# A SEMI–ANALYTICAL APPROACH FOR THE SIMULATION OF DELAMINATION GROWTH UNDER QUASI–STATIC AND CYCLIC LOADING

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ABSTRACT

A semi–analytical approach suitable for the prediction of delamination growth in laminated components is presented. It consists of an analytical part based on linear elastic fracture mechanics and a numerical part. For the latter the finite element method is employed in the present paper. The approach can handle structures loaded by combinations of force, displacement, and temperature loads. For the treatment of quasi–static loads the Griffith crack growth criterion is utilized and equilibrium delamination growth and its stability are predicted. For the treatment of cyclic loads a Paris-type growth law is utilized and structures loaded by combinations of cyclic and constant loads can be investigated. The proposed approach provides systematic and general information about the influence of size and position of a delamination on the growth process and the load carring capacity. Delamination growth and non–linear and non–monotonous structural response, including snap–through and snap–back behavior, can be predicted in a numerically efficient way. To show the capabilities of the proposed approach two laminated structures, an L–shaped component and a T–joint, are investigated in detail.

## 1 Introduction

One of the critical failure modes in laminated composite components is delamination, as it can change the structural stiffness and the load carring capacity significantly. Delamination can be caused by quasi–static or cyclic loading. The delamination growth process caused by quasi–static loading can be stable or unstable and causes a non–linear structural response, which can even be non–monotonous, like snap–through or snap–back.

Typically, Carbon Fiber Reinforced Polymers (CFRP) show a brittle interlaminar fracture behavior. Therefore, localized material non–linearities in the vicinity of the delamination front are neglected and Linear Elastic Fracture Mechanics (LEFM) can be used [1]. Based on LEFM various methods for the prediction of delamination growth have been developed, in CFRP the concept of the energy release rates is employed successfully.

In the present paper a semi–analytical approach is developed for the analysis of delamination growth in structures loaded by quasi–static or cyclic loads [2, 3]. The energy release rate is computed from the change of the structural stiffness caused

by increase of the delaminated area. A numerical procedure is employed to derive the structural stiffness as function of the delamination size and position. Here the Finite Element Method (FEM) is utilized for this purpose.
The semi–analytical approach is combined with the Griffith crack growth criterion to treat quasi–static loads for which equilibrium delamination growth and its stability are predicted [2]. Stable as well as unstable delamination growth can be handled, and non–linear structural response, including snap–through and snap-back, can be predicted in a numerically efficient way. For the treatment of cyclic loading conditions the semi–analytical approach can be combined with delamination growth laws of the Paris–type. In the present paper the law developed in [4] is considered and structures loaded by combinations of cyclic and constant loads can be handled.
Two laminated structures, an L–shaped component and a T–joint, are investigated in detail to show the capabilities of the proposed approach. Quasi–static and cyclic loads superimposed by constant and homogeneous temperature loads are investigated and delamination growth is predicted.

# 2 The Modelling Approach

**Semi–Analytical Approach** The approach is based on the assumption that the energy released at delamination growth is equal to the change of the potential energy of a system effectuated by increase of the delaminated area. For plane problems with through–the–width delaminations the delaminated area is entirely described by a limited number of local delamination front coordinates ${}^{(\mathrm{i})}a$. In Fig. 1 structures with two and three delamination fronts are shown, hence two and three coordinates are required to describe the delaminations. For structures with unit thickness the energy released by a change in one delamination coordinate, ${}^{(\mathrm{i})}a$, while keeping all other coordinates constant is computed as,

$$-\frac{\partial \Pi^{\mathrm{int}}}{\partial {}^{(\mathrm{i})}a} - \frac{\partial \Pi^{\mathrm{ext}}}{\partial {}^{(\mathrm{i})}a} = G \quad , \tag{1}$$

where $G$ is the energy release rate. The potential energy of the structure is the sum of the strain energy, $\Pi^{\mathrm{int}}$, plus the potential of the external forces, $\Pi^{\mathrm{ext}}$.
In three dimensional problems with curved delamination fronts some assumption concerning the shape of the delamination are required to keep the number of delamination coordinates finite. The semi–analytical approach can be used for such problems, however, no information about the local distribution of the energy release rate along the delamination front is obtained.
Structures loaded by prescribed displacements and homogeneous temperature loads are considered here (for a discussion of force controlled loading see [2]). The reaction forces at the load introduction point read,

$$[F_1, F_2, F_3]^T = \mathbf{K}\left([u_1, u_2, u_3]^T - \boldsymbol{\alpha}\Delta T\right) \quad , \tag{2}$$

where $u_1$ and $u_2$ are the displacements in 1–, and 2–direction and $u_3$ is the rotation of the load introduction point, given with respect to the stress free state, see Fig. 1. $F_1$, $F_2$, and $F_3$ are the corresponding reaction forces and the reaction moment. $\mathbf{K}$

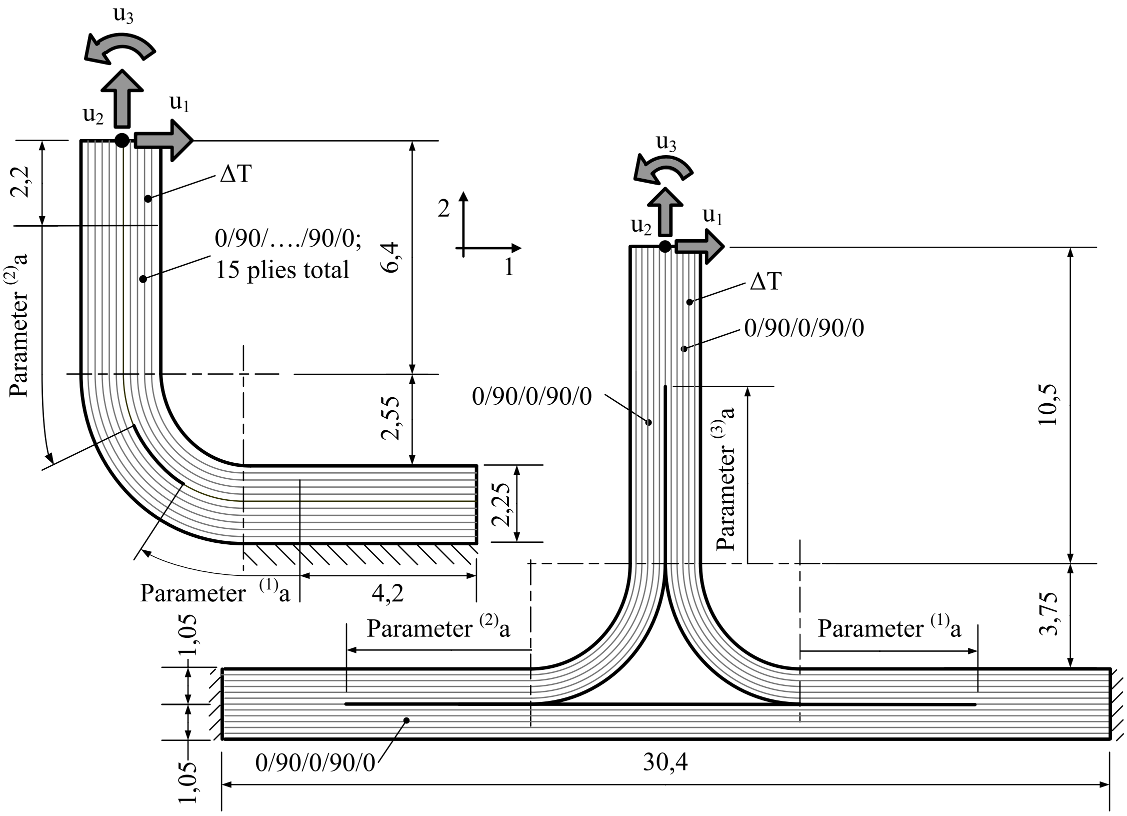


Figure 1: L–shaped component (left) and T–joint component (right) loaded by displacement and constant and homogeneous temperature loads; cross sections and definitions of delamination coordinates.

is the stiffness matrix and $\boldsymbol{\alpha}$ is the vector of the structural coefficients of thermal deformation. $\Delta T$ is the homogeneous temperature load. The sum of the strain energy induced by the displacement load plus the strain energy induced by the temperature difference gives the total strain energy and can be expressed as,

$$\Pi^{\mathrm{int}} = \frac{\mathbf{u}^T \, \mathbf{K}^{\mathrm{int}} \, \mathbf{u}}{2} \quad , \tag{3}$$

where the generalized displacement reads, $\mathbf{u} = [u_1, u_2, u_3, \Delta T]^T$. $\mathbf{K}^{\mathrm{int}}$ is the generalized stiffness matrix, which is defined as $\mathbf{K}^{\mathrm{int}}_{ij} = \mathbf{K}_{ij}$, $\mathbf{K}^{\mathrm{int}}_{i4} = \mathbf{K}^{\mathrm{int}}_{4i} = [\mathbf{K}\boldsymbol{\alpha}]_i \; \forall \, i, j = 1, 2, 3$. $\mathbf{K}_{44}$ is the term related to the strain energy caused by a pure temperature load. Note that the generalized stiffness is a function of the delamination coordinates ${}^{(\mathrm{i})}a$. The potential of external forces is zero in displacement controlled loading and the energy release rate can be computed for each delamination front from Eqs. (1) and (3).

So far only the total energy release rate is accessible, however, for most CFRP the individual mode I and mode II energy release rates, $G_{\mathrm{I}}$ and $G_{\mathrm{II}}$, respectively, are desired. A mode mix variable is introduced, which allows to split the total energy release rates in its mode I and mode II contributions,

$$G_{\mathrm{I}} = \left(1 - {}^{(\mathrm{i})}m\right) G, \quad G_{\mathrm{II}} = {}^{(\mathrm{i})}m G \quad . \tag{4}$$

The mode mix, ${}^{(\mathrm{i})}m$, is a function of the load applied on the structure and is computed at each delamination front as,

$$
{}^{(\mathrm{i})}mG = {}^{(\mathrm{i})}\mathbf{M} \begin{bmatrix} {u_1}^2 & {u_2}^2 & {u_3}^2 & \Delta T^2 & u_1u_2 & u_1u_3 & u_1\Delta T & u_2u_3 & u_2\Delta T & u_3\Delta T \end{bmatrix}^T
$$

$$
{}^{(\mathrm{i})}\mathbf{M} = \begin{bmatrix} M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 & M_9 & M_{10} \end{bmatrix} \quad , \tag{5}
$$

where ${}^{(\mathrm{i})}\mathbf{M}$ are named mode contributions and are functions of the delamination coordinates. An equivalent equation for the mode mix together with a verification is given in [5].

**Numerical evaluation of the stiffness and the mode contributions** The generalized stiffness of the structure and the mode contribution at each delamination front need to be expressed as functions of the delamination coordinates ${}^{(\mathrm{i})}a$. In general cases an analytical solution cannot be given, thus, a numerical procedure is used. The space of delamination coordinates is discretized in a way which allows to describe all delamination sizes and positions of interest. An FEM model parametrized in the delamination coordinates is set up to analyze all combinations of delamination coordinates. For each one the reaction forces and moments, the strain energy, and the mode mix at each delamination front are computed for a set of load cases. For the computation of the mode mix the Virtual Crack Closure Technique (VCCT) is utilized [3, 5]. From the results the generalized stiffness and the mode contributions are derived according to Eqs. (2), (3), and (5). These computations and all FEM analyzes are carried out within a fully automated procedure. The results of the procedure give a pointwise description of the generalized stiffness function, the mode contribution functions, and their difference quotients which are all stored in a database. In the following sections analytical equations for delamination growth under quasi–static and cyclic loading are discussed. The appropriate input for these equations can be recalled from the database. Hence, results can be extracted immediately.

The proposed semi–analytical approach is based on the assumption that the generalized stiffness is independent of the actual load magnitude, thus, only non–linearities caused by the delamination growth process are considered. This means that the displacements need to be small, the material needs to behave linear elastic, and no friction or contact problems occur. However, most problems of practical relevance fullfill these requirements.

**Quasi–static Loading** The Griffith criterion is applied for the prediction of delamination growth caused by quasi–static loading. According to Griffith a crack will grow if,

$$
-G + G_\mathrm{c} \leqslant 0 \quad , \tag{6}
$$

with the critical energy release rate, $G_\mathrm{c}$. It is a function of the actual fracture mode and the critical energy release rates for pure mode I, mode II, and mode III loading, $G_\mathrm{Ic}$, $G_\mathrm{IIc}$, and $G_\mathrm{IIIc}$, respectively. The latter are interface properties and can be determined in experimental testing. The crack growth condition can be written in terms of the potential energies as,

$$\frac{\partial \Pi^*}{\partial {}^{(\mathrm{i})}a} = \underbrace{\frac{\partial \Pi^{\mathrm{int}}}{\partial {}^{(\mathrm{i})}a} + \frac{\partial \Pi^{\mathrm{ext}}}{\partial {}^{(\mathrm{i})}a}}_{-G} + \underbrace{\frac{\partial \Gamma}{\partial {}^{(\mathrm{i})}a}}_{G_\mathrm{c}} \leqslant 0 \quad , \tag{7}$$

where $\partial \Pi^*$ is the total potential energy of the structure and $\Gamma$ is the surface energy of the delaminated area. If $\frac{\partial \Pi^*}{\partial {}^{(\mathrm{i})}a} > 0\ \forall {}^{(\mathrm{i})}a$, no delamination growth takes place. If $\frac{\partial \Pi^*}{\partial {}^{(\mathrm{j})}a} = 0$ for one coordinate ${}^{(\mathrm{j})}a$ and $\frac{\partial \Pi^*}{\partial {}^{(\mathrm{i})}a} > 0$ for all remaining coordinates ${}^{(\mathrm{i})}a$ equilibrium growth along coordinate ${}^{(\mathrm{j})}a$ takes place. Equilibrium growth is stable if,

$$\frac{\partial^2 \Pi^*}{\partial {}^{(\mathrm{i})}a^2} > 0 \quad , \tag{8}$$

otherwise it is neutral or unstable. If $\frac{\partial \Pi^*}{\partial {}^{(\mathrm{i})}a} < 0$ for at least one coordinate ${}^{(\mathrm{i})}a$ dynamic delamination growth takes place along coordinate ${}^{(\mathrm{i})}a$. Such growth processes are always unstable. Note that if the mode mix changes during delamination growth $\frac{\partial^2 \Gamma}{\partial {}^{(\mathrm{i})}a^2} \neq 0$.

From Eqs. (3) and (7) the generalized displacement, ${}^{(\mathrm{i})}\mathbf{u}$, required to cause equilibrium delamination growth along coordinate ${}^{(\mathrm{i})}a$ is computed as,

$$\frac{\partial \Pi^*}{\partial {}^{(\mathrm{i})}a} = \frac{1}{2}\, {}^{(\mathrm{i})}\mathbf{u}^T \frac{\partial\, \mathbf{K}^{\mathrm{int}}}{\partial {}^{(\mathrm{i})}a}\, {}^{(\mathrm{i})}\mathbf{u} + \frac{\partial \Gamma}{\partial {}^{(\mathrm{i})}a} = 0 \quad . \tag{9}$$

The displacement that corresponds to an equilibrium state of the structure, $\mathbf{u}^{\mathrm{eq}}$, is defined as,

$$\mathbf{u}^{\mathrm{eq}} = \min\left({}^{(\mathrm{i})}\mathbf{u}\right) \quad . \tag{10}$$

The stability of this state is computed according to Eq. (8).

In order to take the mode interaction at the delamination front into account various criteria have been developed. Using a quadratic criterion [6] and Eq. (4) the critical energy release rate for mixed mode cases reads,

$$\left(\frac{1}{G_\mathrm{c}}\right)^2 = \left(\frac{1-m}{G_\mathrm{Ic}}\right)^2 + \left(\frac{m}{G_\mathrm{IIc}}\right)^2 \quad . \tag{11}$$

**Cyclic Loading** For the prediction of delamination growth caused by a cyclic load a combination of the semi–analytical approach with Paris–type growth laws is employed. Problems in which all loads vary with the same frequency can be treated as well as problems where some load components are constant. The semi–analytical approach is utilized to compute the minimum and the maximum mode I and mode II energy release rates at each delamination front that occur during one load cycle. These values serve as input for Paris–type growth laws and incremental growth, $\Delta {}^{(\mathrm{i})}a$, at each delamination front is obtained.

In the follwing the law developed in [4] is considered in detail. The growth per load cycle is computed as,

$$\Delta a \frac{1}{g_\mathrm{I}\, \frac{E_\mathrm{y}\, G_\mathrm{Ic}}{Y_\mathrm{t}^2} + g_\mathrm{II}\, \frac{E_\mathrm{y}\, G_\mathrm{IIc}}{S^2}} = A_\mathrm{I}^{g_\mathrm{I}} A_\mathrm{II}^{g_\mathrm{II}} \left[U\left(\frac{G_\mathrm{Imax}}{G_\mathrm{Ic}} + \frac{G_\mathrm{IImax}}{G_\mathrm{IIc}}\right)\right]^{b_\mathrm{I}\, g_\mathrm{I} + b_\mathrm{II}\, g_\mathrm{II}} \tag{12}$$

$$g_i = \frac{\frac{G_{i\mathrm{max}}}{G_{i\mathrm{c}}}}{\frac{G_{\mathrm{Imax}}}{G_{\mathrm{Ic}}} + \frac{G_{\mathrm{IImax}}}{G_{\mathrm{IIc}}}} \quad \forall \;\; i = \mathrm{I}, \mathrm{II} \quad ,$$

where $E_y$ is the transverse ply Young's modulus, $Y_\mathrm{t}$ is the ply transverse tensile strength, $S$ is the in–plane shear strength, $G_\mathrm{min}$, $G_\mathrm{max}$ are the minimum and maximum energy release rates and $G_\mathrm{Imin}$, $G_\mathrm{Imax}$, $G_\mathrm{IImin}$, $G_\mathrm{IImax}$ are its mode I and mode II components. $U$ is a function of $\frac{G_{min}}{G_{max}}$, $\frac{G_{Imax}}{G_c}$, and $\frac{G_{IImax}}{G_c}$. The form of this function depends on whether or not there is shear reversal at the delamination front during the load cycle. In absence of shear reversal $U$ is defined as,

$$U = \left(1 - \frac{G_{min}}{G_{max}}\right) \left[1 + \frac{G_{min}}{G_{max}} \left(1 - \left(\frac{G_\mathrm{Imax}}{G_\mathrm{Ic}} + \frac{G_\mathrm{IImax}}{G_\mathrm{IIc}}\right)\right)\right]^{u_\mathrm{I}\, g_\mathrm{I} + u_\mathrm{II}\, g_\mathrm{II}} \quad . \qquad (13)$$

The parameters $A_\mathrm{I}$, $A_\mathrm{II}$, $b_\mathrm{I}$, $b_\mathrm{II}$, $u_\mathrm{I}$, and $u_\mathrm{II}$ depend on the actual material and have to be determined by cyclic testing.

# 3 Examples

Two laminated structures, an L–shaped component and a T–joint, made of unidirectional reinforced plies are analyzed. Material data and interface properties are taken from the literature [4, 6], see Table 1. The structures have a considerable length in 3–direction, thus, generalized plane strain conditions are assumed. The geometry, the boundary conditions, and the lay–up are shown in Fig. 1. The orientation of the layers is defined with respect to the 1–2 plane. The structures are loaded by prescribed displacements and rotations of the load introduction point and homogeneous temperature changes. In the L–shaped component growth of an existing delamination at the interface between ply five and six is analyzed. This interface is chosen as it faces the highest normal traction in the flawless case [7]. The delamination is entirely described by two delamination coordinates, ${}^{(1)}a$ and ${}^{(2)}a$, see Fig. 1. Note that a decrease in the coordinates is equal to an increase of the delaminated area. In the T–joint delaminations are considered which emerge from the cavity in the center of the structure which is not filled with resin. Three coordinates are required here to describe the delaminated area.
For the computation of the generalized stiffness and the mode contributions FEM

Table 1: *Material and interface data of transversely isotropic carbon/epoxy UD–layer, T300/976, data taken from [4, 6]*

| $E_x$ | $E_y$ | $G_{xy}$ | $\nu_{xy}$ | $\nu_{yz}$ | $\alpha_{xx}$ | $\alpha_{yy}$ | $Y_t$ | S |
|---|---|---|---|---|---|---|---|---|
| $[GPa]$ | $[GPa]$ | $[GPa]$ | $[-]$ | $[-]$ | $[K^{-1}]$ | $[K^{-1}]$ | $[MPa]$ | $[MPa]$ |
| 139.3 | 9.72 | 5.59 | 0.29 | 0.40 | 4.1E-8 | 3.6E-5 | 58 | 100 |

| $G_\mathrm{Ic}$ | $G_\mathrm{IIc}$ | $A_\mathrm{I}$ | $A_\mathrm{II}$ | $b_\mathrm{I}$ | $b_\mathrm{II}$ | $u_\mathrm{I}$ | $u_\mathrm{II}$ |
|---|---|---|---|---|---|---|---|
| 135 $\frac{J}{m^2}$ | 450 $\frac{J}{m^2}$ | 0.16 | 0.13 | 18.0 | 5.3 | -2 | 4 |

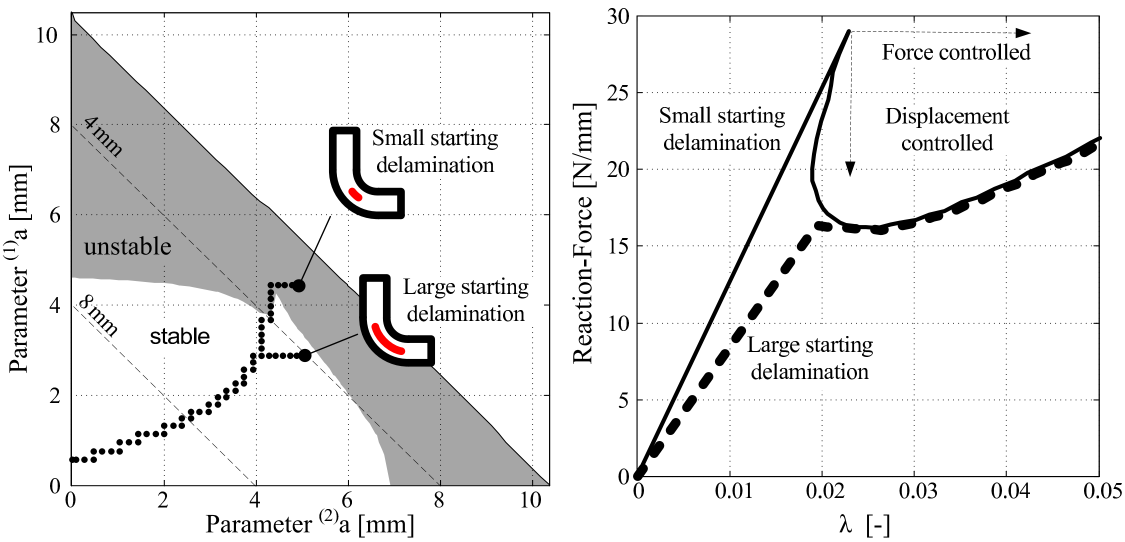


Figure 2: L–shaped component loaded by a quasi–static displacement and a constant and homogeneous temperature load; stability diagram (left); structural response for two different sizes of starting delaminations (right).

models parametrized in $^{(i)}a$ are set up, utilizing the FEM package *ABAQUS/-Standard/V6.6 (ABAQUS Inc., RI, USA)*. Two dimensional generalized plane strain elements with linear shape functions and full integration are used. In the L–shaped component each ply is represented by three elements over the ply thickness, for the T–joint two elements per ply are used. All element aspect ratios are close to one. Discrete values of the delamination coordinates are chosen and each combination is analyzed by a linear FEM analysis. In total 1900 delaminations are analyzed for the L–shaped component and 10584 for the T–joint. The results allow for a pointwise description of the generalized stiffness and the mode contributions as functions of the delamination coordinates. It turns out that the temperature load hardly effects the energy release rate. Thus, the mode contribution is considered to be independent of the temperature load.

**L–shaped component** Loading by an arbitrary generalized displacement $\mathbf{u}$ can now be treated by postprocessing the data generated numerically. As first example a quasi–static displacement load, $\mathbf{u}$=$[\lambda(-0.9\ mm), \lambda(-0.3\ mm), \lambda(0.9), -75\ K]^T$ with variable magnitude, $\lambda$, is considered. For this load the stability of the growth process is presented in Fig. 2 (left) in terms of the delamination coordinates. Delaminations of the same length but different position lie on straight lines with a slope of minus one. The figure shows that large delaminations grow in a stable manner (white area) while small delaminations grow in an unstable manner (gray area). Additionally, the influence of the delamination position on the stability is shown. Two starting delaminations are selected, one that will grow stable and one that will grow unstable, and equilibrium delamination growth is predicted. In Fig. 2 (left, dots) the growth process is shown in terms of the change in the delamination coordinates, indicating that both starting delaminations converge to the same final delamination. In Fig. 2 (right) the structural response is plotted in terms of the resultant of the reaction

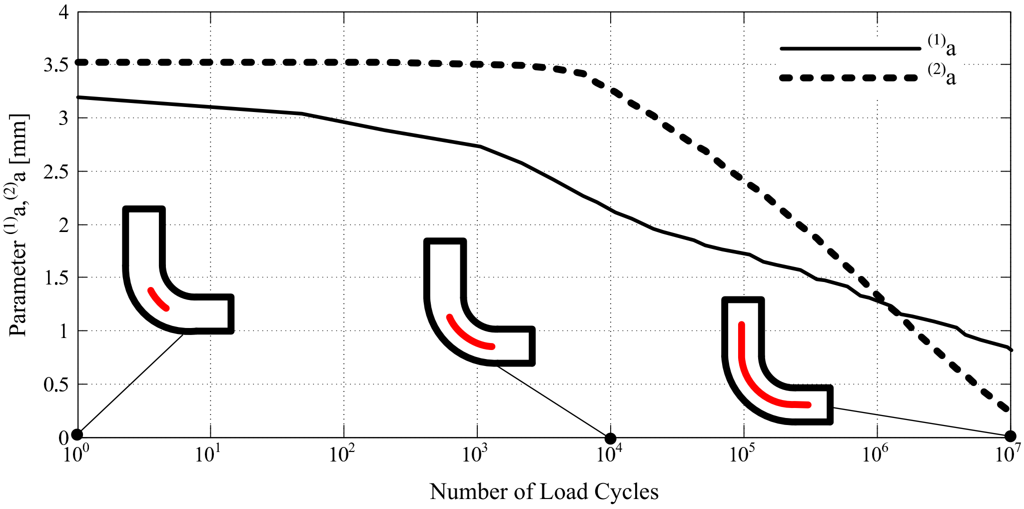


Figure 3: L–shaped component loaded by a cyclic displacement load and a constant and homogeneous temperature load.

force $\sqrt{F_1^2 + F_2^2}$ and the magnitude of the displacement load $\lambda$. At low load levels the structural response is linear until the growth load is reached. The difference in the initial stiffness is due to the different sizes of the starting delaminations. For the structure with the large starting delamination the entire growth process is stable. For the structure with the small starting delamination growth is unstable first. As the delamination size is increasing the equilibrium load is decreasing. As soon as a certain delamination size is reached (at $\lambda = 0.0185$) growth becomes stable and a load increase is required to propagate the delamination further. Such force–displacement curves for unstable equilibrium delamination growth can only be realized theoretically. In structural experiments unstable delamination growth, including dynamic effects, would take place, as indicated by the dashed lines for monotonously increasing displacement and force controlled loading.
Next, cyclic loading is realized by varying the magnitude of the displacement load between $\lambda_{\text{min}} = 0.0075$ and $\lambda_{\text{max}} = 0.018$. The large starting delamination in Fig. 2 is considered as initial configuration and cyclic delamination growth is predicted. The results, Fig. 3, show that the delamination will grow first along coordinate ${}^{(1)}a$ only. At about $5 \cdot 10^3$ load cycles considerable growth in direction ${}^{(2)}a$ starts in addition.

**T–joint** Loading by a quasi–static displacement load in vertical direction, $\mathbf{u} = \begin{bmatrix} 0 & \lambda & 0 & 0 \end{bmatrix}^T$, is considered. The stability of the growth process is shown in Fig. 4 (left), due to the symmetry of the problem the solution is symmetric (i.e. ${}^{(1)}a = {}^{(2)}a$). The figure shows that a wide range of delaminations will grow in a stable manner (white area). As initial configuration just the cavity is considered to form the starting delamination and the delamination growth path is plotted, Fig. 4 (left, dots). It shows that the delamination will grow first in direction ${}^{(3)}a$ in a stable manner. Then unstable growth along ${}^{(1)}a$ and ${}^{(2)}a$ takes place, which again changes to stable growth as soon as a certain delamination size is reached. The structural response is shown in Fig. 4 (right), indicating a slight load decrease during unstable

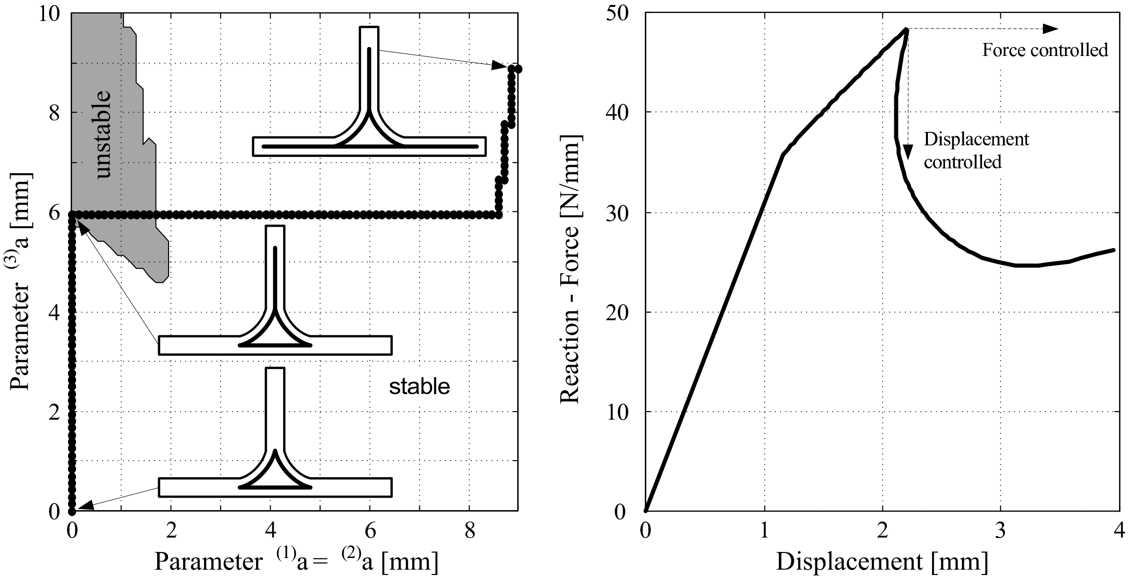


Figure 4: T–joint component loaded by a quasi–static displacement load in vertical direction; stability diagram (left); structural response (right).

growth.
Finally loading by $\mathbf{u} = [\lambda(0.68\ mm),\ \lambda(0.68\ mm),\ \lambda(-0.0002),\ -50\ K]^T$ which is cycled between $\lambda_{\min} = 0.2$ and $\lambda_{\max} = 0.1$ is considered. The cavity is considered as starting delamination and the change in the delamination coordinates is plotted, Fig. 5. The results show that the growth rate is different at each delamination front and considerably changes as the delamination advances. Note that this example was chosen to show the capabilities of the semi–analytical approach, the material data was not verified for high numbers of load cycles.

# 4 Conclusion

A semi–analytical approach for the prediction of delamination growth in laminated composite components is proposed. Structures loaded by a combination of force, displacement, and temperature loads can be handled in a numerically efficient way. A combination with the Griffith crack growth criterion is employed to treat quasi–static loads. Equilibrium delamination growth and its stability are analyzed and a systematic and general understanding of the influence of size and position of a delamination on the the structural response and the load carring capacity is gained. Critical configurations where growth changes from stable to unstable, or vice versa, are determined and non–linear and non–monotonous structural response, including snap–through and snap–back behavior, can be predicted. In order to treat cyclic loads the semi–analytical approach is combined with a Paris–type growth law. Problems in which all loads vary with the same frequency can be treated as well as problems where some load components are constant. Two laminated structures, an L–shaped component and a T–joint, are investigated in detail. Constant and cyclic loads are considered and delamination growth is predicted successfully.

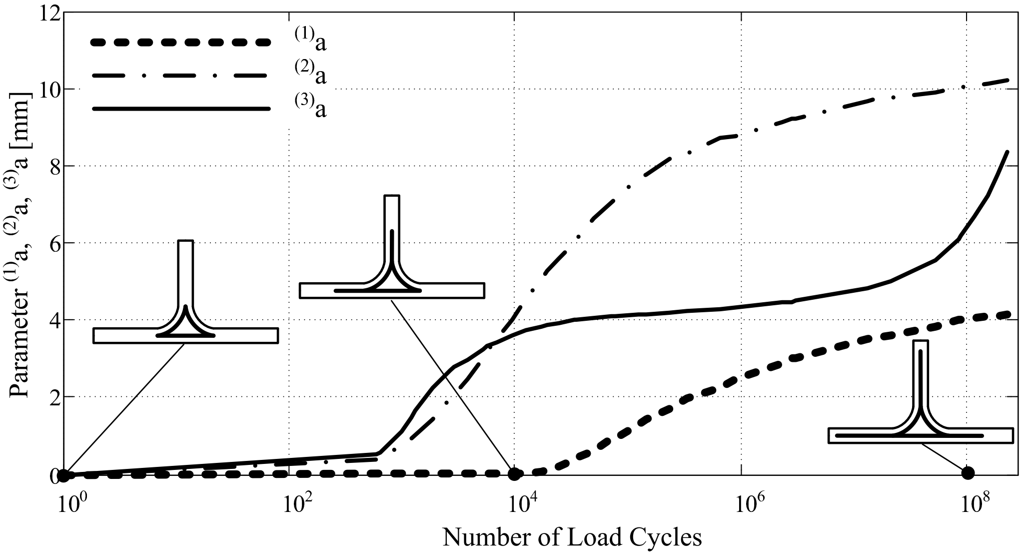


Figure 5: T–joint component loaded by a cyclic displacement load and a constant and homogeneous temperature load.

**Acknowledgment** The funding of the Austrian Aeronautics Research (AAR)/-Network for Materials and Engineering by the Austrian Federal Ministry of Economics and Labor is gratefully acknowledged.

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