A PRESSURE-DEPENDENT CONSTITUTIVE MODEL WITH FAILURE FOR LAMINATED COMPOSITES

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KEYWORDS: Failure, fracture, constitutive model, nonlinearity, pressure, kinking, in-situ

ABSTRACT

This paper describes a constitutive and failure model for composite laminates under 3D stress states, for which the contribution of the hydrostatic pressure is not negligible. While many previous publications in the area have focused on plane-stress, mainly due to its simplicity, real applications of composite structures typically involve 3D stress states. The out-of-plane stress components can be due to the design of the component, e.g. 'L'-shaped beam, or to the application, e.g. hydrostatic pressure in marine or aircraft components. The approach presented here follows previous work by the authors [1, 2], together with new developments carried out for the authors' participation in the second and third World-Wide Failure Exercises [3]. The models have a strong physical basis which provides a solid foundation which not only allows predicting the failure event, but also the failure mode and the consequences of failure in terms of residual materials properties.

1. CONSTITUTIVE MODEL

For polymer-matrix fibre-reinforced composites, the nonlinear behaviour of the matrix results in a nonlinear behaviour for the composite. There are two main failure mechanisms competing for matrix failure in composites: plasticity and brittle fracture, see ***Fig. 1***. While plasticity is more noticeable in shear, compression or in situations involving hydrostatic compression, brittle fracture tends to precede (and thus prevent) significant plasticity in tension.

Experimental data for several composites [4] shows a pressure-dependence of the Young's and shear moduli, see ***Fig. 2***. The model presented here assumes transverse isotropy and a linear pressure-dependence of E_2 and G_{12}:

$$\begin{aligned} E_2 &= E_2^o + \eta_E p \\ G_{12} &= G_{12}^o + \eta_G p \end{aligned} \tag{1}$$

where E_2^o and G_{12}^o are the Young's and shear moduli at atmospheric pressure respectively, and η_E and η_G are slope coefficients for the transverse Young's and shear moduli respectively. The variable p is the transverse hydrostatic pressure, and E_2^o and G_{12}^o are defined experimentally as a function of an equivalent strain.

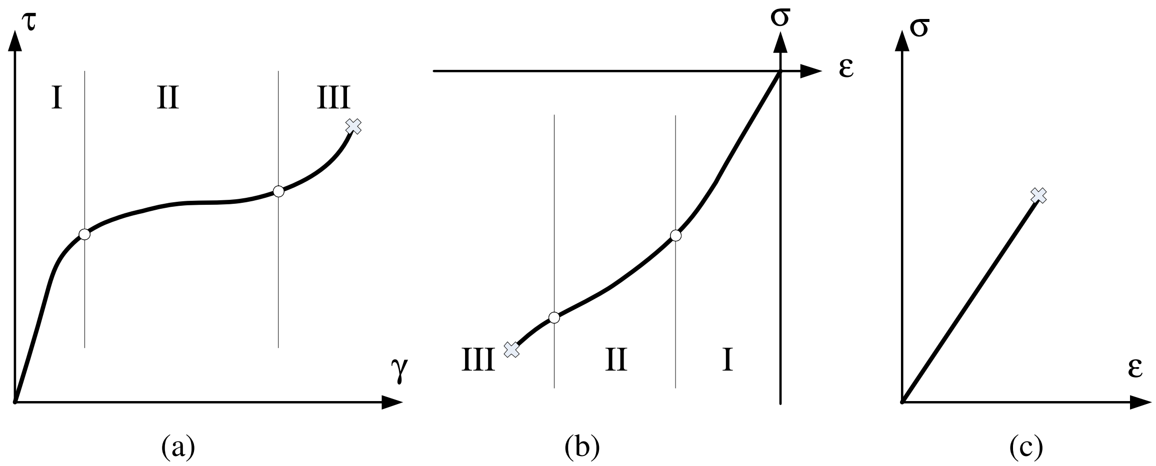


Fig. 1: Typical response of a polymer; (a) three regions in a shear curve; (b) compressive loading, and (c) brittle response in tension, from [3]

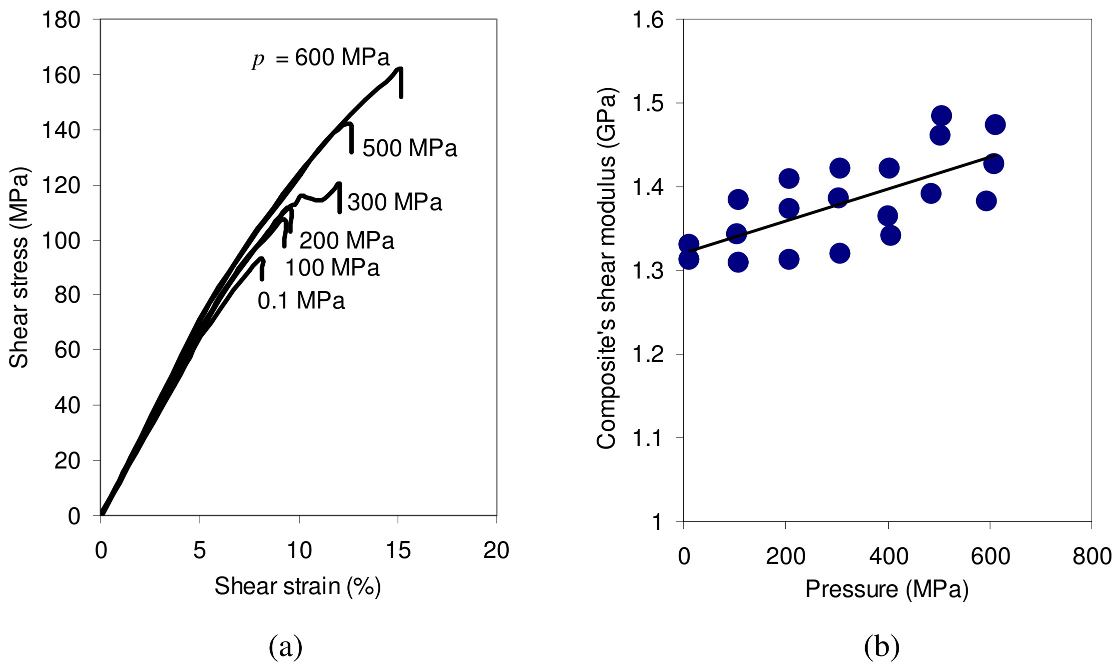


Fig. 2: Effect of pressure on (a) the whole shear response of a composite, and (b) the initial shear modulus, after [5]

2. FAILURE CRITERIA

Matrix failure is predicted using the following equation, which accounts for the strengthening effect of compressive normal stresses and for the weakening effect tensile normal stresses acting on the fracture plane:

$$FI_M = \left(\frac{\tau_T}{S_T^{is} - \eta_T \sigma_N} \right)^2 + \left(\frac{\tau_L}{S_L^{is} - \eta_L \sigma_N} \right)^2 + \left(\frac{\langle \sigma_N \rangle_+}{Y_T^{is}} \right)^2 \qquad (2)$$

with failure being predicted when $FI_M \geq 1$. In Eq. 2, τ_L, τ_T and σ_N are the traction components in the (potential) fracture plane, as shon in ***Fig. 3***.

The strengths S_T^{is}, S_L^{is} and Y_T^{is} are in-situ strengths, i.e., their value is dependent on the thickness of the ply and calculated using a fracture mechanics model. The fracture mechanics model assumes that there is an equivalent micro-crack existing in the composite. Depending on the type of ply, the boundary conditions for the equivalent crack's propagation change, as shown in ***Fig. 4*** are different, and so does the strength. The equivalent crack's dimensions are calculated using the strength from unidirectional tests, and this is dimension is then used to calculate the strength for different laminates.

Following experimental evidence, see ***Fig. 5***, fibre kinking is predicted with a model which assumes fibre-matrix splitting in misaligned fibres, see ***Fig. 6***. Fibre tensile failure is predicted with a maximum stress criterion.

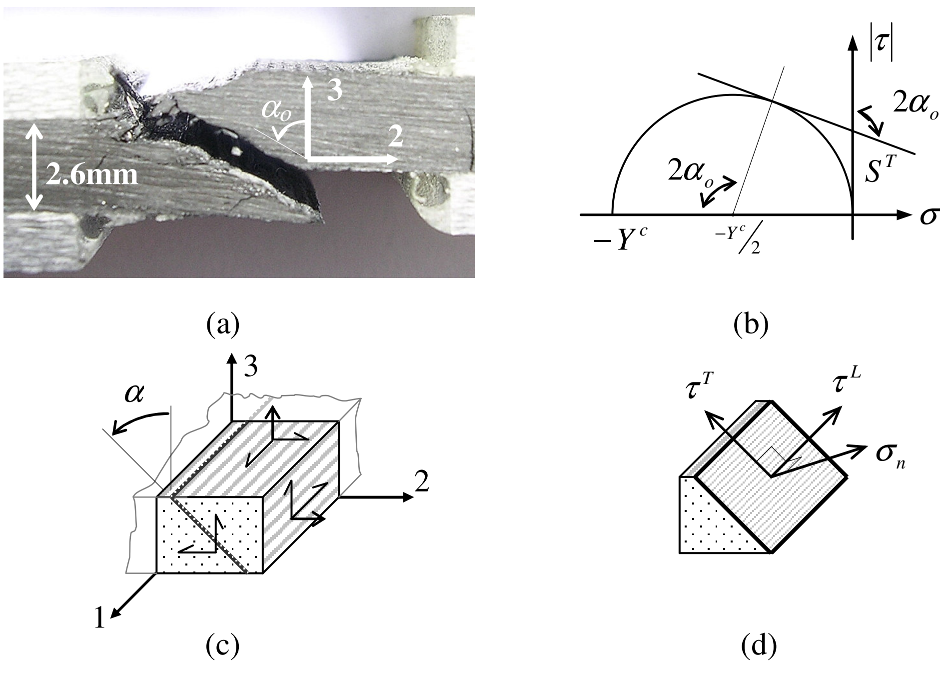


Fig. 3: Traction components acting on the matrix fracture plane, from [1]

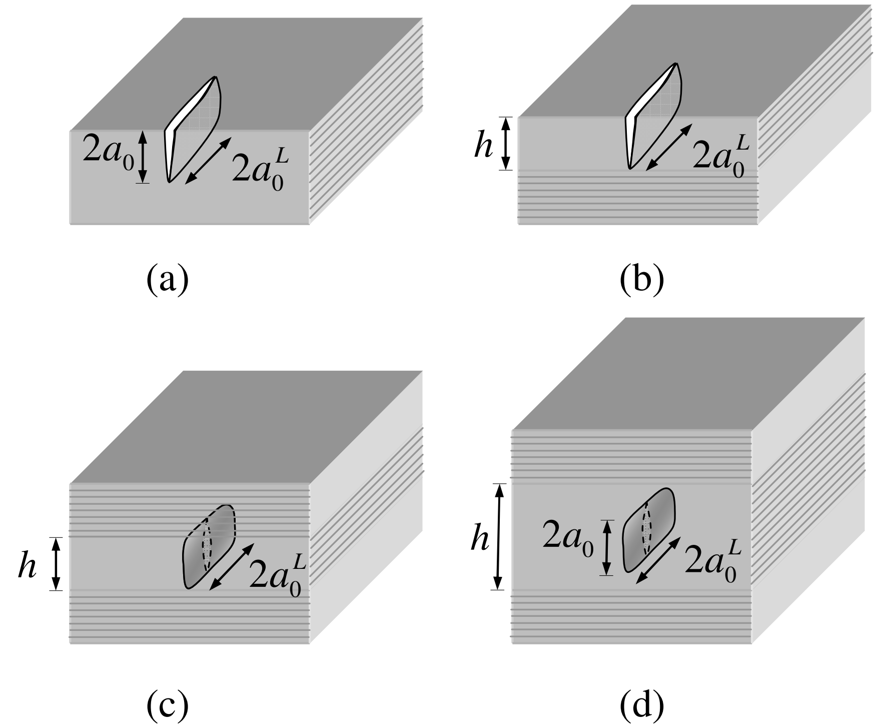


Fig. 4: Slit crack considered for in-situ effects. (a) ply in a UD laminate; (b) thin outer ply; (c) thin embedded ply; (d) thick embedded ply

3. PROPAGATION OF FAILURE

The failure criteria from the previous section have the particularity that they predict the orientation of the fracture plane (or band, in the case of kinking), in addition to the failure event. This particularity is used in the damage propagation model by degrading the traction components precisely on the fracture plane, see ***Fig. 7***. This way, a good physical representation of the actual fracture in the composite is achieved.

Propagation of failure is modelled differently depending on the failure mode predicted. For instance, accumulation of cracks for matrix cracking using a shear-lag model is modelled for a fracture angle $\alpha = 0$, and a wedge effect is simulated for $\alpha > 0$; this is shown in ***Fig. 8***, ***Fig, 9*** and ***Fig. 10***.

The energy absorbed by the model can then be related to the fracture toughness of the failure mode and area fractured. In the finite element implementation of the model, this entails the use of a characteristic length parameter related to the mesh, which has the additional effect of greatly reducing mesh sensitivity. One integration point is used per element, and the characteristic length inside each element is calculated as the ratio of the volume of the element to the area of the fracture surface inside the element.

4. APPLICATION OF THE MODEL

Fig. 11 shows the predicted shear strength and shear strain at failure for a carbon-epoxy. It can be observed that both the shear strength and the shear strain to failure increase with superimposed hydrostatic pressure, with a good agreement between the model and the experiments for both variables. The influence of the transverse stress on the shear response, as predicted by the model, is shown in ***Fig. 12***, presenting good agreement with typical experimental data. The shear response under superposed hydrostatic pressure, as predicted by the model and compared to experimental data, is shown in ***Fig. 13***. A good agreement can be observed.

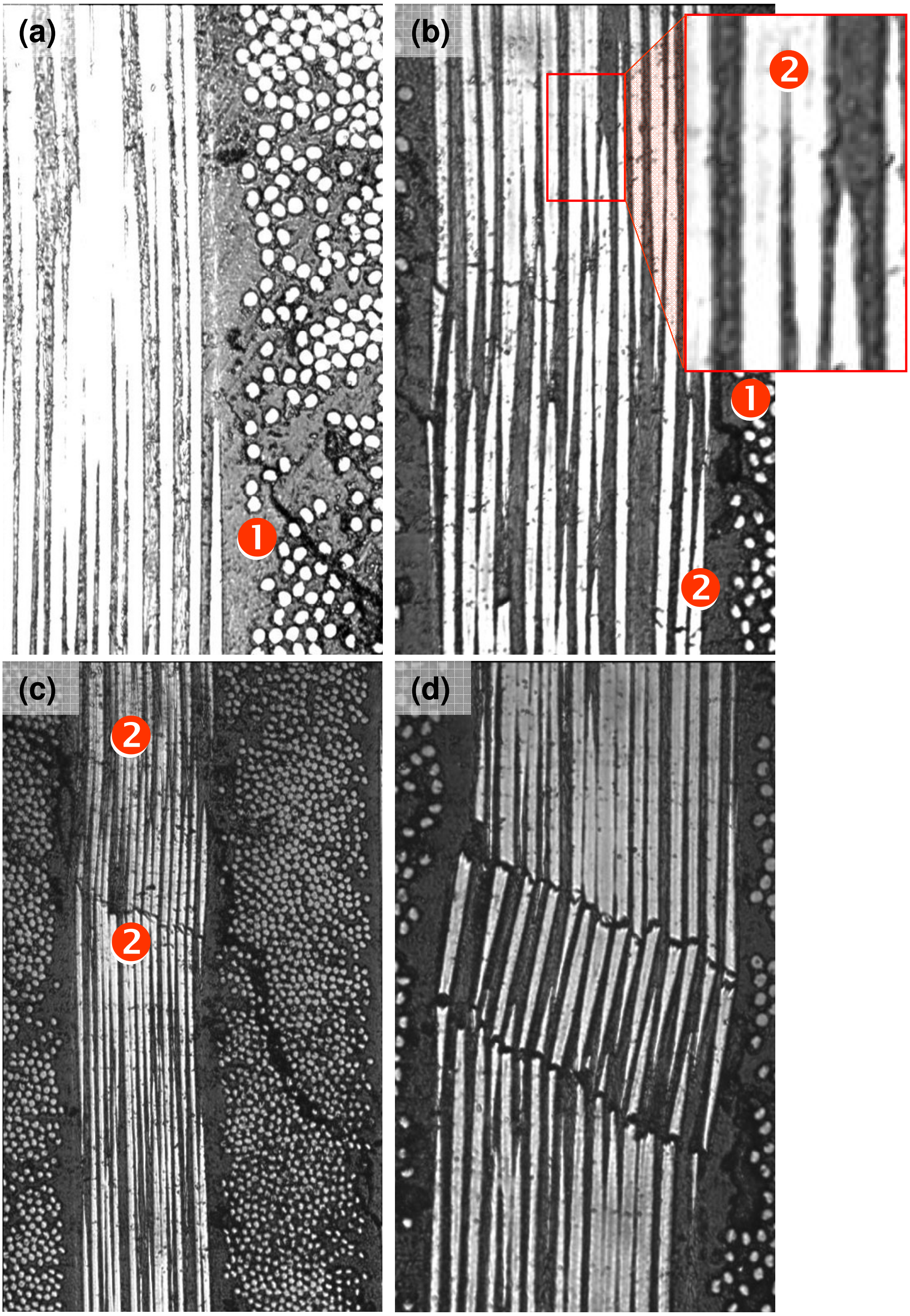


Fig. 5: Sequence of events during kink-band formation. The laminate is being loaded in compression in the vertical direction. (a) Misalignment introduced by a matrix crack in an adjacent layer. (b) Matrix-fibre splitting exists throughout (see zoom); the first fibre failures are indicated. (c) Further fibre failure. (d) Final kink band. Legend: ① Matrix cracking; ② Fibre failure. From [3]

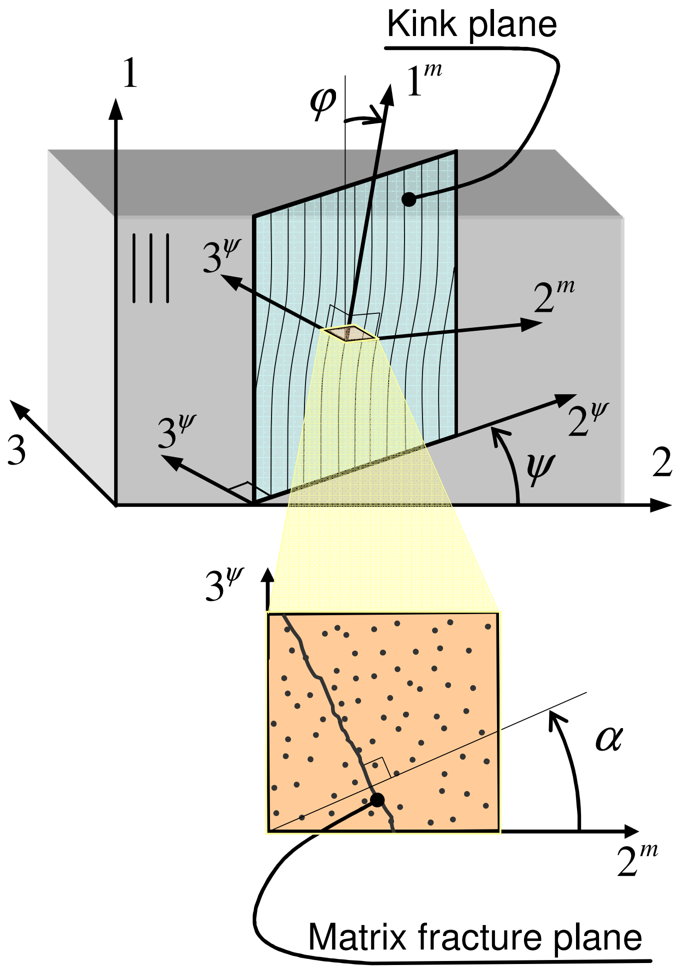


Fig. 6: Physical model for kink-band formation

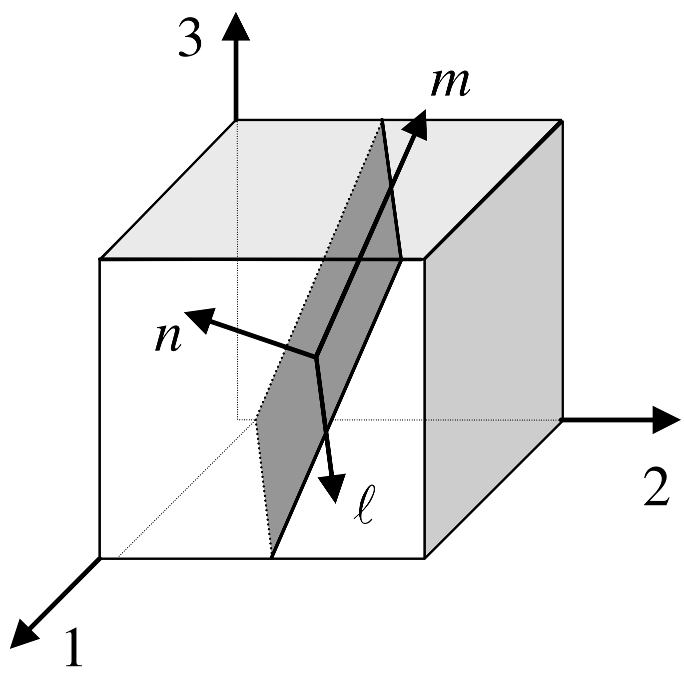


Fig. 7: Coordinate system aligned with the crack

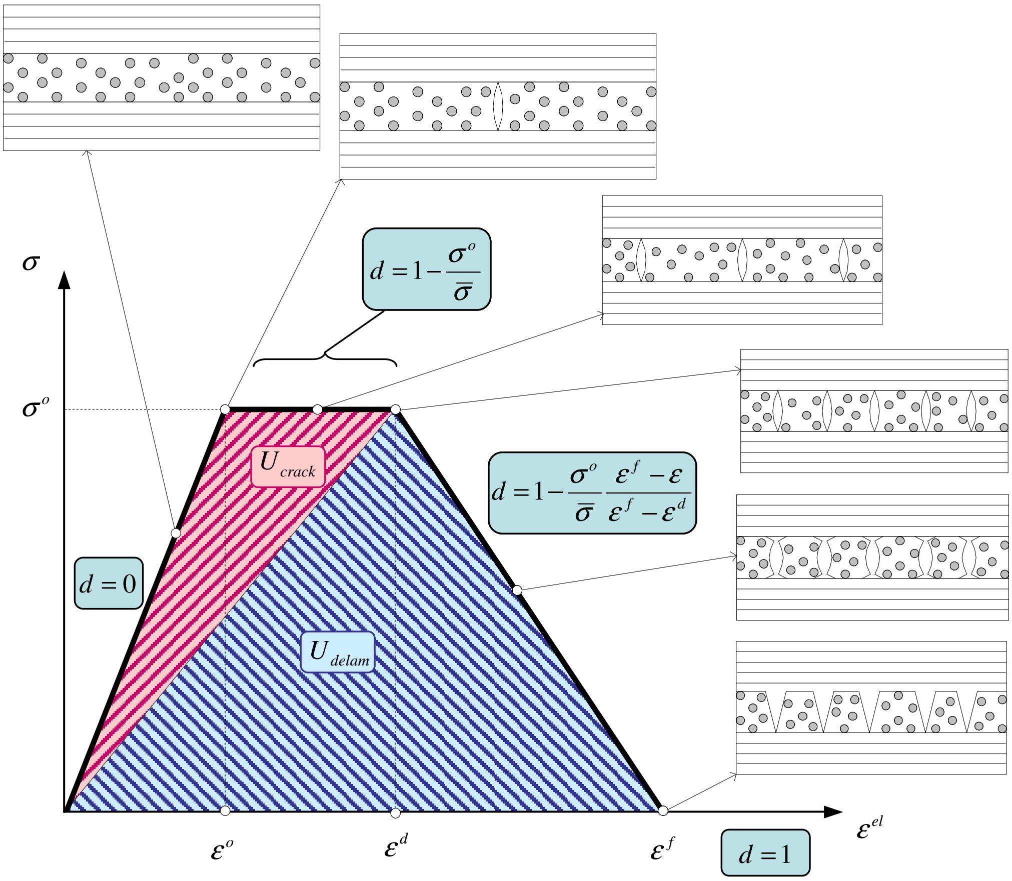
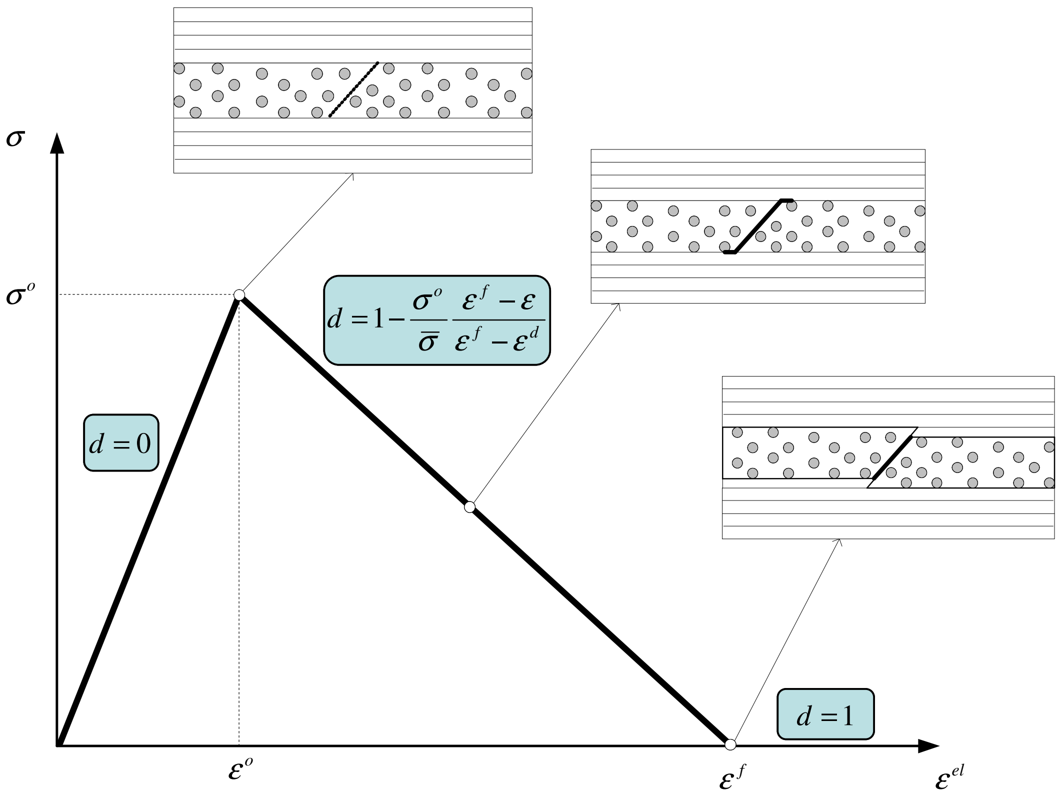


Fig. 8: Progressive cracking for an angle α = 0, from [3]



Fig, 9: Wedge effect for an angle $\alpha \neq 0$, from [3]

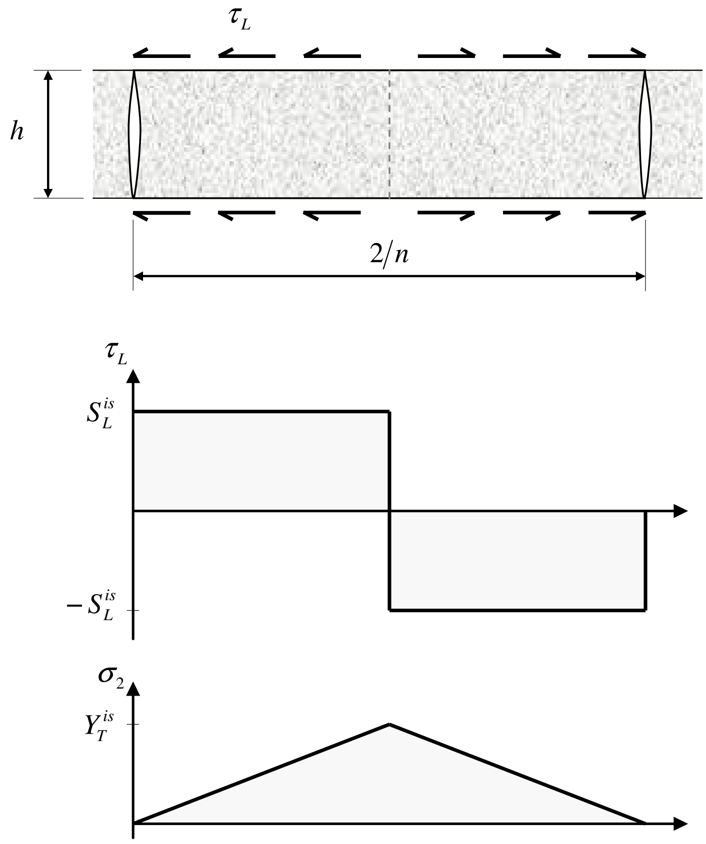


Fig. 10: Shear lag model, [3]

5. CONCLUSIONS

To simulate failure of laminated composites under realistic stress conditions, which might include non-zero hydrostatic pressure, a constitutive model which accurately takes into account the effect of hydrostatic pressure on the material response is absolutely necessary. Without accounting for the effect of hydrostatic stresses on the constitutive response, the stresses in the laminate will not be accurately calculated, for a given imposed strain, and any failure predictions will be subjected to error, even if the failure model is particularly accurate.

Failure of laminated composites is complex, and the prediction of when failure is going to happen requires considerable insight into the physics of the failure mechanisms. Detailed models are required for each failure mechanism. Even for failure mechanisms typically considered to be relatively simple, such as matrix cracking, in-situ effects and interaction between brittle fracture and plasticity lead to complexities that need to be taken into account for predicting failure accurately.

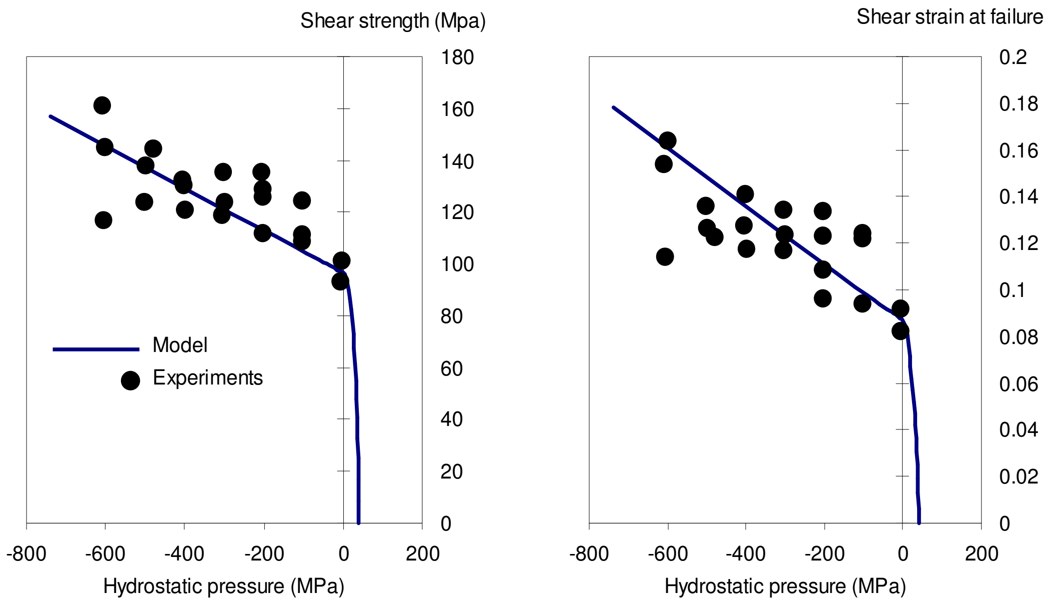
Fibre kinking is arguably the most complex failure mode in laminated composites. This is largely because matrix cracking (with associated complexities), big rotations of the fibres which require non linear geometric models, and the key role of material imperfections for its development.

The accurate kinematic description of the propagation of failure is also complex, and its accurate simulation is a key ingredient for a model successfully modelling the propagation of failure at structural level. Fracture surfaces have to be represented by the failure model, and the correct amount of fracture energy has to be absorbed by the numerical model.

Nevertheless, it is possible to formulate constitutive models which correctly take hydrostatic pressure into account, failure models which are based on robust physical models for each failure mode, and propagation models based on smeared crack approaches which address the dissipation of fracture energy while tackling mesh dependency. The models need further developments and more experimental insight is still desirable, but physically-based descriptions of failure have already established themselves as the way ahead for describing failure in laminated composites.

6. ACKNOWLEDGEMENTS

The contributions from Dr Dávila and Dr Schuecker from NASA Langley Research Center through many discussions on the developments presented in this paper are gratefully acknowledged.



(a) (b)

Fig. 11: (a) Shear strength and (b) shear strain at failure for a carbon-epoxy. Experimental data from [5]

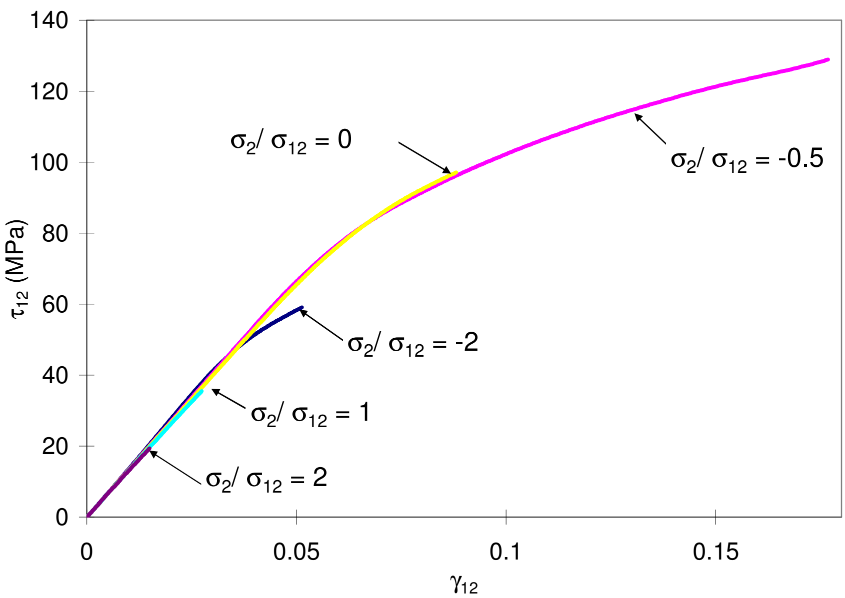


Fig. 12: Influence of the transverse stress on the shear response, as predicted by the model

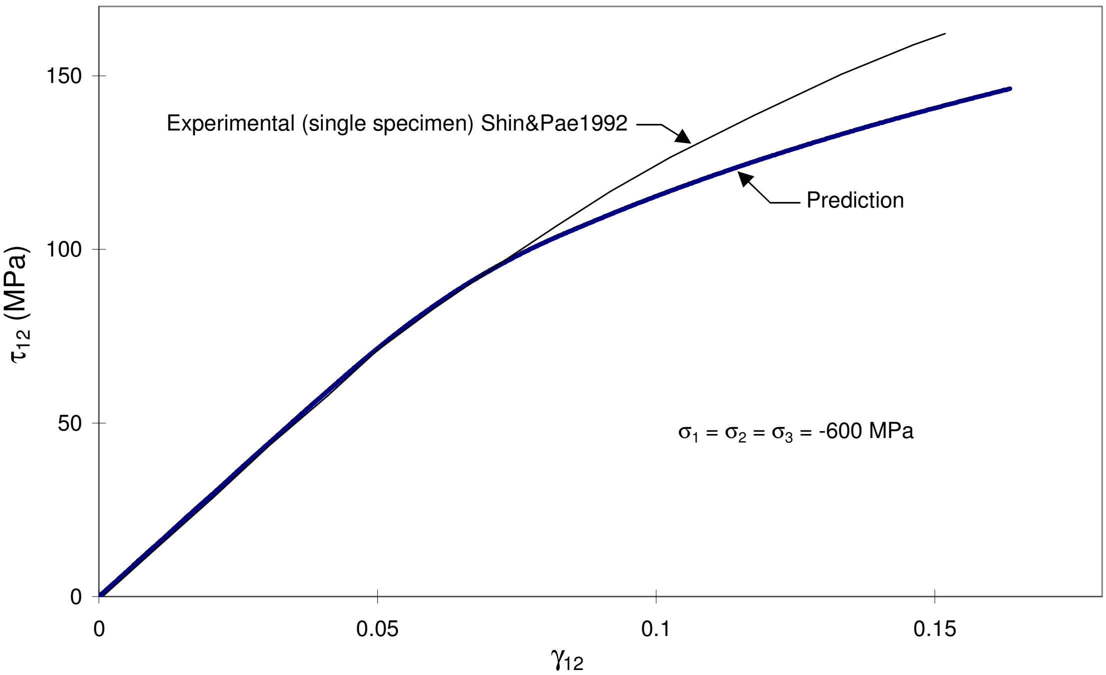


Fig. 13: Shear response under superposed hydrostatic pressure, as predicted by the model and compared against experimental data

REFERENCES

1. Pinho, ST, Dávila, CG, Camanho, PP, Robinson, P, Iannucci, L. Failure Models and Criteria for FRP Under In-Plane or Three-Dimensional Stress States Including Shear Non-Linearity. NASA/TM-2005-213530. NASA Langley Research Center. Hampton, VA 23681, 2005.
2. Pinho, S.T., L. Iannucci, and P. Robinson, Physically based failure models and criteria for laminated fibre-reinforced composites with emphasis on fibre kinking. Part II: FE implementation. Composites Part A: Applied Science and Manufacturing, 2006. 37(5): p. 766-777.
3. Pinho ST, Darvizeh R, Robinson P, Schuecker C, Camanho PP. Material and structural response of polymer-matrix fibre-reinforced composites. Accepted contribution to the 2nd World-Wide Failure Exercise, November 2007. To be published.
4. Hoppel CP, Bogetti TA, Gillespie JW. Literature review - Effects of Hydrostatic Pressure on the Mechanical Behavior of Composite Materials. Journal of Thermoplastic Composite Materials 1995;8:375-409.
5 Shin EES, Pae KD. Effects of hydrostatic pressure on in-plane shear properties of graphite/epoxy composites. Journal of Composite Materials 1992;26(6):828-868.