A Model for Predicting Multiscale Crack Growth Due to Impact in Heterogeneous Viscoelastic Solids

Flavio V. Souza[1]; Victor F. Teixeira[1]; David H. Allen[1]

[1] *College of Engineering, University of Nebraska-Lincoln, Lincoln, Nebraska*

ABSTRACT

This paper presents a multi-scale model for predicting multiple crack growth in heterogeneous viscoelastic solids due to impact. In order to simplify the problem, cracks are considered only at the local scale and a micromechanical viscoelastic cohesive zone model is used to model the fracture process. The multiscale formulation has been implemented in a parallel finite element computer code so that complex problems can be solve numerically. A plate impact problem is considered herein as an example problem and the numerical results are compared to airgun shooting experimental results.

Keywords: Multiscale model, heterogeneous viscoelastic media, impact loading, fracture.

1. INTRODUCTION

It has been observed that many composite materials exhibit inelastic behavior when subjected to impact loading conditions. This inelastic behavior is induced by energy dissipation mostly due to the development of multiple microcracks and the viscoelastic behavior of the matrix. Because the length scales of the structural part and the microstructure of the composite are very far apart, it is not practical to consider the material heterogeneity in a single scale model. On the other hand, multiscale models are able to efficiently solve this kind of problem in an approximate fashion. The main approximations introduced in multiscale models are the assumption of the existence of a Representative Volume Element (RVE) and the use of averaging theorems. However, if one can determine the RVE for a particular composite material and the length scales of the problem are far apart, the errors introduced by the assumptions become negligible.

Even though multiscale models are more computationally efficient than single scale models, they can still be very costly depending mostly on the size and geometry of the RVE. However, due to the development of high speed computers and the advent of parallel programming, it is now possible to solve two- or three-dimensional problems on two, three, and even four length scales. These solutions are obtained by using standard time marching multi-scale algorithms, linked by appropriate homogenization theorems [1,2,3,4,5,6], which are able to predict the evolution of hundreds, even thousands of cracks simultaneously. In this paper, an example solution for a viscoelastic medium with local microcracks is given in order to demonstrate the capabilities of the model.

2. MULTISCALE MODELING

The main hypothesis of multiscale models is that the global constitutive behavior of heterogeneous materials can be determined simultaneously throughout the analysis based on the behavior of the individual constituents and their interactions at the local scale. Multiscale models become very attractive for problems with evolving microstructure due to formation and growth of microcracks, since the evolution of the microstructure is necessarily both spatially and time dependent, otherwise the classical homogenization technique, where average properties of the heterogeneous material are determined *a priori*, may be sufficient and less time consuming. A more detailed description of multiscale models can be found in [7,8,9,10,11,12].

Fig. 1 schematically presents a structural part which is statistically homogeneous at the global scale (superscript 0) but heterogeneous at the local scale (superscript 1), where the microstructure may contain inclusions as well as growing microcracks. In Fig. 1, V^{μ}, ∂V_E^{μ} and ∂V_I^{μ} are the volume, external and internal boundary surfaces of scale μ, respectively, ℓ^{μ} is the length scale associated with scale μ and ℓ_c^{μ} is the length scale associated with the cracks at scale μ.

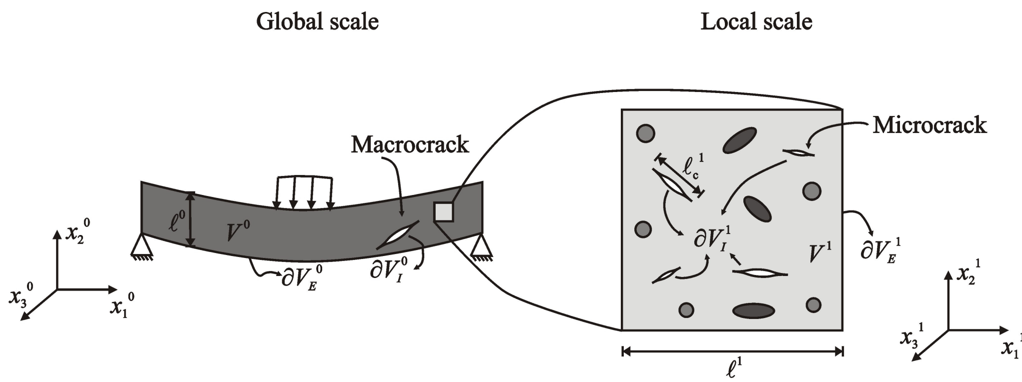

Fig. 1 Schematic description of a two scale problem with cracks

The global scale Initial Boundary Value Problem (IBVP) can be posed as follows:

i) Conservation of linear momentum

$$\sigma^0_{ji,j} + \rho^0 b^0_i = \rho^0 \frac{d^2 u^0_i}{dt^2} \qquad in\ V^0 \tag{1}$$

where σ^0_{ij} is the Cauchy stress tensor, ρ^0 is the mass density of the statistically homogeneous object, b^0_i is the body force vector per unit mass, u^0_i is the displacement vector and V^0 is the volume of the object at the global length scale.

ii) Conservation of angular momentum

$$\sigma^0_{ij} = \sigma^0_{ji} \qquad in\ V^0 \tag{2}$$

iii) Small strain-displacement relation

$$\varepsilon^0_{ij} = \frac{1}{2}\left(u^0_{i,j} + u^0_{j,i}\right) \qquad in\ V^0 \tag{3}$$

where ε^0_{ij} is the infinitesimal strain tensor defined on the global length scale.

iv) Constitutive equations

$$\sigma^0_{ij}(t) = \Omega^{\tau=t}_{\tau=-\infty}\left\{\varepsilon^0_{ij}(\tau)\right\} \qquad in\ V^0 \tag{4}$$

where $\Omega^{\tau=t}_{\tau=-\infty}$ is a functional mapping that accounts for history dependent effects, such as viscoelasticity, and is determined by locally averaging the response at the local scale.

If the problem at hand allows one to assume that: *i*) the global length scale is much larger than the local length scale, $\ell^0 >> \ell^1$, *ii*) the length scale associated with cracks at the local scale, ℓ^1_c, is much smaller than the local length scale, $\ell^1 >> \ell^1_c$, *iii*) these cracks are homogeneously distributed and oriented at the local scale, and *iv*) the length of the wave propagating on the global scale, ℓ^0_ω is much larger than the local scale length, $\ell^0_\omega >> \ell^1$, then the local IBVP can be approximated as a quasi-static problem:

i) Conservation of linear momentum

$$\sigma^1_{ji,j} + \rho^1 b^1_i = 0 \qquad in\ V^1 \tag{5}$$

ii) Conservation of angular momentum

$$\sigma^1_{ij} = \sigma^1_{ji} \qquad in\ V^1 \tag{6}$$

iii) Infinitesimal strain-displacement relation

$$\varepsilon^1_{ij} = \frac{1}{2}\left(u^1_{i,j} + u^1_{j,i}\right) \qquad in\ V^1 \tag{7}$$

iv) Constitutive equations

$$\sigma^1_{ij}(t) = \Omega^{\tau=t}_{\tau=-\infty}\left\{\varepsilon^1_{ij}(\tau)\right\} \qquad in\ V^1 \tag{8}$$

where in the case of the local scale, it is assumed that $\Omega^{\tau=t}_{\tau=-\infty}$ is known a priori, for all constituents.

v) Fracture criterion

$$G^1_i = G^1_{c\,i} \qquad in\ V^1 \tag{9}$$

where G^1_i is the fracture energy release rate at the local scale, $G^1_{c\,i}$ is the critical energy release rate of the material and the index *i* refers to the mode of fracture.

2.1. Homogenization of local fields

Homogenization principles can now be used to establish the relationships connecting both length scales. If all the assumptions made herein hold, it can be shown that,

$$\varepsilon_{ij}^{0} = \frac{1}{V^{1}} \int_{\partial V_{E}^{1}} \frac{1}{2}\left(u_{i}^{1} n_{j}^{1} + u_{j}^{1} n_{i}^{1}\right) dS \qquad (10)$$

$$\sigma_{ij}^{0} = \bar{\sigma}_{ij}^{1} = \frac{1}{V^{1}} \int_{V^{1}} \sigma_{ij}^{1} dV = \frac{1}{V^{1}} \int_{\partial V_{E}^{1}} \sigma_{ki}^{1} n_{k}^{1} x_{j}^{1} dS \qquad (11)$$

$$\rho^{0} = \lim_{\ell^{1}/\ell^{0} \to 0} (\rho^{0}) = \bar{\rho}^{1} = \frac{1}{V^{1}} \int_{V^{1}} \rho^{1} dV \qquad (12)$$

where ∂V_{E}^{1} is the external boundary of the local length scale and n_{i}^{1} is the outward unit normal vector to this external boundary surface.

Finally the constitutive relationship at the global length scale is determined concurrently as the multiscale analysis is performed.

2.2. Crack Modeling at the Local Scale

In order to model crack propagation in viscoelastic media, the micromechanical viscoelastic cohesive zone model developed by Allen and Searcy [13] is used herein. Briefly, the cohesive zone is postulated to be represented by a fibrillated zone that is small compared to the total cohesive zone area. The length scale of this IBVP is one length scale below that of the smallest local scale required in the multiscale problem and its solution has been obtained leading to the following traction-displacement relation in the cohesive zone:

$$T_{i}^{1}(t) = [1-\alpha(t)] \frac{1}{\lambda^{1}} \frac{\delta_{i}^{1}}{\delta_{i}^{*1}} \left[T_{i}^{f} + \int_{0}^{t} E_{c}^{1}(t-\tau) \frac{\partial \lambda^{1}}{\partial \tau} d\tau \right] \quad on \ \ \partial V_{Ic}^{1} \qquad (13)$$

where $E_{c}^{1}(t)$ is the uniaxial viscoelastic relaxation modulus of the undamaged cohesive zone material, ∂V_{Ic}^{1} is the part of the internal boundary on which cohesive zones are active, δ_{i}^{1} is the crack opening displacement vector in the coordinate system aligned with the crack faces, δ_{i}^{*1} is a material length parameter, λ^{1} is the Euclidean norm of the crack opening displacement vector, T_{i}^{f} is the value of traction at which the cohesive zone initiates, and $\alpha(t)$ is the damage parameter, herein assumed to evolve according to the following phenomenological law [13,15]:

$$\dot{\alpha} = A[\lambda(t)]^{m}, \qquad \text{when } \dot{\lambda} > 0 \text{ and } \alpha < 1 \qquad (14)$$

$$\dot{\alpha} = 0, \qquad \text{when } \dot{\lambda} \le 0 \text{ or } \alpha = 1 \qquad (15)$$

where A and m are microscale phenomenological material constants.

Note that equation (13) actually represents the outcome of a homogenization theorem applied to the analytic solution for a cohesive zone, thus creating a third length scale in the current model.

It is important to note that the computational algorithm used herein automatically inserts cohesive zone elements into the finite element mesh at the moment in time at which the criterion for cohesive zone initiation is satisfied, in this case, $T_i^1(t) \geq T_i^f$, thus minimizing the maximum bandwidth of the stiffness matrix as opposed to meshes with cohesive zone elements embedded a priori. Moreover, contact conditions at the cohesive zones are enforced by using the Lagrange multiplier technique in order to avoid cohesive zone interpenetration. Further details of this approach may be found in [16].

3. MULTISCALE ALGORITHM

The multiscale algorithm has been implemented into a finite element code in which the global and local length scales are solved by means of explicit and implicit quasi-static FEM solution schemes, respectively. Since each local scale analysis can be solved independently and simultaneously by different processors, the code has been parallelized using the Open MPI libraries [17], so that the computational time necessary for the solution of multiscale problems can be significantly reduced.

4. EXAMPLE PROBLEM

The example problem herein considered consists of an impact of an elastic cylinder into a heterogeneous viscoelastic plate. The assumed geometry of the local scale used for this example problem is shown in Fig. 2, where units are given in μm. The fiber is assumed elastic while the matrix is assumed linear viscoelastic. The arbitrarily chosen material properties used herein are given in

Table 1. It is herein assumed that the viscoelastic relaxation moduli can be expressed in the form of a Prony series.

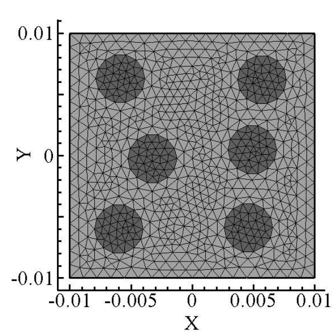

Fig. 2 Geometry and finite element mesh of local scale unit cell

Table 1 Material properties

Viscoelastic materials					
Global viscoelastic			RVE Matrix		
i	E_i (GPa)	ρ_i	i	Ei (GPa)	ρ_i
1	0.6093	2.00E+02	1	0.597577	1.20E+02
2	1.0604	2.00E+03	2	0.573028	1.20E+03
3	1.3809	2.00E+04	3	0.942435	1.20E+04
4	1.8702	2.00E+05	4	0.447273	1.20E+05
5	0.3175	2.00E+06	5	1.495464	1.20E+06
6	1.027	2.00E+07	6	2.090955	1.20E+07
7	2.5923	2.00E+08	7	3.800564	1.20E+08
8	5.7631	2.00E+09	8	2.672005	1.20E+09
9	7.3431	2.00E+10	9	1.948782	1.20E+10
10	6.4029	2.00E+11	∞	0.5	-
11	3.1855	2.00E+12	ν	0.3	
12	0.047	2.00E+13	ρ(kg/m^3)	1170	
∞	1.2	-			
ν	0.3				
ρ(kg/m^3)	1495				

Cohesive zone

	Matrix-matrix interface	Fiber-matrix interface
δ_n(μm)	0.001	0.001
δ_t(μm)	0.001	0.001
T_n^f(MPa)	1000	800
T_t^f(MPa)	1500	1000
A	2.5	5
M	1.5	1.5

Elastic Materials

	Fiber	Cylinder
E(GPa)	230	250
ν	0.3	0.3
ρ(kg/m^3)	1760	7860

4.1. Cylinder/plate impact

This problem consists of the impact of a homogeneous elastic cylinder (12 cm diameter, Young's modulus of 250 GPa, Poisson's ratio of 0.30, and mass density of 7860 kg/m^3) into a rectangular (100 mm long × 6 mm width) heterogeneous plate. Plane stress conditions are herein assumed and symmetry of the problem has been used in order to minimize computational effort. The initial velocity of the elastic ball is 100 m/s. A local mesh (Fig. 2) has been attached to the integration points of the elements with more damage (Fig. 4), totaling 96 local meshes. Lagrange multipliers are also used to enforce the non-interpenetrating frictionless contact conditions between the plate and the cylinder.

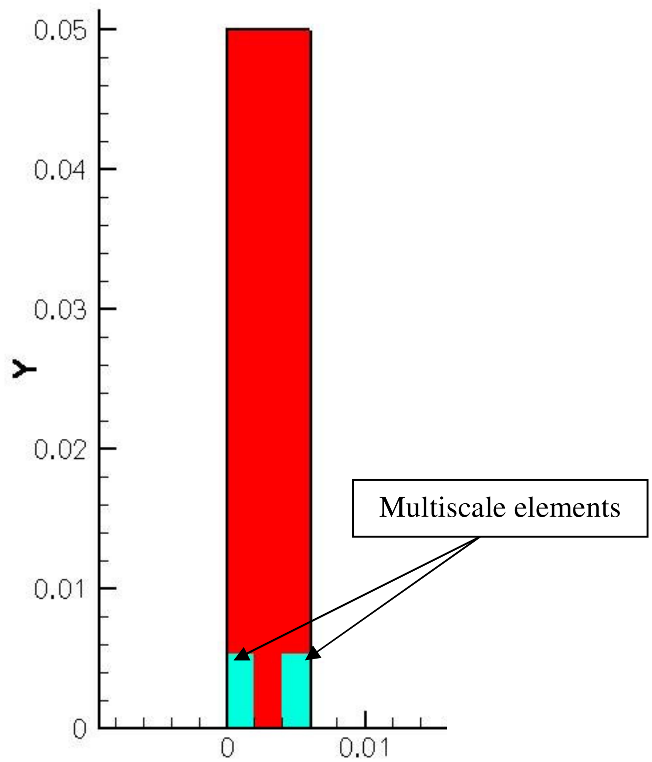

Fig. 4 Multiscale elements

Fig. 5 presents snapshots of the global and selected local meshes at different times. It can be observed that the multiscale model herein presented is capable of predicting spatially dependent (with respect to global position) damage evolution at the local scale. As a consequence, (spatially dependent) damage induced anisotropy has been produced at the local scale due to the orientation of the cracks.

It is important to notice that the bulk viscoelastic behavior of the matrix, as well as the viscoelastic cohesive zones, dissipate a considerable amount of energy so that the regions of the plate that experience late tensile waves experience the formation of few internal boundaries, as can be observed in Fig. 5. This is a desirable feature of computational models intended for the design of viscoelastic impact devices.

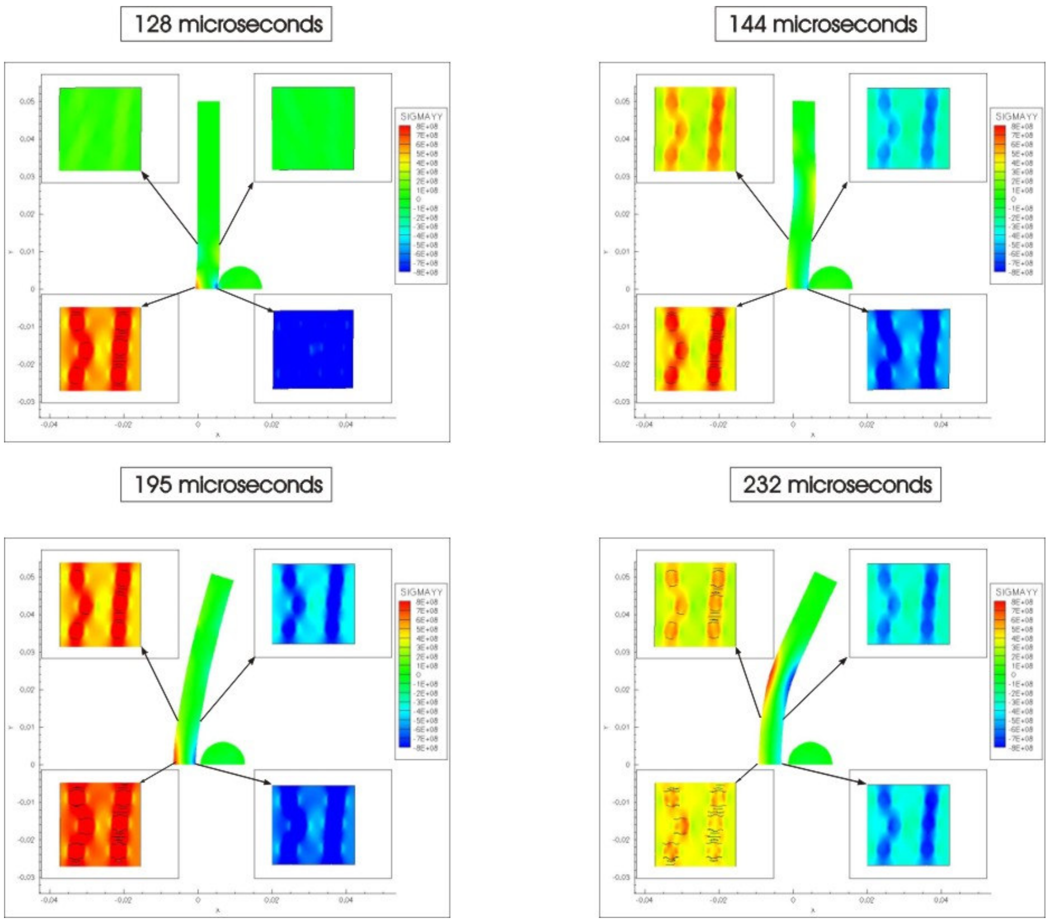

Fig. 5 Snapshots of global and selected local meshes for the viscoelastic impact problem with cracks at local scale

It is also important to note that some of the internal boundaries shown in Fig. 5 are not cracks, but formed cohesive zones which will become cracks when the damage parameter $\alpha(t)$ reaches unity.

5. CONCLUSIONS

This paper has presented a multiscale model for predicting the performance of heterogeneous viscoelastic materials under impact loading. Automatically inserted cohesive zone elements are used in order to model the evolution of microcracks in the material. Material viscoelasticity is another source of energy dissipation accounted for in model. A multiscale finite element code has been developed in order to allow modeling of complex problems that have no available analytic solution.

Some of the capabilities of the model have been demonstrated by an example problem consisting of a plate/cylinder impact, for which multiple energy dissipation mechanisms have been considered, i.e., bulk and cohesive zone viscoelastic dissipation as well as fracture energy. Besides the fact that multiscale models can account for multiple sources of energy dissipation on the local scale, it has other important features in the design of composite materials, such as the fact that material characterization is needed only at the local scale and the fact that important design variables such as constituent volume fractions and particle orientation are incorporated into the model.

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