

FORMULATION AND EXPERIMENTAL IDENTIFICATION OF A NEW FATIGUE DAMAGE MODEL FOR SHORT GLASS FIBRE REINFORCED THERMOPLASTICS

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ABSTRACT

In the present work, a new polycyclic fatigue damage model is formulated and applied for short glass fibre reinforced thermoplastics. The model is able to capture experimental trends observed for the considered composites. The damage growth description involves a set of 20 parameters in the case of a complete 3D – structure. In the current paper, it is considered the particular case of a displacement controlled fatigue tensile test involving 4 damage parameters. The present contribution is a first approach of parameter identification. It is considered a least squares sense based cost function and homogeneous fatigue tests performed on a short glass fibre reinforced polyamide. The identified set of parameters appears to be not depending on the adopted initial values. The model as the parameters determined by the minimisation algorithm, are validated on a fatigue test performed with a different loading condition.

1. INTRODUCTION

The present work is a contribution to the phenomenological modelling of fatigue damage in short glass fibre reinforced thermoplastic matrix composites. In such materials, the fatigue damage kinetic occurs according to three stages: i) material softening and damage initiation ii) coalescence and propagation of micro-cracks iii) macroscopic cracks propagation and material failure, [1, 3]. Different laws have been proposed to predict the damage accumulation in unidirectional or laminated polymer matrix composites [3, 5]. These models do not take into account the first stage of the damage kinetic observed in short glass fibre reinforced thermoplastics. From experimental investigation it is shown that during the first few hundred cycles, the damage kinetic is mainly due to the material softening depending on the applied strain level. Based on the Ladevèze and Ledantec approach [6], the proposed modelling is an attempt to extend previous works by accounting for the important stiffness reduction observed during the early stage of the damage evolution. An inverse analysis is introduced in order to determine the set of parameters introduced in the current work. It is used experimental data obtained from tension-tension fatigue tests performed on a PA6-GF30 and a cost function written in a least squares sense. It appears that the set of parameters computed from the analysis is not depending on the initial values. The modelling with the identified parameters is accurately compared to experimental data that have not been considered for the identification.

2. NEW FATIGUE DAMAGE MODEL

To take into account the observed three damage stages, a new polycyclic fatigue damage model is formulated on the basis of the meso model proposed by Ladevèze and Ledantec [6]. According to the continuum damage mechanics, the damage is introduced as an internal state

variable coupled to elastic behaviour. Assuming a thin structure where $\sigma_{33}=0$, made of an orthotropic material, elastic moduli of the damaged material are given by:

$$\begin{aligned}
E_{11} &= E_{11}^0 (1 - d_{11}) \\
E_{22} &= E_{22}^0 (1 - d_{22}) \\
G_{12} &= G_{12}^0 (1 - d_{12}) \\
G_{13} &= G_{13}^0 (1 - d_{13}) \\
G_{23} &= G_{23}^0 (1 - d_{23})
\end{aligned} \tag{1.1}$$

where E_{11} (resp. E_{22}) is the Young's modulus in the longitudinal (resp. transverse) direction. G_{12} , G_{13} and G_{23} are the shear moduli. d_{ij} are the damage variables associated to the corresponding moduli. The superscript 0 indicates initial values measured when $d_{ij}=0$. In the case where damage is disregarded, the elastic strain energy (W_e) is given by:

$$\begin{aligned}
W_e &= \frac{1}{2} \frac{1}{1 - \nu_{12}\nu_{21}} \left[\left[E_{11}^0 \varepsilon_{11} + \nu_{21} E_{11}^0 \varepsilon_{22} \right] \varepsilon_{11} + \left[\nu_{12} E_{22}^0 \varepsilon_{11} + E_{22}^0 \varepsilon_{22} \right] \varepsilon_{22} \right] \\
&\quad + G_{12}^0 \gamma_{12}^2 + G_{13}^0 \gamma_{13}^2 + G_{23}^0 \gamma_{23}^2
\end{aligned} \tag{1.2}$$

For a damaged material, the elastic strain energy becomes dependent on the state variables d_{ij} . Then, the strain energy (W_d) of a damaged material can be computed from Eq.(1.2) by replacing (E_{ij}^0 , G_{ij}^0) by the corresponding moduli (E_{ij} , G_{ij}) expressed in Eq. (1.1). One obtains:

$$\begin{aligned}
W_d &= \frac{1}{2} \frac{1}{1 - \nu_{12}\nu_{21}} \left[E_{11}^0 (1 - d_{11}) \varepsilon_{11} \langle \varepsilon_{11} + \nu_{21} \varepsilon_{22} \rangle_+ + E_{11}^0 \varepsilon_{11} \langle \varepsilon_{11} + \nu_{21} \varepsilon_{22} \rangle_- \right] \\
&\quad + \frac{1}{2} \frac{1}{1 - \nu_{12}\nu_{21}} \left[E_{22}^0 (1 - d_{22}) \varepsilon_{22} \langle \varepsilon_{22} + \nu_{12} \varepsilon_{11} \rangle_+ + E_{22}^0 \varepsilon_{22} \langle \varepsilon_{22} + \nu_{12} \varepsilon_{11} \rangle_- \right] \\
&\quad + G_{12}^0 (1 - d_{12}) \gamma_{12}^2 + G_{13}^0 (1 - d_{13}) \gamma_{13}^2 + G_{23}^0 (1 - d_{23}) \gamma_{23}^2
\end{aligned} \tag{1.3}$$

where ($\langle A \rangle_+$) and ($\langle A \rangle_-$) stand for the positive and negative parts of A, respectively. Thus, the damage affects E_1 (resp. E_2) only when $\varepsilon_{11} + \nu_{21} \varepsilon_{22}$ (resp. $\varepsilon_{11} + \nu_{21} \varepsilon_{22}$) is positive. This can be explained by the following. When the material is submitted to a compressive loading, transverse matrix cracks are supposed to be closed up and do not have any effect on the damage evolution. On the opposite, during tension loading those cracks are active and contribute to the development of the damage.

The thermodynamic dual variables Y_{ij} associated to the damage variables d_{ij} are deduced from the elastic strain energy W_d of the degraded material:

$$\begin{aligned}
Y_{11} &= -\frac{\partial W_d}{\partial d_{11}} = \frac{1}{2} \frac{1}{1 - \nu_{12}\nu_{21}} E_{11}^0 \varepsilon_{11} \langle \varepsilon_{11} + \nu_{21} \varepsilon_{22} \rangle_+ & Y_{12} &= -\frac{\partial W_d}{\partial d_{12}} = \frac{1}{2} G_{12}^0 \gamma_{12}^2 \\
Y_{22} &= -\frac{\partial W_d}{\partial d_{22}} = \frac{1}{2} \frac{1}{1 - \nu_{12}\nu_{21}} E_{22}^0 \varepsilon_{22} \langle \varepsilon_{22} + \nu_{12} \varepsilon_{11} \rangle_+ & Y_{13} &= -\frac{\partial W_d}{\partial d_{13}} = \frac{1}{2} G_{13}^0 \gamma_{13}^2 \\
&& Y_{23} &= -\frac{\partial W_d}{\partial d_{23}} = \frac{1}{2} G_{23}^0 \gamma_{23}^2
\end{aligned} \tag{1.4}$$

In the proposed modelling, the damage rate is assumed to be the sum of two components:

$$\frac{d(d_{11})}{d(N)} = \frac{\alpha_{11}\beta_{11}}{1+\beta_{11}}(Y_{11})^{\beta_{11}-1} + \lambda_{11}(Y_{11})\left(e^{-(\delta_{11}N)}\right) \quad (1.5)$$

$$\frac{d(d_{22})}{d(N)} = \frac{\alpha_{22}\beta_{22}}{1+\beta_{22}}(Y_{22})^{\beta_{22}-1} + \lambda_{22}(Y_{22})\left(e^{-(\delta_{22}N)}\right) \quad (1.6)$$

$$\frac{d(d_{12})}{d(N)} = \frac{\alpha_{12}\beta_{12}}{1+\beta_{12}}(Y_{12})^{\beta_{12}-1} + \lambda_{12}(Y_{12})\left(e^{-(\delta_{12}N)}\right) \quad (1.7)$$

$$\frac{d(d_{13})}{d(N)} = \frac{\alpha_{13}\beta_{13}}{1+\beta_{13}}(Y_{13})^{\beta_{13}-1} + \lambda_{13}(Y_{13})\left(e^{-(\delta_{13}N)}\right) \quad (1.8)$$

$$\frac{d(d_{23})}{d(N)} = \frac{\alpha_{23}\beta_{23}}{1+\beta_{23}}(Y_{23})^{\beta_{23}-1} + \lambda_{23}(Y_{23})\left(e^{-(\delta_{23}N)}\right) \quad (1.9)$$

The first contribution, scaled by $(Y_{ij})^{\beta_{ij}-1}$ in Eqs. (1.5) to (1.9), is derived from the Norton dissipation potential. Note that in a previous work from Sedrakian et al [1], the damage evolution is only described by this first term. The second component is introduced in order to describe the rapid stiffness reduction occurring during cyclic loading of reinforced thermoplastics. The developed model is then a complete model in the sense that the entire damage process (3 stages) could be described. The instantaneous state variables $d_{ij}(N)$ are obtained by numerical integration of Eqs. (1.5) to (1.9), with the initial conditions $d_{ij}(N=0)=d^{qs}_{ij}$. These initial values are function of the imposed displacement. In the case where the applied strain ε^{app}_{ij} is below a threshold value associated to the beginning of the damage process in the (i,j) direction, $d^{qs}_{ij}=0$. From Eqs. (1.5) to (1.9), it appears that the proposed model involves 20 parameters in the general case of a 3 -D structure. Nevertheless, it should be mentioned that the model is essentially developed for thin composite structures. In that particular case, relationships governing the damage evolution are reduced to Eqs. (1.5) to (1.7) and only 12 parameters have to be identified namely, 4 parameters per damage variable. In the next section, an identification procedure is adopted to reach this goal.

3. EXPERIMENTAL IDENTIFICATION

Material damage parameters identification cannot be achieved without experimental tests involving the three stages of damage. In the present paper, we focus on longitudinal tension-tension tests leading to only a longitudinal damage d_{11} . For simplicity in the notation, d_{11} is denoted hereafter d and corresponds to the longitudinal damage in Eq. (1.5). In the same way, α , β , λ and δ substitute α_{11} , β_{11} , λ_{11} and δ_{11} in Eq. (1.5).

3.1. Experimental procedure

We consider fatigue tests performed in tension-tension. The studied composite material is noted **PA6-GF30**. It is a polyamide matrix reinforced by 30% (weight) of glass fibre. The reinforcement makes the material highly resistant to abrasion, compression, tension or bending. The specimens used in the static and fatigue tensile tests were cut from thin plates. The geometry and dimensions of the tension specimens are shown in figure 1. Fatigue tests

were strain controlled and carried out using a servo-hydraulic machine. The imposed displacement wave was sinusoidal with constant amplitude. All tests were performed at room temperature. The mechanical properties of the material have been extracted from static tension tests. Some of the properties are given in Table 1

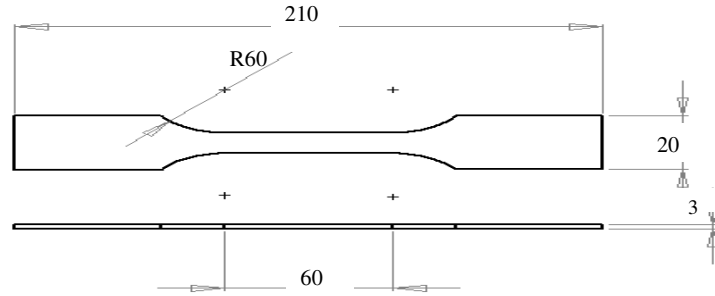


Figure 1: Specimen used for tension tests (dimensions in mm).

Material	E^0 (MPa)	$\epsilon_{SD}(\%)$	$\epsilon_{Max}(\%)$	$\sigma_{UTS}(\text{MPa})$
PA6-GF30 longitudinal direction	7042 (370)	0.7 (0.02)	5 (0.4)	114 (6)

Table 1 : Monotonic mechanical properties of the studied material

Experimental cyclic tests are driven by an applied displacement, see figure 2. It is introduced the maximum strain ϵ_{max} and the minimum strain ϵ_{min} . It is usually defined the fatigue ratio by $R = \epsilon_{min} / \epsilon_{max}$. We denote erupt the maximum strain reached when the material fails. Three strain levels ($\epsilon_{max} = 20\% \epsilon_{rup}$, $\epsilon_{max} = 30\% \epsilon_{rup}$ and $\epsilon_{max} = 40\% \epsilon_{rup}$) have been conducted using three specimens for each different levels. The ratio R remains the same for the different configuration ($R = 0.3$). The frequency is 2Hz. Some properties are summarized in Table 2.

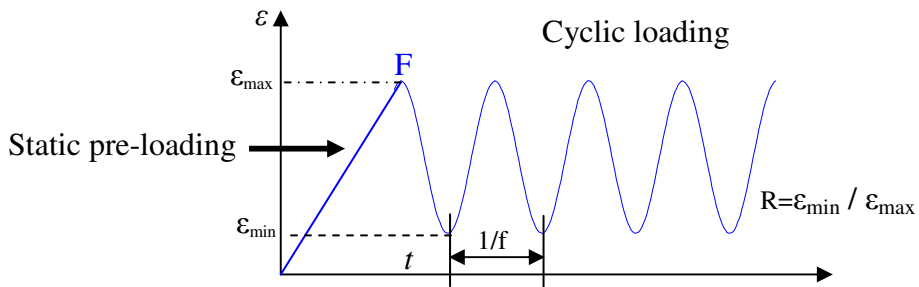


Figure 2: Strain history obtained from an imposed displacement test. Note that the static pre-loading (up to the point F) could give rise to an initial damage representing the quasi-static damage (denoted by d_{qs} in the current work).

	Test 20%	Test 30%	Test 40%
ϵ_{rup}	5	5	5
E^0	7600	7200	6900
d_{qs}	0.05	0.08	0.10

Table 2 Monotonic mechanical properties of fatigue test

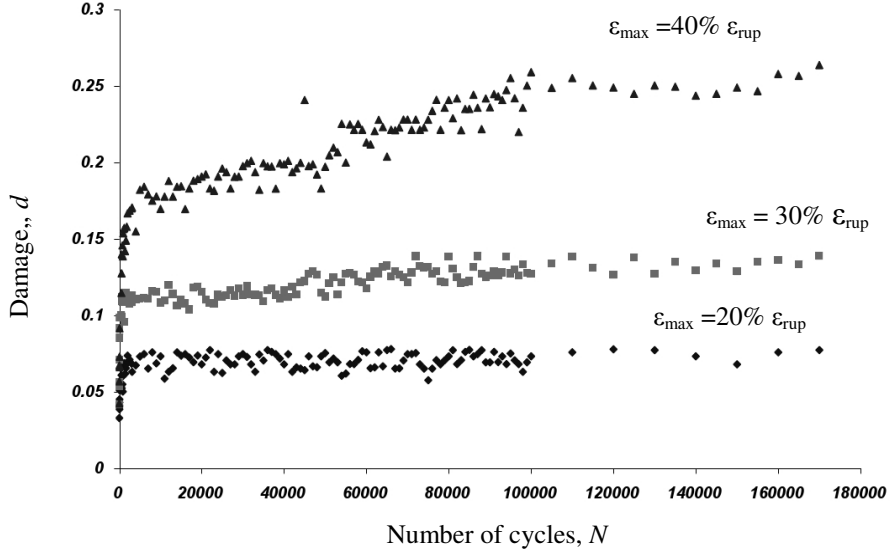


Figure 3: Experimental results of the evolution of the damage d versus the number of cycles. The obtained for the PA6-GF30 subjected to tension-tension displacement controlled fatigue test. Three maximum applied strains ϵ_{\max} are considered: 20%, 30% and 40% of the total fracture strain measured during a static tensile test. The fatigue ratio is $R=0.3$.

It is shown on Figure 3 the evolution of d versus N obtained from experimental fatigue tension-tension tests at different ϵ_{\max} ($\epsilon_{\max}20,30,40$). It is observed that even if data are quite scattered, an increase in ϵ_{\max} leads to an increase in the damage rate, and an enhanced d_{qs} . This is due to a more cumulative damage occurring when the maximum strain passes from $20\%\epsilon_{rup}$ to $30\%\epsilon_{rup}$ to $40\%\epsilon_{rup}$. In figure 2, the point F defines the end of the quasi-static step. It is the highest for $\epsilon_{\max}=40\%\epsilon_{rup}$ and the lowest for $\epsilon_{\max}=20\%\epsilon_{rup}$. In the latter case, less damage is cumulated up to the point F, and d_{qs} is then reduced. From Eq.(1.4) with $Y_{11}=Y$, in the case of a uniaxial strain test ($\epsilon_{22}=0$), the thermodynamic variable Y is increased when ϵ_{11} increases. That leads to an increase in dd/dN , see Eq.(1.5) with $d_{11}=d$.

3.2 Identifications of parameters

The goal is to extract the materials parameter from experimental data. An objective function representing, in the least squares sense, the difference between experimental and numerical data [10] has been built and minimised. The cost function is written as:

$$S(\underline{P}) = \frac{\sum_{i=1}^a [d_i^{comp}(\underline{P}) - d_i^{exp}]^2}{\sum_{i=1}^a (d_i^{exp})^2} \quad (1.10)$$

where a is the number of experimental data. d_i^{exp} measure the longitudinal modulus reduction and d_i^{comp} is the computed value obtained from the modelling (Eq 1.5) with $d_{11}=d$. \underline{P} is the parameter vector that contains $(\alpha, \beta, \lambda, \delta)$.

The minimisation of the cost function is an iterative procedure. At each step, the vector \underline{P} is re-estimated and (\underline{P} becomes $\underline{P} + \Delta\underline{P}$) and the cost function S is re-evaluated. $\Delta\underline{P}$ is computed by using a Gauss-Newton algorithm the iterative procedure is stopped when two consecutive values of the cost function satisfy: $\|S(\underline{P} + \Delta\underline{P}) - S(\underline{P})\| < 10^{-5}$. Namely, the cost function becomes stationary. The parameters α, β, λ and δ are identified by using experimental data

obtained from tests corresponding to $\epsilon_{\max}=30\%\epsilon_{rup}$ and $\epsilon_{\max}=40\%\epsilon_{rup}$, The identified parameters are: $\alpha=1.8473 \cdot 10^{-7}$, $\beta=5.5206$, $\lambda=4.9484 \cdot 10^{-5}$ and $\delta=9.6111 \cdot 10^{-4}$. The comparison between experimental data used for the identification strategy and the computed damages are presented on figure 4.

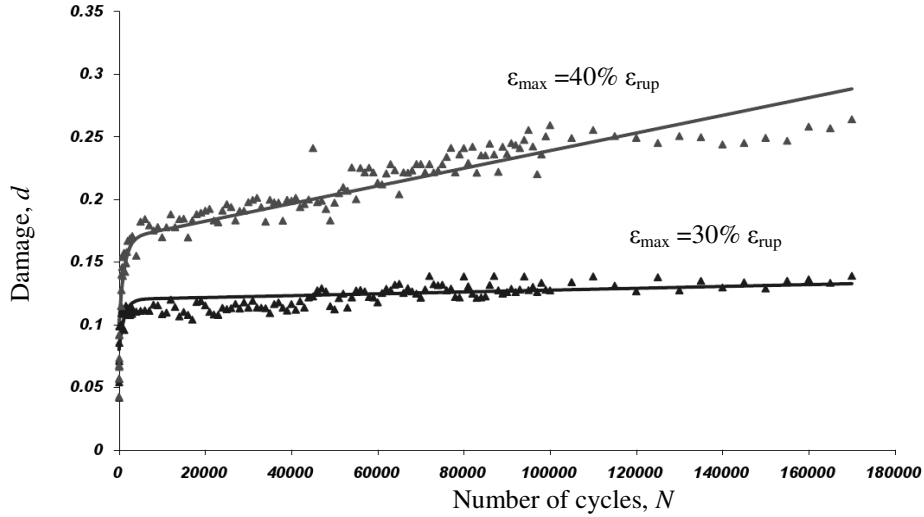


Figure 4: Comparison between experimental and calculated damage data for two configurations ($\epsilon_{\max}=40\%$ and $\epsilon_{\max}=30\%$). These two conditions are used to identify the parameters listed in table 3

Moreover, it has been assessed that the identification procedure does not depend on the initial values of α , β , λ and δ , see Table 3.

	guess values 1 (8 iterations)		guess values 1 (13 iterations)	
	Initial values	Identified values	Initial values	Identified values
α	$1 \cdot 10^{-5}$	$1.8473 \cdot 10^{-7}$	$1 \cdot 10^{-3}$	$1.8473 \cdot 10^{-7}$
β	1	5.5206	$1 \cdot 10^{-2}$	5.5205
λ	$1 \cdot 10^{-4}$	$4.9484 \cdot 10^{-5}$	$1 \cdot 10^{-2}$	$4.9487 \cdot 10^{-5}$
δ	$1 \cdot 10^{-3}$	$9.6111 \cdot 10^{-4}$	$1 \cdot 10^{-1}$	$9.6117 \cdot 10^{-4}$
Cost function values	0.0047		0.0047	

Table 3 Identification results for two different initial parameters

Finally, the set of parameters determined by the inverse procedure is validated on the experimental curve for the case where $\epsilon_{\max}=20\%$. Figure 5 shows the comparison between experimental data and numerical calculations for the parameters listed in table 3. Prediction based on the modelling is able to capture the damage evolution.

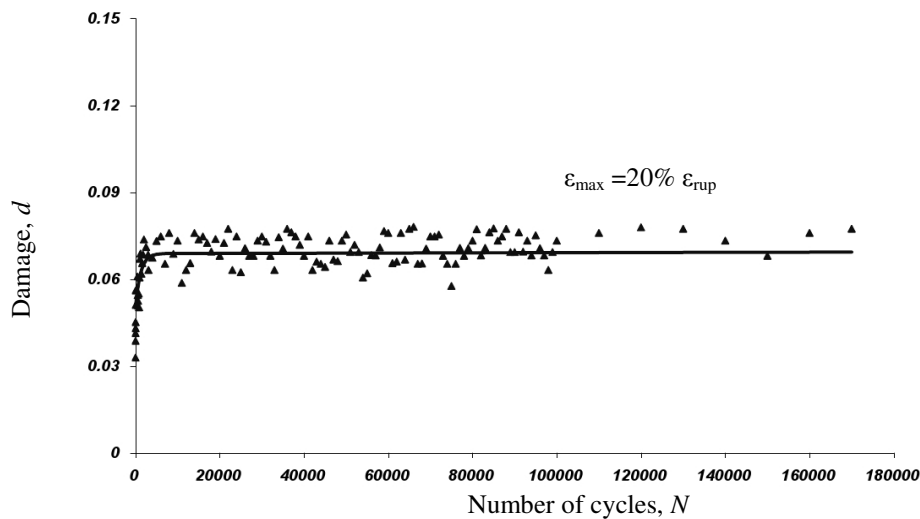


Figure 5: Comparison between experimental and calculated damage data for $\epsilon_{\max}=20\%$. This validate the identified set of parameters

4. CONCLUSION

In this work, a new cumulative fatigue damage model in short glass fibre reinforced thermoplastics is developed. These composites are characterised by an evolution of the damage in three stages. The new model is able to capture the first stage of damage specific to the composite materials with thermoplastic matrix.

This paper presents a first identification strategy: homogeneous tensile fatigue tests have been performed in the longitudinal direction. Three strain controlled fatigue tests at different levels have been carried-out to identify the four parameters governing the evolution of the longitudinal damage. The parameters are identified by using experimental data from 2 different tests at different strain level. The model, insensitive to the initial values, is validated on a third fatigue test

A second identification strategy is currently ongoing. It is based on the use of optical whole-field displacement/strain measurements by digital image correlation coupled to an inverse method from one single coupon. It is worth noting that the mechanical test must give rise to heterogeneous stress/strain fields. Indeed, in this case, the constitutive parameters are expected to be all involved in the response of the specimen.

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