 R_2)) - k_2^2 R_2^2]\frac{\mu_2 \nu_1^0 \cos(k_2 R_2)}{\mu_1 \nu_2^0 \cos(k_1 R_2)},$$

$$a_{43} = [k_2 R_3 - \tan(k_2 R_3)]\frac{\cos(k_2 R_3)}{\cos(k_3 R_3)}, \quad a_{45} = \tan(k_3 R_3) - k_3 R_3,$$

$$a_{44} = -[1 + k_2 \mathrm{R}_3 \tan(k_2 R_3)]\frac{\cos(k_2 R_3)}{\cos(k_3 R_3)}, \quad a_{46} = 1 + k_3 R_3 \tan(k_3 R_3),$$

$$a_{53} = [(\nu_2^0 - k_2^2 R_3^2)\tan(k_2 R_3) - \qquad a_{55} = k_3 R_3 \nu_3^0 + (k_3^2 R_3^2 - \nu_3^0)\tan(k_3 R_3),$$

$$-k_2 R_3 \nu_2^0] \frac{\mu_2 \nu_3^0 \cos(k_2 R_3)}{\mu_3 \nu_2^0 \cos(k_3 R_3)}, \qquad a_{56} = k_3^2 R_3^2 - \nu_3^0[1 + k_3 R_3 \tan(k_3 R_3)],$$

$$a_{54} = [\nu_2^0(1 + k_2 R_3 \tan(k_2 R_3)) - \qquad a_{65} = k_3 R_4 \nu_3^0 + (k_3^2 R_4^2 - \nu_3^0)\tan(k_3 R_4),$$

$$-k_2^2 R_3^2] \frac{\mu_2 \nu_3^0 \cos(k_2 R_3)}{\mu_3 \nu_2^0 \cos(k_3 R_3)}, \qquad a_{66} = k_3^2 R_4^2 - \nu_3^0[1 + k_3 R_4 \tan(k_3 R_4)].$$

(8)

In eqn (8) the wave number $k_i = 1/\Lambda^{(i)}, i = 1,2,3$ correlates with the layer acoustic velocity c_i on the frequency ω which is identical within all layers. The non-dimensional elastic parameters are expressed in the following form

$$\nu_i^0 = \frac{2(1 - 2\nu_i)}{1 - \nu_i}, \qquad \frac{\mu_i}{\mu_j} = \frac{E_i(1 + \nu_j)}{E_j(1 + \nu_j)}, \qquad \mu_{ij} = \frac{\nu_j^0 \mu_i}{\nu_i^0 \mu_j} = \frac{\lambda_i + 2\mu_i}{\lambda_j + 2\mu_j}, \qquad i,j = 1,2,3 \qquad (9)$$

3.3 Determinant of the general matrix

Furthermore we determine the algebraic expression of the $det\,\boldsymbol{A}$. After cumbersome analytical procedure associated with calculation of the 6^{th} order determinant we finally get

$$det\,\boldsymbol{A} = \frac{[1 + \tan(k_1 R_1)\tan(k_1 R_2)]\cos[k_2(R_3 - R_2)]}{[1 + \tan(k_3 R_3)\tan(k_3 R_4)]\cos(k_1 R_2)\cos(k_3 R_3)} D(k_1, k_2, k_3), \qquad (10)$$

In eqn (10) D is defined as the sum of the terms consisting of multivariate polynomials multiplied by trigonometric tangents, thus the derived functions are given by:

$$D(k_1, k_2, k_3) = \mu_{23} P_{34}(k_3)[\mu_{21} P_{12}(k_1) Q_{23}(k_2) - P_{32}(k_2) Q_{12}(k_1)] -$$

$$-Q_{34}(k_3)[\mu_{21} P_{12}(k_1) P_{23}(k_2) - Q_{12}(k_1) Q_{32}(k_2)],$$

$$P_{12}(k_1) = k_1^3 R_1^2 R_2 - k_1(R_2 - R_1)\nu_1^0 +$$

$$+[k_1^2 R_1(R_2 \nu_1^0 - R_1) + \nu_1^0]\tan[k_1(R_2 - R_1)],$$

$$P_{23}(k_2) = k_2^3 R_2^2 R_3 - k_2(R_3 - R_2)\nu_2^0 +$$

$$+[k_2^2 R_2(R_3 \nu_2^0 - R_2) + \nu_2^0]\tan[k_2(R_3 - R_2)],$$

$$P_{32}(k_2) = -k_2^3 R_3^2 R_2 - k_2(R_3 - R_2)\nu_2^0 +$$

$$+[k_2^2 R_3(R_2 \nu_2^0 - R_3) + \nu_2^0]\tan[k_2(R_3 - R_2)],$$

$$P_{34}(k_3) = -k_3^3 R_4^2 R_3 - k_3(R_4 - R_3)\nu_3^0 +$$

$$+[k_3^2 R_4(R_3 \nu_3^0 - R_4) + \nu_3^0]\tan[k_3(R_4 - R_3)],$$

$$Q_{12}(k_1) = k_1^3 R_1 R_2(R_2 - R_1)\nu_1^0 + k_1(R_2 - R_1)(\nu_1^0)^2 -$$

$$-\{k_1^4R_1^2R_2^2 + k_1^2[R_1R_2(\nu_1^0)^2 - (R_1^2+R_2^2)\nu_1^0] + (\nu_1^0)^2\}\tan[k_1(R_2 - R_1)],$$

$$Q_{23}(k_2) = k_2^3R_3R_2(R_3 - R_2)\nu_2^0 + k_2(R_3 - R_2)(\nu_2^0)^2 -$$

$$-\{k_2^4R_3^2R_2^2 + k_2^2[R_3R_2(\nu_2^0)^2 - (R_3^2+R_2^2)\nu_2^0] + (\nu_2^0)^2\}\tan[k_2(R_3 - R_2)],$$

$$Q_{32}(k_2) = k_2(R_3 - R_2) - (1 + k_2^2R_2R_3)\tan[k_2(R_3 - R_2)],$$

$$Q_{34}(k_3) = k_3^3R_3R_4(R_4 - R_3)\nu_3^0 + k_3(R_4 - R_3)(\nu_3^0)^2 -$$

$$-\{k_3^4R_3^2R_4^2 + k_3^2[R_3R_4(\nu_3^0)^2 - (R_3^2+R_4^2)\nu_3^0] + (\nu_3^0)^2\}\tan[k_3(R_4 - R_3)].$$

(11)

3.4 Eigenvalue equation

Upon further analysis of the expressions given by eqn (10) and eqns (11), we concluded that the necessary and sufficient condition for $det\,\boldsymbol{A} = 0$ reduces to the equation $D = 0$. The acoustic velocities and wave numbers obey the law $\omega = c_ik_i, i = 1,2,3$ (no summation over index i) subjected to the condition that the eigenfrequency ω is constant for the whole system of layers. It follows from this that but one independent variable enters into the expression of *D* in eqns (11). Denoting this variable in dimensionless form by

$$\xi = \omega\frac{R_3 - R_2}{c_2}, \tag{12}$$

we specify the dimensionless parameters and acoustic velocity in each layer as

$$n_2 = \frac{R_2}{R_3 - R_2},\quad n_3 = \frac{R_3}{R_3 - R_2},\qquad m_1 = \frac{c_2R_1}{c_1(R_3 - R_2)},\qquad m_2 = \frac{c_2R_2}{c_1(R_3 - R_2)},$$

$$m_3 = \frac{c_2R_3}{c_3(R_3 - R_2)},\qquad m_4 = \frac{c_2R_4}{c_3(R_3 - R_2)},\qquad c_i = \sqrt{\frac{E_i(1 - \nu_i)}{\rho_i(1 + \nu_i)(1 - 2\nu_i)}}\quad i = 1,2,3 \tag{13}$$

According to eqns (10), (11) through the instrumentality of eqns (12), (13) we finally obtain the eigenvalue equation relative to ξ such that $-D = 0$

$$\mu_{21}\mu_{23}p_1q_2s_3 + \mu_{21}p_1p_2q_3 + \mu_{23}q_1s_2s_3 + q_1q_3r_2 = 0 \tag{14}$$

In eqn (14) the polynomially related tangent functions are expressed by formulae

$$p_i = m_i^2m_{i+1}\xi^3 - \nu_i^0(m_{i+1} - m_i)\xi - (m_i^2\xi^2 - \nu_i^0m_im_{i+1}\xi^2 - \nu_i^0)\tan[\xi(m_{i+1} - m_i)]\,, i = 1 \tag{15}$$

$$q_i = m_im_{i+1}(m_{i+1} - m_i)\nu_i^0\xi^3 + (m_{i+1} - m_i)(\nu_i^0)^2\xi -$$

$$-[m_{i+1}^2m_i^2\xi^4 - \nu_i^0(m_{i+1}^2 + m_i^2)\xi^2 + m_im_{i+1}(\nu_i^0)^2\xi^2 + (\nu_i^0)^2]\cdot$$

$$\cdot\tan[\xi(m_{i+1} - m_i)]\,, i = 1,3 \tag{16}$$

$$r_2 = -\xi + (1 + n_2 n_3 \xi^2)\tan\xi \tag{17}$$

$$s_i = p_i(m_{i \rightleftarrows i+1}), \;\; i = 3 \tag{18}$$

Following the notation of eqn (18) the functions s_i can be determined by simple permutation of the subscripts of the only two parameters entering into eqn (15). The functions p_2, q_2, s_2 are resulted from eqns (15), (16), (18) under the index $i = 2$ and it is necessary to exchange the parameters m_2, m_3 for n_2, n_3 at that.

3.4.1 Degenerate cases for homogeneous sphere structure

An equation for determining the eigenfrequencies of radial vibrations of a homogeneous spherical shell with free surfaces is obtained from eqn (14) in two obvious cases: (a) the face layers are excluded from the model that is $R_3 = R_4, \; R_2 = R_1$, (b) the three layers are identical in physical properties then $\nu_i^0 = \nu^0, c_i = c, \; \mu_{ij} = 1, i, j = 1,2,3; \; m_2 = n_2, \; m_3 = n_3$. The variant (b) of degeneration of the eqn (14) is more involved due to algebraic calculus of polynomial multiplication and thereby enables us to check for the accuracy of the analytic relationships described above. The both variants result in the identical form of eigenvalues equation

$$\tan\xi = \frac{\xi^3 R_1 R_4 + \xi\nu^0 (R_4 - R_1)^2}{\xi^4 R_1^2 R_4^2/[\nu^0 (R_4 - R_1)^2] - \xi^2 (R_1^2 + R_4^2 - \nu^0 R_1 R_4) + \nu^0 (R_4 - R_1)^2} \tag{19}$$

The variant of calculating eigenvalues in the case of fixed boundary surfaces of a sphere can be obtained by applying eqn (4) to the case of $u(R_1) = u(R_4) = 0$. As it is, the determinant of the system of two homogeneous relations connecting the constants C_1, C_2, equals zero and we come to an equation:

$$\tan\xi = \frac{\xi (R_4 - R_1)^2}{\xi^2 R_1 R_4 + (R_4 - R_1)^2} \tag{20}$$

The transition to the model of a monolithic elastic sphere is realized by assuming that $R_1 = 0$ in eqns (19), (20). In particular, for the problem on radial vibration of an elastic monolithic sphere with a fixed surface, we obtain from eqn (20) the equation $\tan\xi = \xi$, that is also the equation for eigenvalues of the boundary-value problem on radial vibration of gas in a spherical vessel [6].

3.4.2 Approximate equation for a sandwich sphere

An equation in simpler terms than the given eqn (14) which approximates to it can be obtained for a sphere of sandwich-type material. An approximate approach is based on a shell model with thin stiff layers enveloping a thick soft core layer. In that case the argument of tangent functions entering into the eqns (15), (16), (18) decreases from the one entering into eqn (17) by more than an order of magnitude as far as $c_1, c_3 \gg c_2$, $(m_{i+1} - m_i) \ll 1, \; (R_{i+1} - R_i) \ll R_3 - R_2, \; i = 1; 3$.

This ratio analysis of the parameters in the domain of least values of ξ corresponding to the smallest eigenfrequencies allow us to use the limit relationships of first order between the functions: $\tan[\xi(m_{i+1} - m_i)] = \xi(m_{i+1} - m_i), i = 1; 3$. Under the circumstances and with regard to a symmetric sandwich layup the eigenvalue equation is adjusted in accordance with eqns (14) – (18) to the following notation

$$\mu_{21}\mu_{23}(m_1^3+\nu^0 m_0 m_1 m_2)(m_4^3-\nu^0 m_0 m_3 m_4)[\nu_2^0 n_2 n_3 \xi^3+(\nu_2^0)^2\xi -$$

$$-(n_2^2 n_3^2\xi^4-\nu_2^0 n_2^2\xi^2-\nu_2^0 n_3^2\xi^2+(\nu_2^0)^2 n_2 n_3\xi^2+(\nu_2^0)^2)\tan\xi]+$$

$$\mu_{21}(m_1^3+\nu^0 m_0 m_1 m_2)(\nu^0(m_4^3-m_3^3)-m_0 m_3^2 m_4^2\xi^2-(\nu_2^0)^2 m_0 m_3 m_4)\cdot$$

$$\cdot[n_2^2 n_3\xi^3-\nu_2^0\xi-(n_3^2\xi^2-\nu_2^0 n_2 n_3\xi^2-\nu_2^0)\tan\xi]+$$

$$\mu_{23}(m_4^3-\nu^0 m_0 m_3 m_4)(\nu^0(m_2^3-m_1^3)-m_0 m_1^2 m_2^2\xi^2-(\nu_2^0)^2 m_0 m_1 m_2)\cdot$$

$$\cdot[n_3^2 n_2\xi^3+\nu_2^0\xi+(n_3^2\xi^2-\nu_2^0 n_2 n_3\xi^2-\nu_2^0)\tan\xi]-$$

$$-[\xi-(1+n_2 n_3\xi^2)\tan\xi](\nu^0(m_4^3-m_3^3)-m_0 m_3^2 m_4^2\xi^2-(\nu_2^0)^2 m_0 m_3 m_4)\cdot$$

$$\cdot(\nu^0(m_2^3-m_1^3)-m_0 m_1^2 m_2^2\xi^2-(\nu_2^0)^2 m_0 m_1 m_2)=0, \qquad (21)$$

in which $m_0 = m_4 - m_3 = m_2 - m_1 = (h/h_c)(c_2/c)$ and h, c is accordingly the thickness and sound velocity of the restraining layers as opposed to the same h_c, c_2 of the midlayer (core). The denotation of $\nu^0 = \nu_1^0 = \nu_3^0$ corresponds to the first formula of eqns (9). It is obvious that the parameter $m_0 = 0$ if the thickness h is configured-out. Then the eqn (21) reduces to (19). A zero value of the last parameter retains in the case of rigid fastening of boundary surfaces of the core, i.e., at $E = E_1 = E_3 \to \infty$. Moreover in this case $m_i \sim E^{-1/2}$, $\mu_{21} = \mu_{23} \sim E^{-1}$, and we obtain the eqn (20) in limiting case accurate within the "core's" indexes.
The case of $\rho_1^0 = \rho_3^0 = \rho^0 \to 0$ is not simple for explicit calculation. However the assumption of the invariable coefficients of elastic foundation on the spherical surfaces of a core allows of analyzing the core free vibration under the tentative variant of imponderable material of the restraining layers [7].

4. CALCULATED VARIANTS FOR THE BASIC FREQUENCY

The analysis showed that, with growing ξ the roots of the eqn (21) tend to values $\xi_k = k\pi$ if $k \to \infty$. Of greatest interest are the initial values of the roots, in particular, their smallest value, to which there corresponds the basic frequency of free vibrations.
The extent of the region of the wave process – the thickness of a shell – affects the basic frequencies differently according to the values of a boundary curvature. The basic (lowest) frequencies of radial vibrations of a monolithic sphere and of a homogeneous shell have been derived with the use of eqns (19), (20) and their respective curves are illustrated in Fig. 2.
It should be recorded that $R_1 = 0$ for the curves 1-3 in Fig. 2 and the upper curve (1) corresponds to rigid fixation of boundary surfaces whereas the rest curves correspond to open boundary surfaces at $\nu = 0.4$ (2,4) and $\nu = 0.3$ (3).

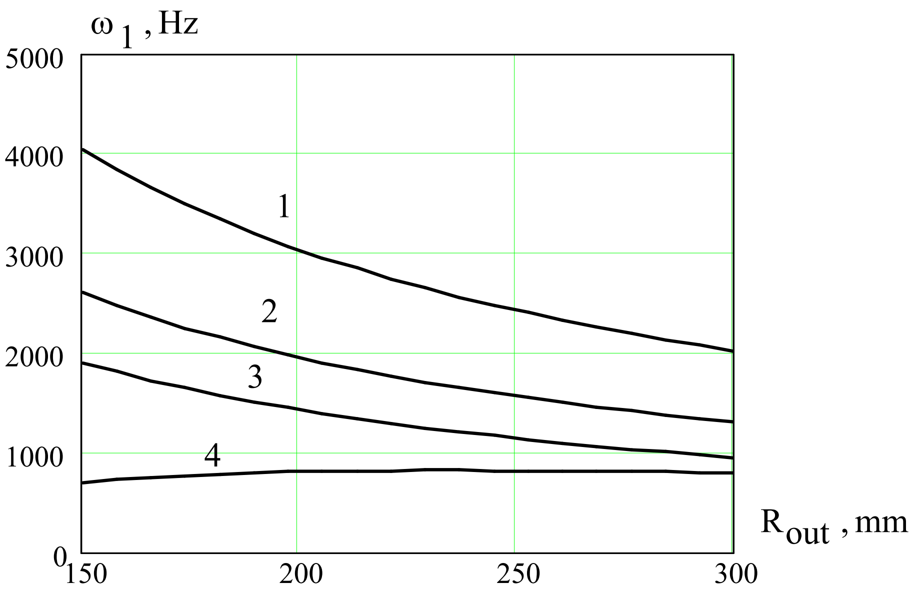


Figure 2: First eigenfrequency of radial vibrations of a monolithic sphere, 1-3, and of a sphere with an internal cavity of radius $R_{inn} = 50\ mm$, 4, $E = 40\ MPa, \rho = 119\ kg/m^3$

The effect of physical constants on the first eigenfrequency dependence of a layered sphere is analyzed with regard to a shell with a symmetric sandwich wall. Structure parameters for the wall of sandwich type are cited in Table 1. A search of the minimal nonzero magnitude of ξ from eqn (14) provided us with calculation data to obtain the eigenfrequency in terms of elastic parameters of the midlayer from eqn (12). The basic frequency grows by about an order of magnitude compared with that for a homogeneous shell made of the core material with the same geometric parameters, sees Fig. 3.

It is worthy of note that the abscissa in Fig. 3a, b characterizes continuous variations in relative physical characteristics of the outer layers (Young's modulus and Poisson's ratio of the material). The unchangeable mechanical properties of the core relating to Fig. 3*a* are given in the middle row and to Fig. 3*b* in the extreme row of the Table 1 under consideration that the face layer and midlayer thicknesses are dimensionally stable.

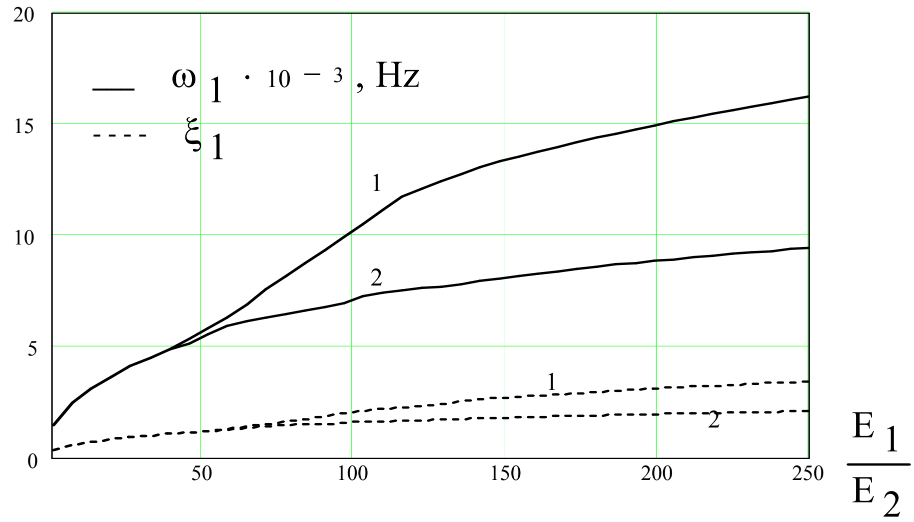
The effect of specific density is also seen from Fig. 3a, b, where the curve 1 refers to the value of $\rho_1 = \rho_{core} = const$ whereas the curve 2 allows for a value of density directly proportional to a formal parameter.

The basic frequency and eigenvalue rapidly grows as the Young's module of face layers increases so long as the core remains soft, Fig. 3a, with only a moderate increase when the core is essentially stiffened as compared to outer layers, Fig. 3b.

Table 1: Parameters of a sandwich three-layer sphere.

Layer	R_1, $100\ mm$	E , GPa	ν	ρ, kg/m^3
$R_4 - R_3$	5	10	0.30	2687
$R_3 - R_2$	30	0.04	0.40	119
$R_2 - R_1$	5	10	0.30	2687

a



b

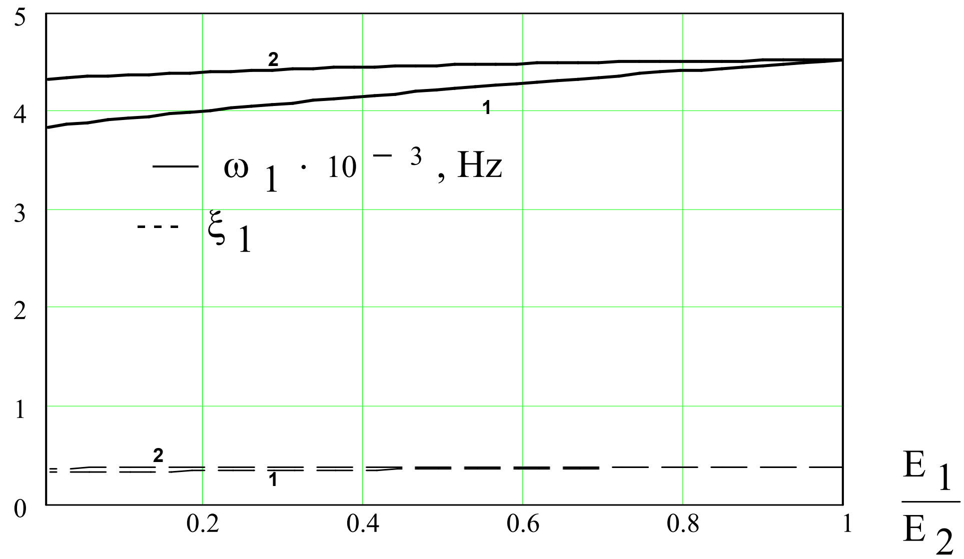


Figure 3: First eigenfrequency ω_1 and the corresponding eigenvalue ξ_1 vs. the Young's modules ratio of face layers and core. $E_2 = E_{core}$, a - physical constants of a core are given in the middle row of the Table 1, b - in the extreme row.

5.CONCLUSIONS

1. A transcendental equation for eigenvalues of the boundary-value dynamic problem at hand was derived and the variants of its degeneration for the cases of a homogeneous sphere and a spherical sandwich shell have been demonstrated. Eigenfrequencies are proportional to eigenvalues with the coefficient dependent on acoustic velocity of the material and a layer thickness.

2. The least eigenfrequency of transverse vibrations varies over a wide high-frequency range by several fold subject to physical properties and location of layers within the thickness of a sandwich shell.
3. The outer radius of a solid sphere and Poisson's ratio of the material exert primary control over natural frequencies, but for a hollow sphere with $6 \geq R_{out}/R_{inn} \geq 3$ the thickness affects insignificantly.

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