MODELLING FATIGUE LIFE OF COMPOSITE LAMINATES WITH ANFIS

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ABSTRACT

Adaptive Neuro-Fuzzy Inference System (ANFIS) has been successfully used for the modelling of fatigue behaviour of a multidirectional composite laminate. The neuro-fuzzy model has been initially introduced in a previous work by the authors through the use of data base containing 257 valid fatigue data points. Coupons were cut at 0^o on-axis and 15^o, 30^o, 45^o, 60^o, 75^o, and 90^o off-axis directions from an E-glass/polyester multidirectional laminate with a stacking sequence of [0/(±45)2/0]T. The modelling accuracy of this novel computational technique, in this field, is very high. For all cases studied, it has been proved that a portion of around 50% of the available data is adequate for accurate modelling of the fatigue behaviour of the material under consideration. The new technique is a stochastic process which leads to the derivation of a multi-slope S-N curve based on the available experimental data without the need for any assumptions. Employment of this technique can lead to a substantial decrease of the experimental cost for the determination of reliable fatigue design allowables. New ANFIS models were developed for the purposes of this second approach to model the fatigue life of composite laminates. Two different material systems were used in the frames of this paper. The modelling accuracy is very high verifying the applicability of the proposed method as a reliable fatigue life modelling technique.

1. INTRODUCTION

During the last decade novel computational methods have been introduced in a number of areas of engineering science. After proving their abilities in pattern recognition, data clustering, signal processing and other scientific applications, artificial neural networks (ANN) and genetic algorithms (GA) where used in engineering. They were initially used as tools for optimizing the design methods. Novel computational methods like Artificial Neural Networks (ANN), Genetic Programming (GP) and Adaptive Neuro-Fuzzy Inference System (ANFIS) have been used to model/predict fatigue behaviour of different material systems. During those years, a limited number of articles have been published on the subject of modelling composite material fatigue using artificial intelligence methods, e.g., [1-9].

Lee and co-workers, [1], tried to model fatigue lives of a unidirectional and a $[(\pm45/0_2)_2]_S$ multidirectional composite laminate, under constant amplitude at different R-ratios and also under block loading. Their research findings proved that, at least for the case of constant amplitude loading, the ANN modelling ability is equivalent if not better than the modelling ability of other conventional modelling techniques. Two advanteges of ANN modelling were recognized; less calculation labour and effective modelling even for relatively limited data bases. Unfortunately, for block loading conditions the modelling ability of the evolved ANN was poorer. Later on, Al-Assaf and El Kadi, in a series of papers [2-4] achieved the modelling of fatigue life of unidirectional composite materials using different ANN paradigms. They also discussed the possibility of improving the modelling efficiency by using other types of ANN besides the classical feed-forward algorithm. Efficient modelling of ANN was also shown in another article by Vassilopoulos et al [5]. In that work it was pointed out that ANN could be used in order to reduce experimental effort and cost and determine fatigue design allowables easier, compared to other, conventional methods, e.g., linear regression analysis. ANN was also proved to be a useful tool for constructing constant life diagrams from a reduced data set as reported in [6-8]. Constant life diagrams are very important in design when variable amplitude loading patterns are taken into account. In a recent work, [9], genetic programming, a new method in this scientific field capable of evolving computer programs, was presented. It was described as an evolutionary method that can be successfully used for fatigue life prediction of composite materials. Genetic programming was finally qualified against three traditional methods for the fatigue life modelling of composite materials; linear regression analysis, Weibull statistics and wear-out model. Genetic programming is a stochastic tool which "follows" the trend of the available experimental data without the need to make any assumptions, e.g., about the type of the S-N curve or the statistical distribution of the data.

The present research focuses on the application of the Adaptive Neuro-Fuzzy Inference System (ANFIS) for the fatigue life modelling of composite laminates. The basic idea has been previously introduced by the authors in [10]. It has been shown that ANFIS method can be used to model the fatigue behaviour o a multidirectional composite laminate with a stacking sequence of $[0/(\pm45)_2/0]_T$. Comparison of the produced S-N curves with the curves produced by linear regression analysis or Weibull statistics proves the validity of the proposed methodology. Herein, the method is applied on two new material systems. The examined ata bases were retrieved from the literature and fatigue data was used for the establishment of the ANFIS models.

2. FATIGUE DATA

In addition to the data base that has been used in [10] for the introduction of the ANFIS modelling technique, two more data bases were retrieved from the literature, [11-12]. They refer to multidirectional GL/Polyester [11] and GL/Epoxy [12] laminates tested under several constant amplitude loading patterns. Coupons cut from a multidirectional Gl/Polyester laminate were tested under 12 R-ratios for a comprehensive representation of a constant life diagram in [11]. Reading counter-clockwise on the constant life diagram the following R-ratios can be identified: 0.9, 0.8, 0.7, 0.5, 0.1, -0.5, -1, -2, 10, 2, 1.43, and 1.1. Herein, for the comparison of the modelling techniques, experimental data collected under tensile (R=0.1) and under entirely compressive loading (R=10) were selected. In total, 150 valid constant amplitude fatigue data points were retrieved from the DOE/MSU database [11]. Another 147 valid fatigue data points were found in database Optidat [12] for the predetermined material tested at three different constant amplitude conditions; 47 coupons for tension-tension loading at R=0.1, 64 coupons for tension-compression loading at R=-1, and 36 coupons at compression-compression loading, R=10.

3. ANFIS MODELLING APPLICATION

3.1 Fuzzy logic methods

Fuzzy logic methods have been used to model various highly complex and nonlinear systems based on a set of sample data and fuzzy "if-then rules". A fuzzy inference system can model the qualitative aspects of human knowledge without employing any quantitative analyses. The following notation is common in fuzzy logic modelling and is adapted to serve the needs of the present study:

(a) *Linguistic variables:* Form the basic concept underneath fuzzy logic i.e. a variable whose values are words rather than numbers. The input linguistic variables specified in the previous paper [10] for the specific problem of fatigue life modelling are the following: orientation angle (θ), stress ratio (R), maximum cyclic stress ($\sigma_{\max}$) and cyclic stress amplitude (σ_{a}). The number of cycles to failure (N) was used as the only output variable. When examining the new data sets however, the orientation angle of all the coupons is the same, equal to 0^{o}. Also, since maximum cyclic stress, cyclic stress amplitude and stress ratio are dependent to each other it was decided to use only one stress parameter. Therefore the new ANFIS models are developed based on two input parameters, stress ratio and maximum cyclic stress.

(b) *Fuzzy sets:* In contrast to a classical set, a fuzzy set does not have a crisp boundary, i.e. the transition from the case "belong to a set" to the case "not belong to a set" is gradual. Normally this smooth transition is characterized by a *membership function* which gives flexibility to the fuzzy sets in commonly used modelling linguistic expressions. For the case studied herein, a linguistic expression could be: "maximum cyclic stress ($\sigma_{\max}$) is low" or "stress ratio (R)" is high etc.

(c) *Membership function (MF)*: It is the curve which defines the way each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The membership function type can be any appropriate parameterized membership function like triangle, Gaussian or bell-shaped.

(d) *Linguistic rules:* A set of linguistic "If-then" rules applied on the defined linguistic variables. A single fuzzy "If-then" rule assumes the form "If x is A then y is B", where A and B are linguistic values defined by fuzzy sets on the ranges X and Y, respectively. The if-part of the rule "x is A" is called the *antecedent* or *premise*, while the then-part of the rule "y is B" is called the *consequent* or *conclusion*. Fuzzy "If-then" rules with multiple antecedents like the following are often used.

Rule: If stress ratio is low and maximum stress is low, then specimen life is long.

The resulting output after the described fuzzy logic method has to be *defuzzified* or else converted to a crisp value by using any of the available defuzzification methods, like the centre of gravity method etc. The membership functions used to represent linguistic variables may have important effect on modelling performance as the type of the MF being used determines when a given rule is to be put in effect or not (in fuzzy logic "the rule is fired"). Three types of membership functions, namely the triangular type, the Gaussian type and the bell-shaped type have been used in this study to examine the influence of each one of them on the produced data

3.2 Adaptive Neuro-Fuzzy Inference System

Although the fuzzy inference system has a structured knowledge representation in the form of fuzzy "if-then" rules, it lacks the adaptability to deal with a changing external environment. Therefore neural network learning concepts have been incorporated in fuzzy inference systems, resulting in adaptive neuro-fuzzy modelling. The adaptive inference system is a network which consists of a number of interconnected nodes. Each node is characterised by a node function with fixed or adjustable parameters. The network is "learning" the behaviour of the available data during the training phase by adjusting the parameters of the node functions to fit that data. The basic learning algorithm, the back propagation, aims to minimize a set measure or a defined error, usually the sum of squared differences between the desired and the actual model outputs. The fuzzy modelling was first explored by Takagi and Sugeno [13]. The ANFIS architecture that was used in the previous study was based on the first order Takagi-Sugeno model. It was assumed that the number of cycles to failure (N) under fatigue loading was a function of the orientation angle (θ) of the fibres, the stress ratio (R), the maximum stress ($\sigma_{\max}$) and the stress amplitude (σ_{a}). Thus (θ, R, $\sigma_{\max}$, σ_{a}) were the input parameters, while the number of cycles which corresponds to each combination of the four input parameters was the unique output of the ANFIS model. In this model, the i$^{\text{th}}$ rule for the prediction of fatigue life could be expressed as follows;

Rule I :

$$\textit{If } \theta \textit{ is } A_j,\ R \textit{ is } B_k,\ \sigma_{\max} \textit{ is } C_l,\ \sigma_a \textit{ is } D_m,\ \textit{then } f_i = n_i\theta + o_iR + p_i\sigma_{\max} + q_i\sigma_a + r_i$$

Where: $j = 1,...,N_1,\ k = 1,...,N_2,\ l = 1,...,N_3,\ m = 1,...,N_4,\ \textit{and } i = 1,...,N_1N_2N_3N_4$

$A, B, C, \textit{and } D$ are the fuzzy sets defined for $\theta, R, \sigma_{\max}, \textit{and } \sigma_{\mathrm{a}}$ respectively. $N_1, N_2, N_3, \textit{and } N_4$ indicate the number of membership functions defined by the indicated fuzzy input variables, f is a linear consequent function defined in terms of the input variables and $n, o, p, q, \textit{and}, r$ are the consequent parameters of the Takagi-Sugeno fuzzy model [13]. In that model, nodes of the same layer had similar functions, as described below. The output of the i$^{\text{th}}$ node in layer l is denoted as $O_{1,i}$. The ANFIS models that were employed for this work are based on the same idea, although being simpler since they use only two input parameters. A schematic representation is presented in Fig. 1.

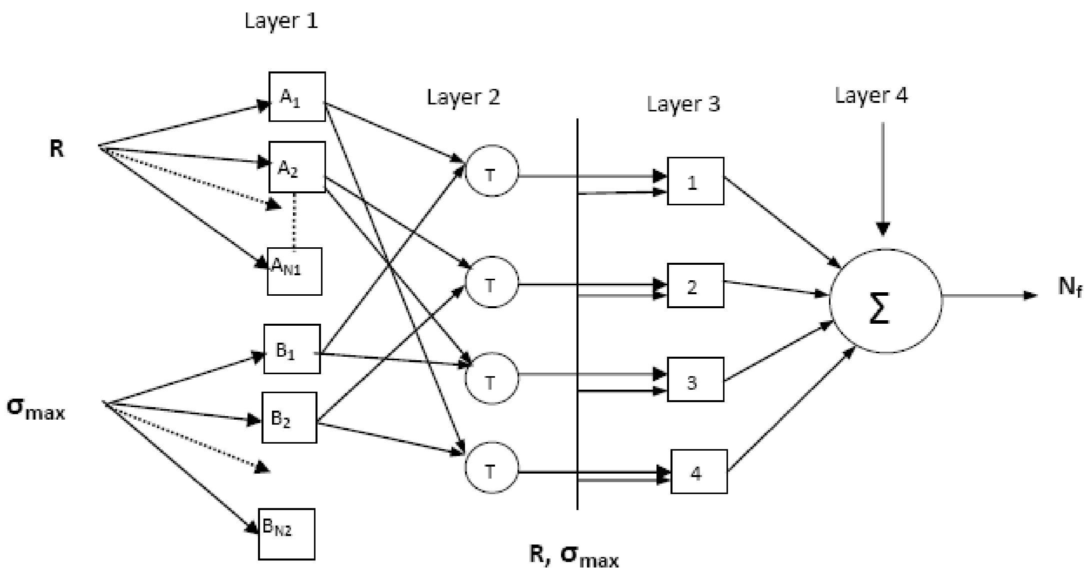

Figure 1. ANFIS architecture based on the Takagi-Sugeno model.

The fuzzy inference system shown in Fig. 1 is composed of four layers. Each layer involves several nodes. The output signals from the nodes of the previous layer will be accepted as the input signals in the current layer. After manipulation by the node function in the current layer, the output will be served as input signal for the subsequent layer.

Layer 1: The first layer of this architecture is the fuzzy layer. Each node of this layer makes the membership grade of a fuzzy set.

Layer 2: Every node in layer 2 is a fixed node, marked by a circle, whose output is the product of all the incoming signals i.e. T-norm operation.

The output signal denotes the firing strength of the associated rule. The firing strength is also called "degree of fulfilment" of the fuzzy rule, and represents the degree to which the antecedent part of the rule is satisfied.

Layer 3: Every node in layer 3 is an adaptive node marked by a square node with a node function.

The consequent parameters in this layer are to be adapted in order to minimize the error between the ANFIS outputs and their experimental results.

Layer 4: Every node in layer 4 is a fixed node marked by a circle node. The node function computes the overall output by summing all the incoming signals.

This ANFIS structure represents a four dimensional space partitioned into $N_1 \times N_2 \times N_3 \times N_4$ regions, each one governed by a fuzzy "If-then" rule. In other words, the premise part of a rule defines the fuzzy region, while the consequent part specifies the output within the region.

A hybrid learning algorithm is used to adapt the parameters of the 1st layer, called premise or antecedent parameters, and the parameters of the 3rd layer, referred to as consequent parameters, in order to optimize the network. The network uses a combination of back-propagation and the least squares method to estimate membership function parameters. More specifically, in the forward pass of the hybrid learning algorithm, node outputs go forward till layer 3 and the consequent parameters are identified by the least squares method. In the backward pass, error signals propagate backwards and the premise parameters are updated by a gradient descent method.

4. MODELLING AND DISCUSSION

The pre-processing of experimental data was necessary in order to achieve adequate modelling performance. The values of $R \text{ and } \sigma_{\max}$ were normalised with respect to the corresponding maximum values. All stress ratios were divided by 10, while maximum cyclic stress values were divided by the corresponding maximum value for each one of the data sets. The number of cycles to failure (N), which constitutes the system output, was converted to its logarithmic value. This operation helped to avoid poor modelling performance of ANFIS due to extreme differences in the values of the recorded cycles to failure.

From the resulting data set, after the pre-processing procedure, a training set was constructed by selecting a portion of the data in a random way. The remaining portion was used for the construction of the test set. As one of the objectives of this work was to investigate the effect of the size of the training set to the modelling ability of the generated ANFIS model, the portion of data used for training and testing was varying. It was decided to start with allocating a portion of 90% of the data for training and the remaining 10% for testing. This case is identified in the sequel as 90-10. During the analysis stages that followed, the portion of data used as a training set was gradually decreased while that used as a testing set was increased, reaching the extreme case of having 10% of the data used for training and 90% of them used for testing (case 10-90). The performance outcome of ANFIS in all these cases was evaluated. The scatter of the input values is critical for the quality of the evolved model. Less scatter leads to higher modelling efficiency. However, as it was also reported in [10], for composite material's fatigue data the modelling accuracy is not considerably affected by the usual scatter that is presented. On the other hand, the stochastic nature of the method characterizes the input-output process, since different in general output data should be expected, even for identical input values processed by the same ANFIS model.

For all the types of data sets described above, ANFIS was constructed using the three types of MF, i.e. the triangular type, the Gaussian type and the Bell type MF. Different ANFIS structures were generated for each case, from 90-10 to 10-90 using all the different architectures. In order to improve the generated models' performance, the available experimental data were clustered by the subtractive clustering algorithm.

A clustering technique can be used to generate a Takagi-Sugeno type fuzzy inference system which best models data behaviour using a minimum number of fuzzy rules, thus preventing the explosion of rules. The rules themselves can be partitioned according to the fuzzy qualities associated with each one of the data clusters. The subtractive clustering method [14] which is an extension of mountain clustering method has been used in this paper to estimate the number of clusters and cluster centres in the fatigue life data set. This method first assumes that each data point can be a potential cluster centre. It then calculates a measure of the likelihood that each selected point could really define the cluster centre, based on the density of surrounding data points. The steps of the subtractive clustering algorithm can be summarized as follows:

- select the data point with the highest potential to be the first cluster centre,
- remove all data points in the vicinity of the first cluster centre as determined by the range of influence (radius), and finally
- iterate on this process until all the data are within the radii of a cluster centre.

Data clustering was performed herein in order to improve ANFIS modelling performance. The advantage of using data clustering in the proposed solution is the improvement of modelling accuracy by the development of a simpler neuro-fuzzy model.

The statistics of ANFIS application after data clustering on all the examined cases, from 10-90 to 90-10 are presented in Table 1, for both material systems. As shown in this table, a small data set (e.g., 10% in case 10-90) can easily be fitted during the training process, but then the trained model cannot be used to produce reliable results. An optimum selection of the training set for reliable modelling and elimination of the experimental cost was found to be between 60%-70% of the available experimental data.

Although several problems were met during the calibration of the ANFIS parameters, the final results were very promising. As depicted in Fig. 2, even for the case where only small portions, e.g., 40%, of the available experimental data taken from [11] were used for training the model, the predicting ability of ANFIS was excellent. However, when the training set was less than 50% of the available experimental data, ANFIS behaviour was not consistently good. Nevertheless, since a randomly selected 60% or more of the available experimental data were used for training, all ANFIS predictions were corroborated very well by the experimental data, as it is schematically shown in Figs. 3-4 for typical cases of the different material systems tested under different loading conditions.

Table 1. Modelling accuracy of the ANFIS models as a function of the data used for training.

	DOE/MSU [11]		Optidat [12]	
Training set-test set (% of data points)	Training R^2	Testing R^2	Training R^2	Testing R^2
10-90	0.987	0.720	0.998	0.370
20-80	0.973	0.700	0.997	0.476
30-70	0.980	0.923	0.974	0.586
40-60	0.956	0.941	0.985	0.443
50-50	0.950	0.949	0.971	0.546
60-40	0.952	0.903	0.953	0.637
70-30	0.962	0.929	0.930	0.837
80-20	0.960	0.913	0.929	0.845
90-10	0.954	0.949	0.921	0.834

Compared to traditional methods for interpretation of composite materials' fatigue data, i.e., linear regression and Weibull statistics among others, ANFIS is equivalent if not superior since it can derive S-N curves that are not based on any assumptions and simply "follow" the trend of the given experimental data.

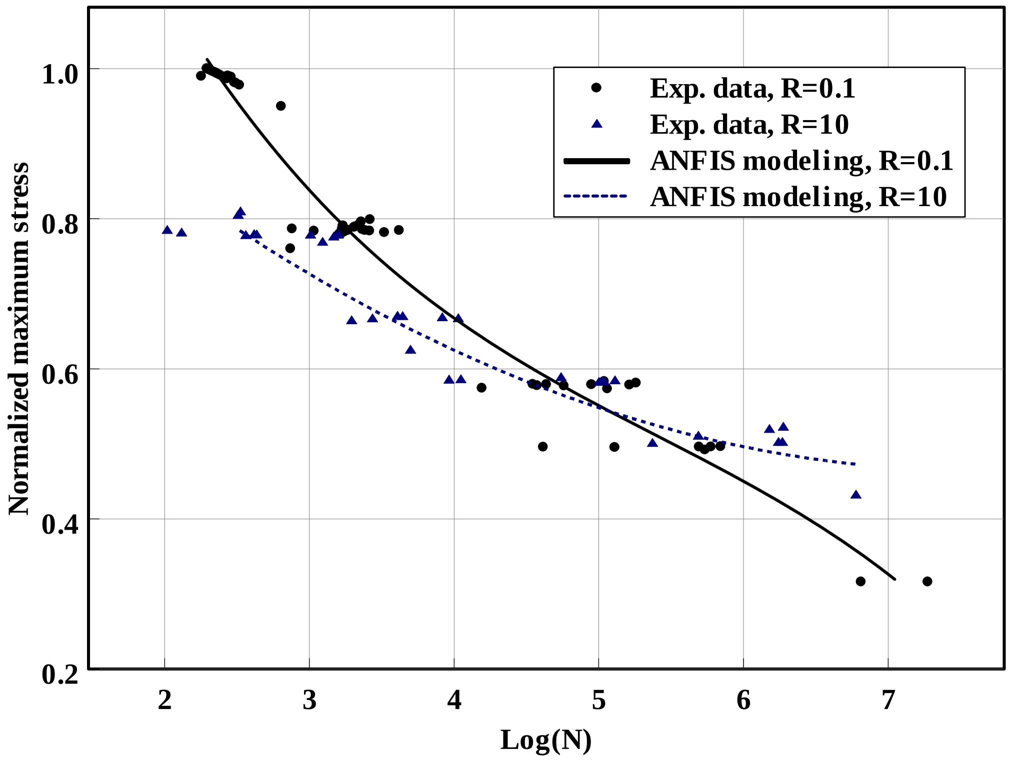

Figure 2. Modeling efficiency of ANFIS. Data from [11], 40% used for the training.

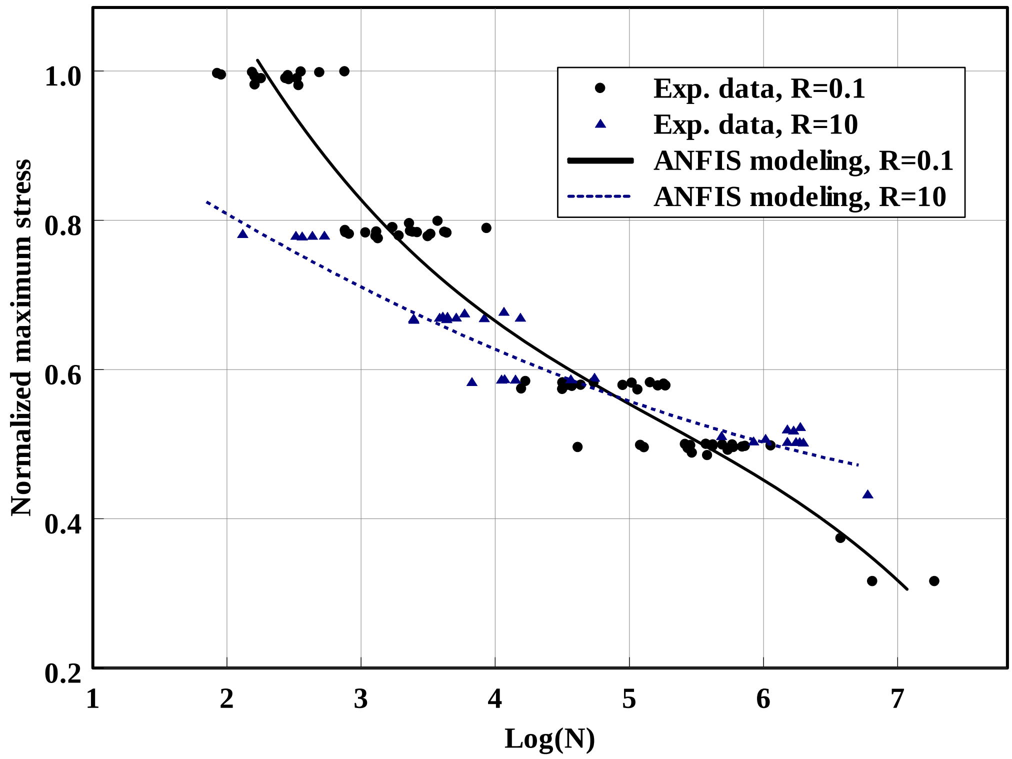

Figure 3. Modelling efficiency of ANFIS. Data from [11], 70% used for the training.

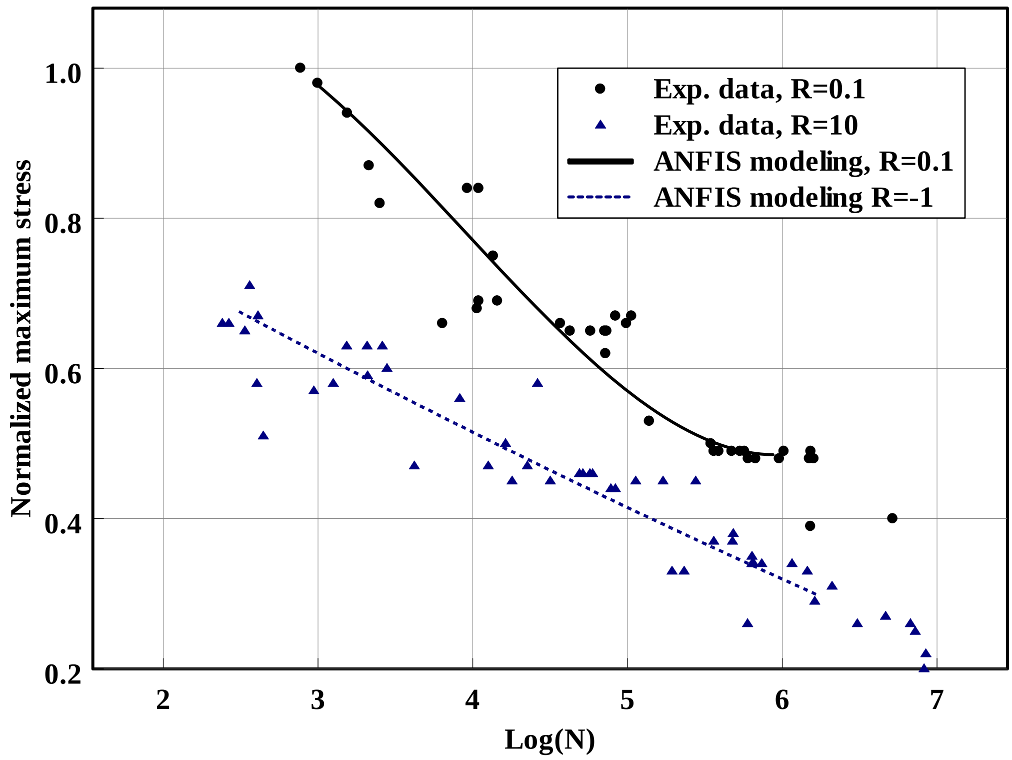

Figure 4. Modelling efficiency of ANFIS. Data from [12], 80% used for the training.

5. CONCLUSIONS

Adaptive Neuro-Fuzzy Inference System (ANFIS) was used in this work for modelling fatigue behaviour of two different multidirectional composite materials. Modelling efficiency of this novel computational tool was evaluated. The R-ratio, R and the maximum cyclic stress, σ_{max} were used as input parameters. The corresponding number of cycles to failure was considered as the only output. ANFIS can be used to model the fatigue behaviour of the examined composite materials and compares favourably with other modelling techniques as also pointed out in another work by the authors, [10].

Three different ANFIS model structures were developed, each one of them based on a different type of the employed membership function: a Gaussian, a bell-shaped or a triangle membership function has been used for each one of the three different ANFIS models. Additionally, subtractive clustering technique was applied on the available experimental data. The investigation revealed that ANFIS modelling accuracy is not considerably affected by the type of membership function employed. However, it becomes more effective when the available experimental data are clustered prior to training. The ANFIS model with Gaussian membership function and data clustering was qualified as the most accurate among the examined cases.

The proposed method for the interpretation of fatigue life data based on ANFIS is a material independent method. ANFIS modelling is not based on any assumptions, for example that the data follow a specific statistical distribution, or that the S-N curve is a power curve equation. Moreover, the process does not take the mechanics of each material system into account. It is a material-independent data-driven method that correlates input with output values in order to establish a model describing the relationship between them. In that context the proposed method can be easily applied on any material, provided that an adequate amount of data exists.

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