LIFETIME PREDICTION OF POLYMER-MATRIX COMPOSITE STRUCTURES FOR CIVIL ENGINEERING APPLICATIONS

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ABSTRACT

The lack of full understanding the fundamental parameters controlling long-term of PMC's (Polymer-Matrix Composites) performance necessarily leads to over-design and, furthermore, inhibits greater utilization. In this context, lifetime prediction of these structures is an important issue to be solved before wider dissemination of civil engineering applications happened. As an example, standards dealing with certification of GRP pipes require at least 10000 hours of testing for a high number of specimens. Even though these strong requirements may be foreseen as reasonable, concerning the safety of civil engineering applications, they severely restrict the improvement and innovation of new products.

In this context the present article reviews some theoretical approaches for long-term criteria. Time-dependent failure criteria for viscoelastic materials are presented and illustrated with experimental cases.

1. INTRODUCTION

The published experimental results and theoretical description of creep rupture of PMC's is not so well documented as for the polymers. This could be explained, in part, by the extremely large variety of material systems offered by the manufacture industry. The other reason is because the systems are naturally very complex, with a great number of boundaries between the constituents, given origin to a large number of local defects, such as debonds and cracks.

According to Reifsneider et al. [1], all creep rupture analyses could be divided into two major categories: the local and direct analysis of the growth of the defects and the global and homogeneous analysis. The late analysis concerns only the summation of all micro-processes effects acting concomitantly and often designated as accumulation or quasi-homogeneous damage models. The local and direct analysis of the growth of the defects has shown promising results. This type of analysis has the advantage to allow a deeper understanding of the mechanisms responsible for the rupture and creep-rupture. Nevertheless the global and homogeneous analysis, simpler to formulate and solve, is more appropriated for practical applications.

Miyano et al. [2-5] showed experimental evidence that flexural and tensile strength of CRFP (Carbon Fiber Reinforced Composite) depend on rate loading and temperature, even near room temperature. It also proved that time-temperature superposition principle (TTSP) was applicable to obtain master curves for strength of CFRP. Moreover, the same TTSP was applicable for static, creep and fatigue strengths, which presented the same failure mechanism over a wide range of time and temperature. Hiel [6-7] stated that the failure should be a part of a complete constitutive description of the material. Brinson [8] argued that this approach could simplify the procedure to predict the delayed failure in structural polymers without loosing the necessary accuracy. Being failure part of the complete constitutive description of the viscolastic material it is easy to demonstrate that failure criterion benefits form the Time-Temperature and Time-Stress Superposition Principle (TTSP and TSSP) procedures.

2. FAILURE CRITERIA FOR VISCOELASTIC MATERIALS UNDER STATIC FATIGUE (CREEP)

The Reiner-Weissenberg [9] criterion states that the work done during the loading by external forces on a viscoelastic material is converted into a stored part (potential energy) and a dissipated part (loss energy). Basically the criterion says that the instant of failure depends on a conjunction between distortional free energy and dissipated energy, a threshold value of the distortional energy is the governing quantity.

Let us assume that the unidirectional strain response of a linear viscoelastic material, under arbitrarily stress $\sigma(t)$, is given by the power law as,

$$\varepsilon(t) = D_0\sigma(t) + D_1\int_0^t \left(\frac{t-\tau}{\tau_0}\right)^n \frac{\partial\sigma(\tau)}{\partial\tau}d\tau \,, \tag{1}$$

where D_0, D_1, n = material constants; and τ_0 = time unity (equal to 1second or 1hour or 1day, etc.). Accordingly these time-dependent failure criteria [10] predict the lifetime under constant load, as a function of the applied load σ_0 and the strength under instantaneous conditions σ_R:

R-W Criterion

$$\left(\frac{t_f}{\tau_0}\right) = \left(\frac{1}{2-2^n}\right)^{\frac{1}{n}} \left(\frac{D_0}{D_1}\right)^{\frac{1}{n}} \left(\frac{1}{F_a^{\,2}}-1\right)^{\frac{1}{n}} . \tag{2}$$

Maximum Work Stress Criterion (MWS)

$$\left(\frac{t_f}{\tau_0}\right) = \left(\frac{1}{2}\frac{D_0}{D_1}\right)^{\frac{1}{n}} \left(\frac{1}{F_a^{\,2}}-1\right)^{\frac{1}{n}} . \tag{3}$$

Maximum Strain Criterion (MS)

$$\left(\frac{t_f}{\tau_0}\right) = \left(\frac{D_0}{D_1}\right)^{\frac{1}{n}} \left(\frac{1}{F_a^{\,2}}-1\right)^{\frac{1}{n}} . \tag{4}$$

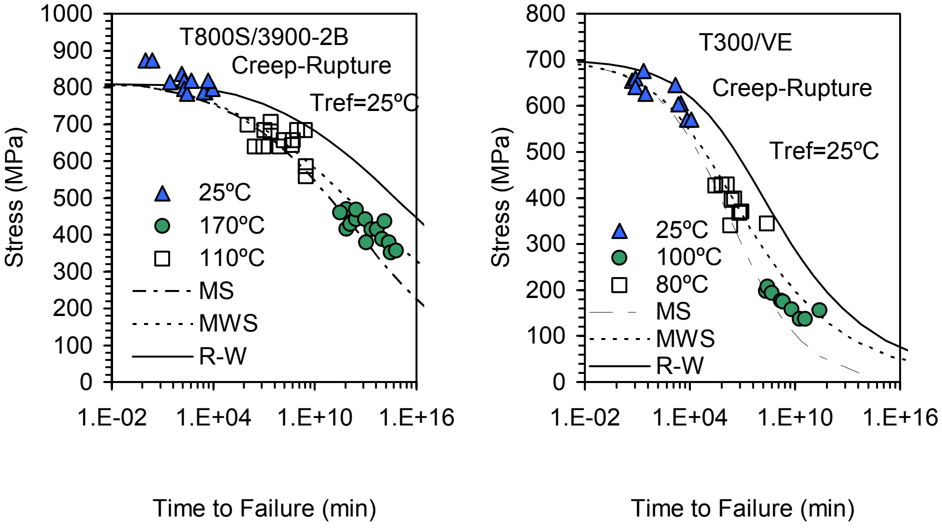
where $F_a = \sigma_0/\sigma_R$ is the normalized applied stress.

The creep strength for two different composites is used to illustrate the potential applicability of the previous theoretical approaches. One is a carbon fiber reforced polymer, consisting of nine layers of plain woven cloth of carbon fiber and matrix vinylester resin (T300/VE) [11]. The other is the T800S/900-2B which consists of unidirectional T800 carbon fiber and 3900 epoxy resin with toughened interlayer [12] and with the following staking sequence $[45/0/90/\text{-}45/90]_S$. The creep properties for the vinylester resin and for the T800S/900-2B composite are presented in Table 1.

The experimental procedure [11-12] was based on the Time-Temperature-Superposition Principle (TTSP) to accelerate the tests. In Figures 1a and 1b are depicted the experimental creep strength and the theoretical predictions. As it can be observed the theoretical predictions, based time-dependent failure criteria are in fairly good agreement with experimental results.

Table 1: Viscoelastic creep properties.

Material	Ref.	T (ºC)	Creep Compliance (1/MPa)				Instantaneous Failure Stress (MPa)
			D_0	D_1	n	τ_0	σ_R
VE	Miyano (2005)	25º	3.60E-04	1.69E-05	0.209	1 min	700
T800S/3900-2B	Miyano (2006)	25º	1.85E-05	6.33E-07	0.119	1 min	830



(a) (b)

Figure 1: Creep lifetime prediction for (a) T800S/900-2B and (b) T300/VE.

In both cases R-W, MWS and MS failure criteria predict dissimilar lifetimes and R-W predict systemically higher lifetimes for each stress load. The experimental data fall systematically between R-W and MS failure criteria.

3. RELATIONSHIP BETWEEN CONSTANT STRESS/STRAIN RATE (CSR) UNTIL FAILURE AND CREEP FAILURE

It is not difficult to deduce the lifetime under constant stress rate (CSR) loading for the previous criteria, as a function of the stress rate R and the strength under instantaneous conditions σ_R:

R-W Criterion

$$\left(\frac{t_f}{\tau_0}\right)=\left(\frac{(n+1)(n+2)}{(n+2)+\left(1-2^{n+1}\right)}\right)^{\frac{1}{n}}\left(\frac{D_0}{D_1}\right)^{\frac{1}{n}}\left(\frac{1}{F_a^{\;2}}-1\right)^{\frac{1}{n}}, \tag{5}$$

Maximum Work Stress Criterion

$$\left(\frac{t_f}{\tau_0}\right)=(n+2)^{\frac{1}{n}}\left(\frac{D_0}{D_1}\right)^{\frac{1}{n}}\left(\frac{1}{F_a^{\;2}}-1\right)^{\frac{1}{n}}, \tag{6}$$

Maximum Strain Criterion

$$\left(\frac{t_f}{\tau_0}\right)=(n+1)^{\frac{1}{n}}\left(\frac{D_0}{D_1}\right)^{\frac{1}{n}}\left(\frac{1}{F_a}-1\right)^{\frac{1}{n}}, \tag{7}$$

where $F_a=\left(Rt_f/\sigma_R\right)$ is the normalized applied stress.

These results, by comparison with previous ones, show us that all time-dependent failure criteria predict the same curve shape for stress-time to failure curves under creep and CSR loading, i.e for each criterion both curves differ only by a constant parameter. In Figures 2a and 2b are depicted the experimental CSR strength and the theoretical predictions. As it can be observed the theoretical predictions, based time-dependent failure criteria are in fairly good agreement with experimental results. The exception goes to short-time prediction for the T800S/900-2B composite, for which was not predicted the strength increase in the high strain rate range.

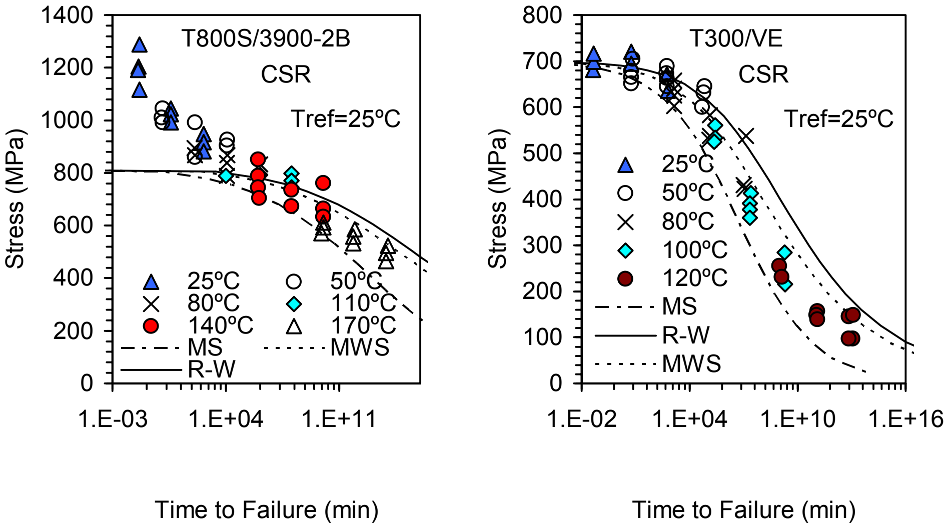


Figure 2: CSR lifetime prediction for (a) T800S/900-2B and (b) T300/VE.

3.1 Continuum damage mechanics

A classical approach to consider the degradation of mechanical properties is provided by the method of continuum damage mechanics (CDM). Following the original ideas of Kachanov [13], the net stress, defined as the remaining load bearing cross section of the material is given [14] by,

$$\tilde{\sigma} = \frac{\sigma}{1-\omega}, \tag{8}$$

where $0 \le \omega \le 1$ is the damage variable. At rupture no load bearing area remains and the net stress tends to infinity when $\omega \to 1$.

Kachanov [13] assumes the following damage growth law

$$\dot{\omega}(t) = C\left(\frac{\sigma(t)}{1-\omega(t)}\right)^{\nu}, \tag{9}$$

where C and ν are material constants. This equation leads to a separable differential equation for $\omega(t)$, assuming $\omega(0)=0$

$$\left(1-\omega(t)\right)^{\nu}\dot{\omega}(t) = C\sigma^{\nu}(t) \Rightarrow 1-\left(1-\omega(t)\right)^{1+\nu} = C(1+\nu)\int_0^t \sigma^{\nu}(\tau)\,d\tau\,. \tag{10}$$

The damage growth law is given as,

$$\omega(t) = 1-\left[1-C(1+\nu)\int_0^t \sigma^{\nu}(\tau)\,d\tau\right]^{\frac{1}{1+\nu}}. \tag{11}$$

The damage speed according to time can then be readily obtained as

$$\dot{\omega}(t) = C\sigma^{\nu}(t)\left[1-C(1+\nu)\int_0^t \sigma^{\nu}(\tau)\,d\tau\right]^{\frac{1}{1+\nu}-1}. \tag{12}$$

Assuming failure when $\omega = 1$ then the following expression is obtained

$$C(1+\nu)\int_0^t \sigma^{\nu}(\tau)\,d\tau = 1\,. \tag*{13)}$$

From the previous relationship, the time to failure for creep is readily obtained assuming $\sigma(t)=\sigma_0$,

$$t_c = \frac{1}{C(1+\nu)\sigma_0{}^{\nu}}. \tag{14}$$

A similar expression is obtained for CSR (Constant Stress/Strain Rate) loading condition, assuming $\sigma(t)=Rt$,

$$t_s = \frac{1}{C\sigma_0{}^{\nu}}, \tag{15}$$

where $\sigma_0 = Rt_s$.

Therefore this result implies that the creep lifetime curves differ from the CSR lifetime curves by one constant, i.e. both curves can be superimposed by shifting a constant amount on the log time scale.

The previous expression, Equation (15) can be rearranged in order to obtain a Monkman-Grant type relationship

$$t_s R^{\frac{\nu}{\nu+1}} = \left(\frac{1}{C}\right)^{\frac{1}{\nu+1}}. \tag{16}$$

It was demonstrated by Odqvist [15] that the Kachanov's version of the continuum damage mechanics (CDM) theory implies the Linear Cumulative Damage law (LCD) also known as the Miner's Rule [16]. The LCD law provides a simply way to account damage accumulation due to creep cycle at multiple stress levels, based creep master curves. The mathematical statement of LCD law in the integral form is written as

$$\int_0^{t_f} \frac{dt}{t_c\left[\sigma(t)\right]} = 1, \tag{17}$$

where t_c is the lifetime under creep and t_f is the lifetime under variable stress (or strain).

Stigh [14] proved, by extension, that any separable evolution damage law also implies the LCD. That is, if

$$\dot{\omega}(t) = f(\sigma)g(\omega), \tag{18}$$

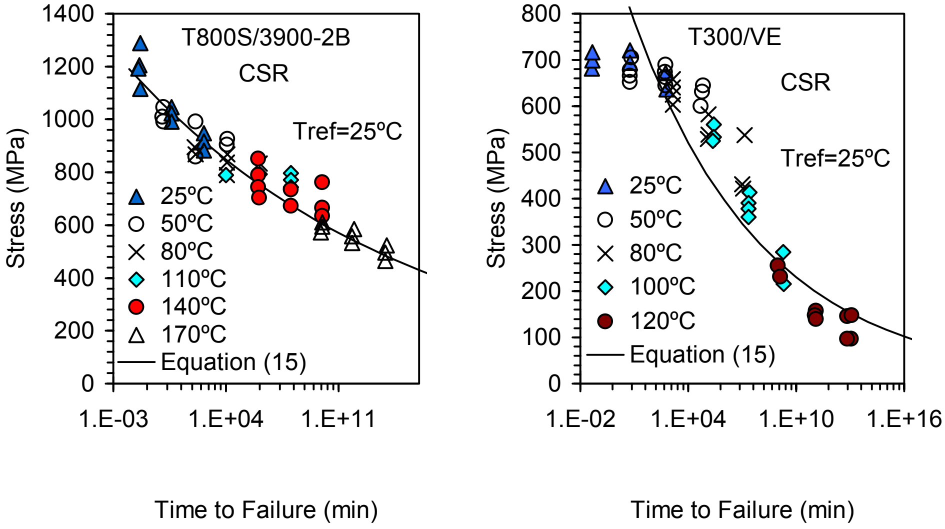
This approach can be used to analyze the previous data. In table 2 are presented the material parameters, *v* and *C*, obtained by curve fitting the CSR experimental data. In Figures 3a and 3b are depicted the obtained results.

Table 2: Material parameters obtained for Equation (15).

Material	C	v
T300/VE	9.9354E-51	16.934
T800S/3900-2B	5.458E-124	40.726

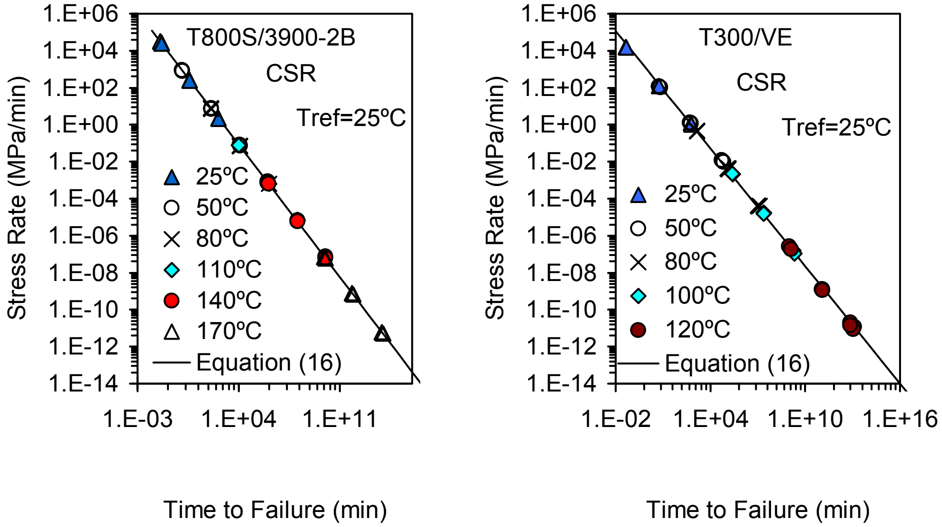
In Figures 4a and 4b are depicted the relationship between the time to failure and the stress rate which obeys to a Monkman-Grant relationship, as predicted by Equation (16). Following the deduced relationships between CSR and creep lifetimes, the creep strength curves can be calculated using Equation (14) and the material parameters given in Table 2. In Figures 5a and 5b are shown the calculated creep strength curves, using Equation (14), compared with experimental data. The predictions made using Equation (16) are in good agreement with experimental data.

The simple relationship obtained between creep and CSR lifetime was used to calculate the creep strength based on CSR data. All theoretical approaches predict a constant amount for shifting along the log time scale. In Table 3 are shown these shift quantities given by the ratio between lifetime under CSR and lifetime under creep, t_s/t_c.



(a) (b)

Figure 3: Curve fitting of CSR lifetime for (a) T800S/900-2B and (b) T300/VE using Equation (15).

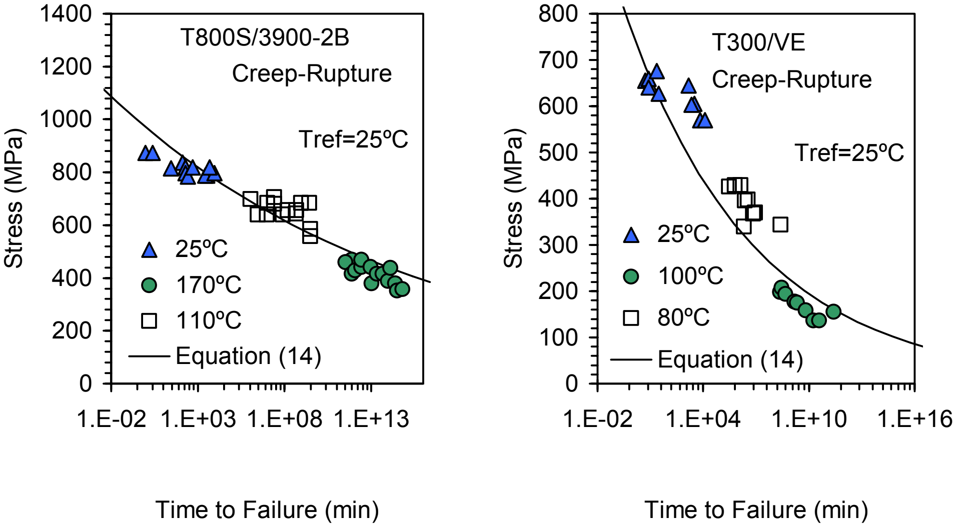


(a) (b)

Figure 4: Time to failure evolution in function of stress rate for (a) T800S/900-2B and for (b) T300/VE.

Table 3: The ratio between lifetime under CSR and lifetime under creep predicted by the theoretical models.

Theoretical Model	T300/VE	T800S/3900-2B
	t_s/t_c	t_s/t_c
CDM & LCD	17.93	41.73
R-W	81.76	1074.35
MSW	44.16	562.67
MS	2.479	2.573



(a) (b)

Figure 5: Calculated creep lifetime for (a) T800S/900-2B and (b) T300/VE using Equation (14).

3.2 Curtin-McLeen model

The conditions of applicability of Miner's rule were discussed by Christensen [17]. In this work Christensen [17] provided a theoretical validation of the use of LCD law when relating creep failure conditions to constant stress (or strain) rate failure conditions. For that purpose Christensen [17] developed a simple kinetic crack growth theory based following the Schapery [18] formalism and based on a generalization to viscoelastic material of the Griffith result for elastic material. Guedes [19-20] extended this validation by using other existing theoretical frameworks.

Du and McMeeking [21] predicted the creep rupture time in unidirectional composites under tensile loads. The model assumed that when the composite strain (or stress) of the McLean [22] model had reached the rupture strain (or stress) of the Curtin [23] model, the composite failed. Latter on Koyanagi et al. [24] proposed a modified version applied to a unidirectional glass fiber/vinylester composite, which was in part experimentally validated.

Such model can be used to demonstrate the applicability of LCD law to relate static strength (CSR) to creep failure. Following Du and McMeeking [17] and Koyanagi et al. [24], the Curtin-McLeen model (CML) lifetime expressions are deduced for creep and constant stress rate. The McLeean model was derived considering the fiber was elastic and the matrix was viscoleastic;

Constant stress:

The fibers strain is (elastic)

$$\varepsilon = \frac{\sigma_f}{E_f}, \tag{19}$$

and the matrix strain (viscoelastic)

$$\dot{\varepsilon} = \frac{\dot{\sigma}_m}{E_m} + B\sigma_m{}^n, \tag{20}$$

where ε is the total strain, B and n the creep coefficients, σ_m the matrix stress, σ_f the fiber stress, and E_m, E_f are the matrix and fiber modulus, respectively

The composite stress is given by the rule of mixtures,

$$\sigma = \left(1 - V_f\right)\sigma_m + V_f \sigma_f , \tag{21}$$

where V_f is the fiber volume fraction. From the above equations the composite creep strain is derived

$$\varepsilon\left(t\right) = \frac{\sigma}{V_f E_f} - \frac{1 - V_f}{V_f E_f}\left(\left(\frac{E_m}{\left(1 - V_f\right)E_m + V_f E_f}\sigma\right)^{1-n} + B\frac{\left(n-1\right)V_f E_m E_f}{\left(1 - V_f\right)E_m + V_f E_f}t\right)^{1/(1-n)} \tag{22}$$

Assuming the allowable maximum fiber stress as

$$\varepsilon_{\max} = \frac{S_{\max}}{E_f} , \tag{23}$$

The creep lifetime expression is given by

$$t_c = \frac{\left(1 - V_f\right)E_m + V_f E_f}{\left(1 - n\right)V_f E_m E_f B}\left[\left(\frac{1 - V_f}{\sigma - V_f S_{\max}}\right)^{n-1} + \left(\frac{\left(1 - V_f\right)E_m + V_f E_f}{\sigma E_m}\right)^{n-1}\right]. \tag{24}$$

Constant Stress Rate:
Following the same procedure, if *n*=2, an explicit expression is obtained for the strain

$$\varepsilon\left(t\right) = \frac{\dot{\sigma}_0}{V_f E_f}t - \frac{\sqrt{B\left(V_f E_f\right)^3 \dot{\sigma}_0 \left(1 - V_f\right)^2}}{B\left(V_f E_f\right)^3}\tanh\left(\frac{E_m\sqrt{B\left(V_f E_f\right)^3 \dot{\sigma}_0\left(1 - V_f\right)^2}}{V_f E_f\left(1 - V_f\right)\left(E_m\left(1 - V_f\right) + V_f E_f\right)}t\right) \tag{25}$$

Assuming the allowable maximum fiber stress as

$$\varepsilon_{\max} = \frac{S_{\max}}{E_f} , \tag{26}$$

the time to failure under constant stress rate can be predicted, by solving numerically the resulting equation.
In Table 4 are indicated the model properties used to simulate glass fiber-reinforced polymer.

Table 4: Elastic and viscoelastic properties used to simulate a glass fiber-reinforced polymer.

E_c (MPa)	E_f (MPa)	E_m (MPa)	V_f	B	n	S_{max} (MPa)
10500	70200	3867	0.1	2.50E-08	2	1100

Using the numerical procedure devised by Miyano et al. [3], based on LCD law, the creep strength was predicted based on CSR strength predicted by CML. The CSR strength predicted by CML obeys to a Monkman-Grant relationship. These results were compared with creep strength directly obtained via the CML model. The comparison is depicted in Figure 6 and a significant deviation between both creep strengths is clear. This result shows that the methodology proposed by Miyano et al may not always work properly. We have to bear in mind that this is a mere model simulation. As for all models it is an idealization which considers the fiber elastic and are perfectly aligned to a single direction, the load direction, and the fiber fracture do not affect the creep of the matrix.

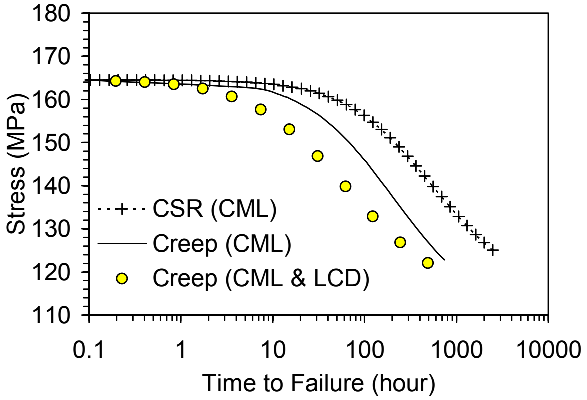


Figure 6: Comparison of creep strength obtained directly from the CML model and indirectly by applying the LCD law to the static strength (CSR) data obtained from the CML model.

4. CONCLUSIONS

It is shown that the time-dependent failure criteria predict the same curve shape for stress-time to failure curves under creep and CSR loading, i.e. both curves differ only by a constant parameter. However the amount of shifting is dependent on the theoretical approach. Creep and CSR strength curves for two different composites are used to illustrate these results in comparison with LCD (Linear Cumulative Damage) law used by Miyano et al. [2-5].

Finally, using [21] and [24] developments the Curtin-McLeen model (CML) lifetime expressions are deduced for creep and constant stress rate. This model is used to analyse the applicability of LCD (Linear Cumulative Damage) law to relate static strength (CSR) to creep failure for unidirectional laminates. It is verified that the relationship between the time to failure and the stress rate obeys to a Monkman-Grant equation. Using the numerical procedure devised by [3], based on LCD law, the creep strength was predicted based on CSR results. These were compared with creep strength directly obtained from the CML model. The results show that the methodology proposed by Miyano et al may not always work properly.

ACKNOWLEDGEMENTS

The author wishes to dedicate this work to the memory of Professor Albert H. Cardon (VUB, Brussels).

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