A PLY SCALE NON LOCAL APPROACH TO DESCRIBE THE FAILURE OF LAMINATED STRUCTURE

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ABSTRACT

To describe the failure of structures which present stress gradients, a non local approach is proposed [9]. This model is based on the definition of a Fracture Characteristic Volume (*FCV*) corresponding to a cylinder, of ply thickness and of section of a few square millimetres, on which average stresses are calculated. The choice of this FCV instead of the more classical characteristic area (the ply thickness multiplied by the characteristic length) is consistent with the fracture zone observed. This volume depends on the meso-structures of the ply, that it is balanced or unbalanced woven plies with various sizes of carbon or glass yarns. It is defined at the ply scale in order to be able to study any lay up and the section is identified by two tests: a homogeneous tension test and a test on a sample which present a stress gradient (a [0°] plate with a hole, for example).

This non local approach combining with a damage model [8] which describes the non linear behaviour of laminated composites was implemented into the Abaqus Software. This behaviour model is based on a diffuse damage kinematics, which describe the apparition of micro-cracks running parallel to fibres in the warp and weft direction and on a kinematic hardening model to account for the inelastic strains in the shear direction. Comparison between the results of simulations and the structural tests show the efficiency of this approach in case of static loading conditions.

1. INTRODUCTION

Fracture in composite materials involves several mechanisms operating on various scales [1]. Models based on continuum damage mechanics account for the gradual diffuse damage induced by small cracks running parallel to the fibre direction [2], [3]. Even if the size of these cracks corresponds to the thickness of the ply, they do not generally lead to the rupture of the laminate. On the other hand, rupture of a ply in the fibre direction has generally catastrophic effects for the laminate and consequently for the structure. In a large class of laminates and structures, the first rupture of the ply in the fibre direction leads to the rupture of the structure in the case of static loading conditions.

In the case of laminates with brittle behaviour (quasi-isotropic laminates for example), methods based on classical fibre rupture criteria work quite efficiently if the stresses in the structure are quasi-homogeneous. In the case of high stress gradients (as in the case of plate with a hole[4]), these local criteria are no longer suitable and non local criteria, such as point-wise and average stress criteria [5] are more appropriate.

Following the work of Isupov and Mikhailov [6], we studied non local criteria based on a characteristic length to predict the failure of quasi-isotropic plates with open elliptical holes [7]. These criteria consisting for example on considering a critical average stress over the length, give suitable results. Nevertheless, these conventional criteria are difficult to use for complex structures (such as multi-perforated plate) and any laminates. Moreover, in the case of the laminates we studied, the fracture zone look as a volume (thickness of the ply multiplied by a characteristic surface) and not as a surface (thickness multiplied by the characteristic length). We proposed in [8] a non local approach based on mean quantities over a ply scale characteristic volume we called the *Fracture Characteristic Volume* (FCV), corresponding to a cylinder of ply thickness and a few square millimetres section. This area, which is taken to be circular, depends on the meso-structures of the ply and could be determined by performing two tests on structures showing different stress distributions. The FCV was integrated into the ABAQUS Software and numerical results were compared to experimental data for woven ply laminated structures [9].

This paper presents this new non local approach through two cases of application: the first is a simplified approach which can apply to the laminates presenting an elastic linear behaviour, the second, is a more complete approach associating a model of behaviour with the non local approach in order to predict the failure of any laminated structure. Comparisons between experimental data and simulated data performed on plates with holes, notches and saw cuts show the efficiency of this approach, even for structures with very high stress gradients. Experimental strain field and mesh influence are analysed.

2. NON LOCAL APPROACH

In the case of a composite structure with geometric irregularities, high strain gradients appear near the singularities. Tests on woven ply CFRP laminates were performed and strain maps were measured using the Digital Image Correlation method [10]. Figure 1 shows the longitudinal strain map for a [0°] laminated plate (45 mm x 200 mm of length) with a saw cut (1 mm x 3 mm of length) on both side of the specimen. The level of strain is very high at the crack tip, comparing to that obtained with a homogenous test (around 1.5%) with no visible crack in the fibre direction. This observation, which has to be confirmed by other experiments, indicates that the fibre rupture strain level is much higher in presence of strain gradient and justifies the use of a non local approach. A local criterion would underestimate the failure of the structure.

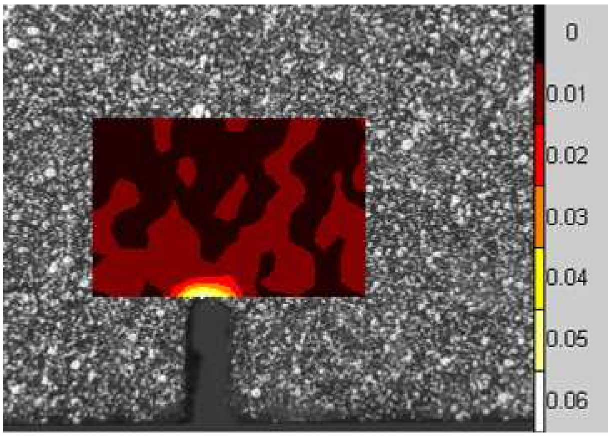

Fig. 1: Longitudinal strain map before failure for the saw cut plate and [0°] laminate.

2.1. Fracture Characteristic Volume (FCV)

In order to study any lay up, it was proposed to use a non local approach based on a ply scale characteristic volume: $V=hS$. One of the dimensions of this Fracture Characteristic Volume *(FCV)* corresponds to the thickness h of the ply and the area S in the plane depends on the meso-structure of the ply. In the case of studied woven plies, we adopted a circular area. A circular area is characterised by the diameter but does not need the definition of directions (contrary to a square for example). Then, mean quantities (stresses, strains, damage …) can be calculated over this volume in order to define fracture criteria.

In function of the material, failure criteria have to be chosen and diameter of the circle which constitute the FCV have to be identified. The identification of this diameter required two tests on structures showing different stress distributions, a homogeneous tension test and a plate with a circular hole for example. The process of identification also required structure computations (plate with a hole) and the choice of a laminate ([0°] laminates for example). In the case of balanced woven plies, the diameter of the circular area was found to be close to the size of the yarn (mm order). This simple approach can be easily applied everywhere in the structure, with any laminate.

2.2. Implementation into the Abaqus Software

The non local approach based on the definition of a Fracture Characteristic Volume *(FCV)* was integrated into the Abaqus Software by using an URDFIL routine. The failure criterion (such as maximal stress or strain criterion or a Tsai criterion) is also evaluated in a post-computation step relative to the incremental calculation. It is computed with the stresses and internal variables resulting from the total increment. The criterion is computed at all Gauss points in the structure, in order to obtain an intrinsic criterion, from the values of the close Gauss points included in surface S. The non local approach implemented is slightly enriched since the criterion is tested at points at least at a given distance from the border (the radius of the circular area, see Figure 2).

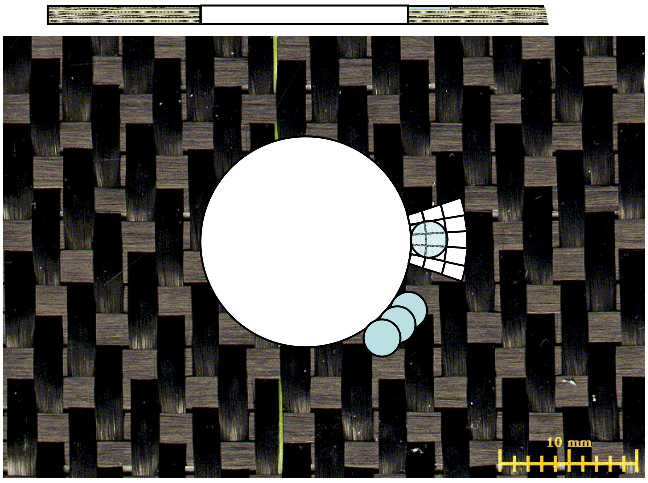

Fig. 2: Non local approach based on a Fracture Characteristic Volume (FCV)

Currently, the integration in ABAQUS is limited to plates in tension and the laminated plate theory (the strains are homogeneous over the thickness and only one Gauss point is needed in the thickness).

The size of the mesh has to be smaller than the zone in which the mean associated force is calculated. We observed that even in the high stress gradient zones, upward of ten Gauss points in the zone, the mean value no longer changes [9]. The computation is stopped when the FCV criterion is reached for one Gauss point (first ply failure criterion).

The use of the non local approach can be enriched by associating to the FCV a model of behaviour of material according to the type of material, orientation of the plies... In the case of woven ply laminates loaded in the fibre direction, a simple elastic model can be use to describe the behaviour of the material. But, in the case of a material which presents a non linear behaviour, like a [±45°] laminate, a more precise model of behaviour which integrates the description of the damage process and the inelastic strains, is necessary to obtain suitable results. This model could be implemented in a UMAT routine and associated to the URDFIL routine [9]. We present this both cases in the next part of this paper.

3. APPLICATION

Two types of material were studied. The first is a balanced woven ply laminate reinforced by carbon fibres (CFRP laminate) and the second is an unbalanced woven ply laminate reinforced by glass fibres (GFRP laminate). A lot of tests on various laminates with different geometric irregularities were performed to analyse the validity of the non local approach in various cases of stress concentration.

3.1. Use of the FCV in association with a simple model of the behaviour of elastic material and a Tsai criterion

Both materials present an elastic behaviour for various orientations of the plies, like in the fibre direction ([0°] and [90°]) or near to the fibre direction, such as a [±15°] laminate, or for a quasi-isotropic (QI) laminate. It is the case for a lot of industrial problems, which the direction of the loads is known and the plies are then oriented in these directions. The behaviour of the CFRP laminate can be easily describe by a transverse isotropic model of behaviour and the GFRP laminate, by an orthotropic model of behaviour.

For this approach, we choose the Tsai criterion [11] to describe the failure of the laminated structure. In the case of woven plies, if the behaviour of the material is similar under both tension and compression conditions, the Tsai criterion can be written with only three coefficients (*X* = *X'*, *Y* = *Y'* and *S*):

$$\frac{\sigma_{xx}^2}{X^2}+\frac{\sigma_{yy}^2}{Y^2}-\frac{\sigma_{xx}\sigma_{yy}}{X^2}+\frac{\sigma_{xy}^2}{S^2}=1 \tag{1}$$

It needs the identification of three parameters in the case of GFRP laminate -by realising tension tests on homogeneous specimens, two in the warp and weft directions and one in the shear direction- and only two in the case of CFRP laminate (X = Y).

An average of the Tsai criterion is evaluated on the FCV and the calculation will be stopped when it reaches to 1.

3.2. Use of the FCV in association with a model of the behaviour of woven ply laminate

Case of CFRP laminate: In paper [9], a complete model based on continuum damage mechanics, which describes the behaviour of balance woven ply laminate, is presented. Damage process is taken into account by defining three internal variables which describe the lost of rigidity in the warp (d_1), weft (d_2) and shear direction (d_{12}).

This lost of rigidity could be gradual due to the development of micro-cracks running parallel to the fibre direction. It is the case in the shear direction. The evolution of internal variables depends on the thermodynamic forces, based on the strain energy potential, or more specifically on their maximum values during the loading history. Assuming the existence of plane stresses and small perturbations, the strain energy in the woven ply can be written as follows:

$$E_D^{ps}=\frac{1}{2}\left[\frac{\langle\sigma_1\rangle_+^2}{E_1^0(1-d_1)}+\frac{\langle\sigma_1\rangle_-^2}{E_1^0}+\frac{\langle\sigma_2\rangle_+^2}{E_2^0(1-d_2)}+\frac{\langle\sigma_2\rangle_-^2}{E_2^0}-2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2+\frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})}\right] \tag{2}$$

where < . >$_+$ is the positive part and < . >$_-$ is the negative part. The tension energy and the compression energy are split in order to describe the unilateral nature of the damage process due to the opening and closing of the micro-defects.

So, the thermodynamic forces could be written as follow:

$$Y_{d_i} = \frac{\partial E_D^{ps}}{\partial d_i} = \frac{\langle \sigma_i \rangle_+^2}{2E_i^0 (1 - d_i)^2}; \qquad Y_{d_{12}} = \frac{\partial E_D^{ps}}{\partial d_{12}} = \frac{\sigma_{12}^2}{2G_{12}^0 (1 - d_{12})^2} \tag{2}$$

. To account for the tension/shear coupling during the development of d_{12}, we define the equivalent thermodynamic force and the maximum value of this force during the history of the loading as follows:

$$Y = \alpha_1 Y_{d_1} + \alpha_2 Y_{d_2} + Y_{d_{12}} \qquad and \qquad \underline{Y}(t) = \sup_{\tau \le t} (Y(\tau)) \tag{3}$$

where α_1 and α_2 are the tension/shear coupling coefficients. A classical ([2]) linear law with respect to the square root of $\underline{Y}$ is used to describe the damage variable development:

$$d_{12} = \left\langle \frac{\sqrt{\underline{Y}} - \sqrt{Y_O}}{\sqrt{Y_c} - \sqrt{Y_O}} \right\rangle_+ ,$$

d_1=d_2=0 if (d_{12}<1 and Y_{d1}<Y_{1f} and Y_{d2}<Y_{2f}) (4)
else d_{12}=d_1=d_2=1

where the constant parameters Y_o and Y_c correspond to the threshold and the critical value of the development of d_{12} (which ranges from 0 to 1).

In case of brittle behaviour like in the fibre direction, the lost of rigidity is instantaneously. The damage is null if the material is healthy and it reaches to 1 if the laminate is broken.

In the shear direction, inelastic strains are observed and the model describes them by a kinematics hardening law.

Failure criteria are based on the evolution of the thermodynamic forces (Y_d). If the evolution of the damage is sudden with no plasticity (as the brittle behaviour), this criterion is equivalent to a mean stress or strain criterion over the volume V. This criterion can be applied to plies undergoing gradual damage processes. In this case, the associated force (2) is proportional to the elastic strains and this criterion will be similar to a maximum mean elastic strain criterion.

In the case of the non local approach, we consider the mean value of the force over the volume. The failure criterion in the fibre direction is the following:

$$\overline{Y}_{d_i} = \frac{1}{V} \int_V Y_{d_i} \, dV \qquad and \qquad \overline{Y}_{d_i} < Y_{i_f} \tag{5}$$

with Y_{1f} and Y_{2f} are the parameters defining the ultimate forces in the warp and weft directions.

Case of GFRP laminate: A new model, based on the continuum damage mechanics too, was developed at the LMA in order to describe the behaviour of every laminate [12] such as unbalanced woven ply laminate. It is describes in the paper of Y. Thollon.

This model was associated to the non local approach to predict the failure of GFRP laminated structure. Failure criteria are based on the evolution of the thermodynamic forces like in the case of CFRP laminate. The different results in the definition in two Fracture Characteristic Volumes: the first to describe the failure in the warp direction, the second, to describe the failure in the weft direction. The identification of both failure criteria, in the warp and in the weft direction, and both FCV need four tension tests. Failure criteria were identified by performing tension tests on homogeneous specimen, on [0°] and [90°] laminates. FCV are cylinders like in the case of balance woven ply. Diameter of each circular area was evaluated with structure computation which was compared to experimental data obtained for a [0°] or a [90°] perforated plate.

4. RESULTS

In order to test the validity of the non local approach in the case of structures showing very high stress gradients, the force at failure was measured experimentally with various sequences ($[0°]_8$, $[\pm45°]_{4s}$, $[\pm15°]_{4s}$ and $[15°]_8$ CFRP laminates, ($[0°]_8$, $[90°]_8$, and (QI) GFRP laminates) and different stress distributions (plates with a hole, plates with notches and saw cuts on both sides), as shown in Figures 3 and 4. The plates have the dimension of 200x45x2mm, which represents an 8 plies laminate. The hole has 13mm diameter. The notches on both sides were 3 mm radius and the saw cuts were 1mm x 3mm in size.

The experimental results (Exp) and numerical results are normalized. The results obtained with the proposed approach, coupling the local non linear behaviour and the non local criterion (FCV/CDM model), show a quite good match, even at the high stress concentrations corresponding to the notches and saw cuts. Failure is appearing for the GFRP specimens and the [0°] and [±15°] CFRP specimens when the FCV criterion is reached and for the [15°] and [±45°] CFRP specimens when an instability of the structure is detected.

For the [0°] and [±15°] CFRP laminates, the global behaviour is linear and the use of a linear behaviour and the classic Tsai criterion calculated with the mean stress on the FCV is sufficient to predict the failure of the specimen (Figure 3).

For the GFRP laminate, the global behaviour is linear too. The use of linear behaviour and the Tsai criterion is not suitable for [90°] and (QI) because the diameter of the FCV was identified on a [0°] plate, and the mechanical properties are not the same in the warp and in the weft directions, contrary to the balance woven ply.

The results obtained with the non local approach combined to the model we developed are relevant compared to the experimental data (Figure 4).

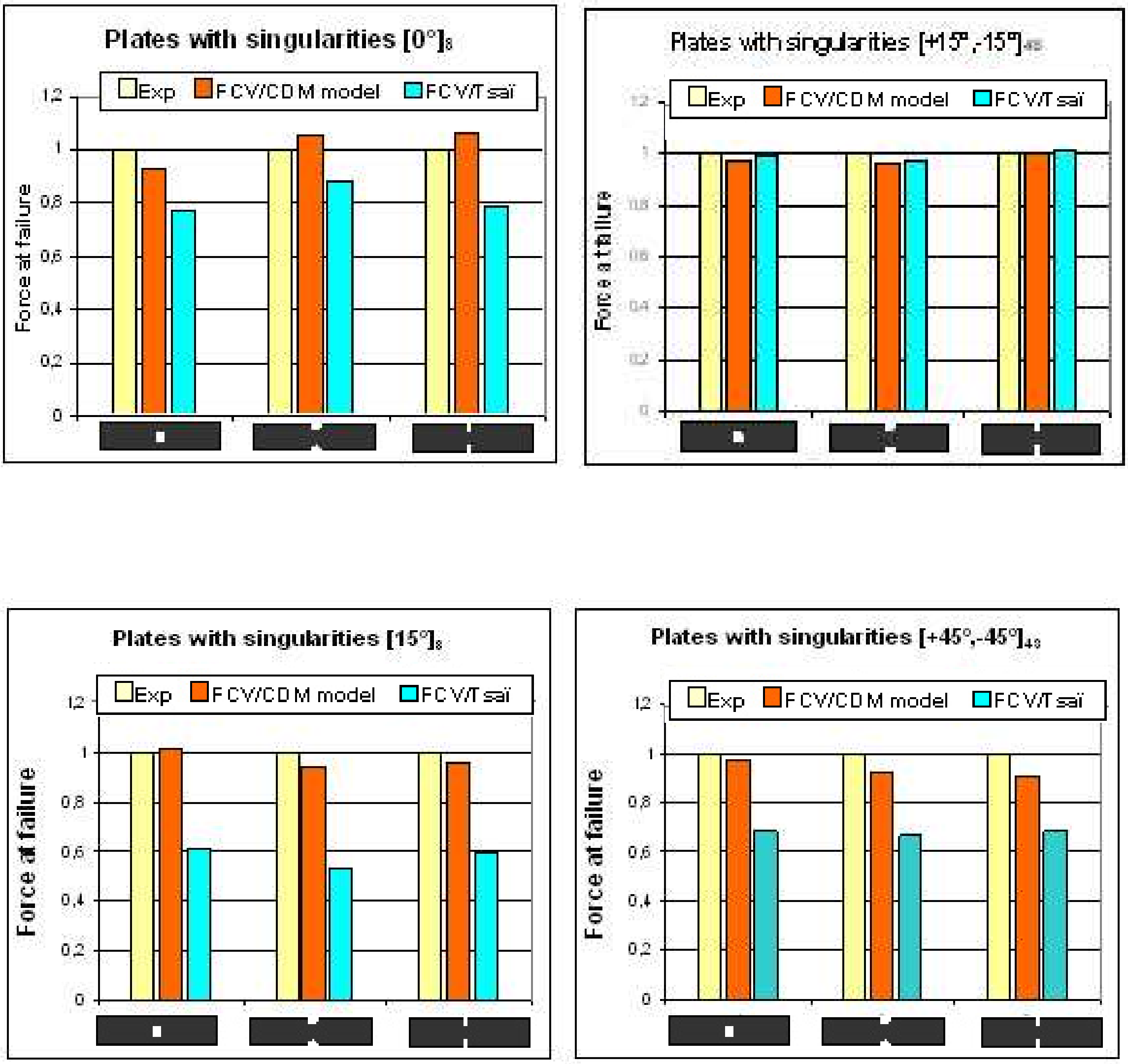

Fig.3: Comparison between simulations and tests on various balanced laminates reinforced by carbon fibres
[0°], [±15°],[45°],[15°]

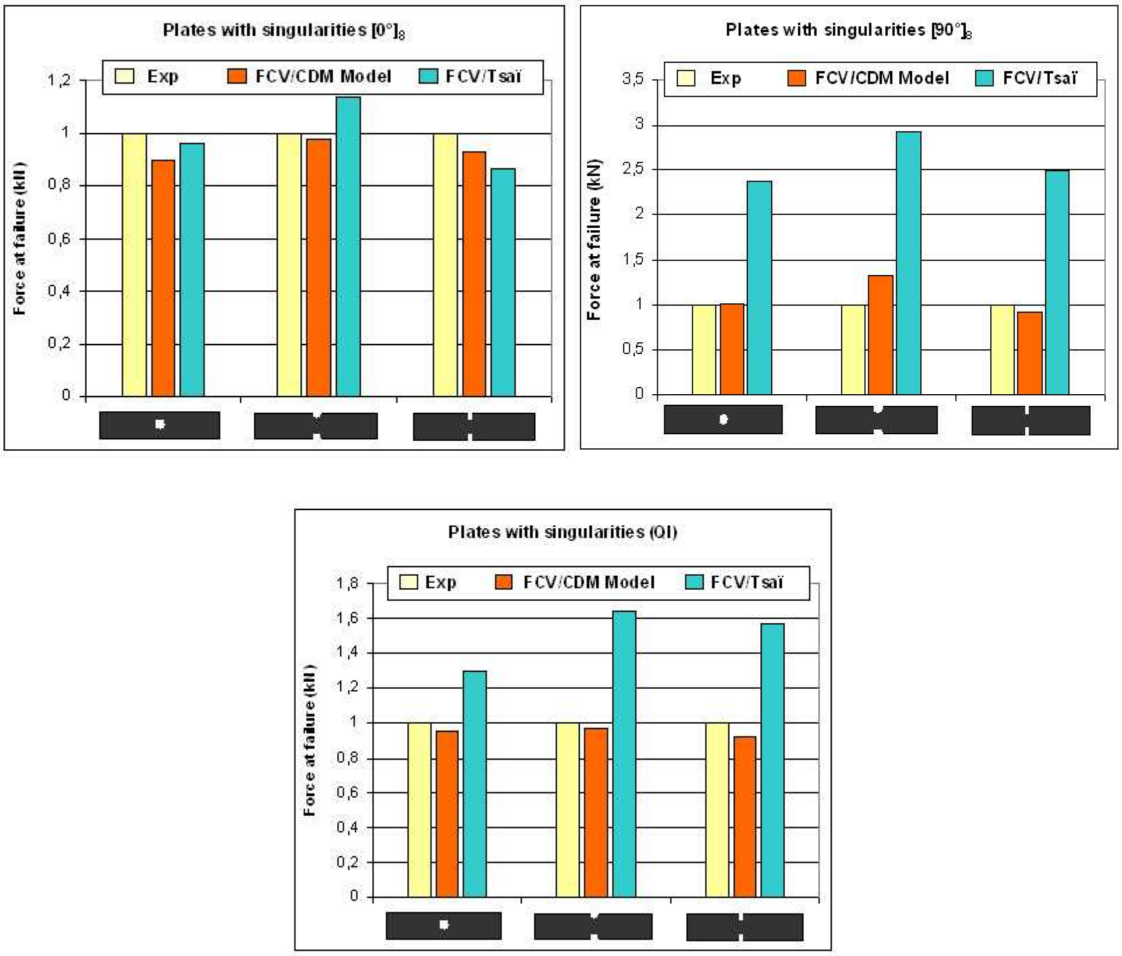

Fig4.: Comparison between simulations and tests on various unbalanced laminates reinforced by glass fibres
[0°], [90°],[QI]

5. CONCLUSION

In the model recalled here, diffuse damage and inelastic strain evolution laws were associated with a simple non local ply scale approach based on a *Fracture Characteristic Volume*. In general, this first ply failure model satisfactorily describes the failure of woven ply laminated structures.

This approach was implemented into the ABAQUS Software. This model is able to predict with a relevant precision the failure of different types of laminates in the presence of various singularities, which can generate high stress gradients.

The non-local approach based on the Fracture Characteristic Volume was successfully tested on two different materials: on balance woven ply laminate reinforced by carbon fibres and unbalance woven ply laminate reinforced by glass fibres.

This study concerns only structures in tension. We are still working on compression (material and structure). If necessary, we will consider a difference between tension and compression (in particular in the fibre direction for the FCV).

Finally, we would to extend this approach at the case of fatigue loading conditions.

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