REASONABLE KNOCKDOWN FACTORS FOR SANDWICH FACE SHEET WRINKLING

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ABSTRACT

Design rules and formulas used to predict wrinkling failure of sandwich structures often include a knockdown factor to compensate for "imperfections" in the sandwich structure. The knockdown is usually based on experience and empirically derived from compression testing. It can be substantially large and clearly affect the design. For example the most widely used Hoff's formula suggests a reduction of 45%, from 0.91 to 0.5, due to the effect of imperfections. The wrinkling formulae are also often simplified, thus neglecting the effect of the stacking sequence of the face sheet laminate. These types of rough “rule of thumb” design constraints must be re-evaluated and questioned. Today’s modern design methods and highly optimised structures seek a weight reduction of the few last percent. This demands a higher level of accuracy in the design process and yesterday’s methods must be improved to meet the demand.

In this paper an alternative method to derive a reasonable knockdown factor is presented and it is shown that this factor can also be predicted based on constituent material properties and assumed amplitude of initial imperfections. The design method previously published by the authors in [1] is extended to fully anisotropic materials and a first ply failure criterion. The results have thus far been confirmed by tests and good agreement has been achieved.

KEYWORDS:
wrinkling, buckling, imperfections, compression strength, stability, sandwich.

NOMENCLATURE:

D_f	Bending stiffness of the face sheet
E_c	Young’s modulus of the face core
E_f	Young’s modulus of the face sheet
G_c	Shear modulus of the core
P	Applied load
P_{Allen}	Critical wrinkling load according to Allen

P_{cr}	Critical load
$P_{Plantema}$	Critical wrinkling load according to Plantema
P_{φ}	Applied load in each φ direction
$P_{\varphi,cr}$	Critical load in each φ direction
$P_{\varphi,face}$	Load in φ direction necessary to cause face failure
$P_{\varphi,core}$	Load in φ direction necessary to cause core failure
$P_{\varphi,cr}$	Critical wrinkling load in each φ direction
t_f	Face thickness
w_0	Shape of the initial wrinkling wave
W_0	Amplitude of the initial wrinkling wave
w_t	Shape of amplified wrinkling wave
l	Natural wrinkling wavelength
α	Fibre angle
σ_{Hoff}	Critical face sheet wrinkling stress according to Hoff and Mautner
σ_z	Stress perpendicular to face sheet
φ	Strip direction
φ_{cr}	Critical wrinkling direction
v_c	Poisson's ratio of the core
λ	Load factor
λ_{cr}	Critical Load factor

1. INTRODUCTION

Wrinkling is a failure mode specific to sandwich structures. It is associated with the loss of stability of the face sheets under compressive loading. Wrinkling is typically critical for a sandwich with relatively thin and stiff face sheets on a thick core not providing sufficient support to the face sheet. One of the papers most commonly referred to on wrinkling is the work by Hoff and Mautner [2] where the Hoff's formula (1) is presented.

$$\sigma_{Hoff} = 0.5\sqrt[3]{E_f E_c G_c} \tag{1}$$

This formula includes a knockdown factor and the 0.5 constant should according to the analytical assumptions by Hoff and Mautner be 0.91. Hoff and Mautner did however perform some test and found that their derived formula was non-conservative and therefore suggested the present knockdown factor. The Hoff's formula is still widely used in composite industry and recommended by for example DNV [3] as well as the ISO standard [4] for design of small crafts. Although the DNV High-Speed LightCraft rules are the most developed of similar ship design rules, they still need to be refined for wrinkling failure analysis.

Wrinkling has also been addressed by authors like Plantema [5], see equation (2), and Allen [6], equation (3) and (4), who derived the wrinkling load by similar means but incorporated the effect of the local bending stiffness of the face sheet. Plantema assumed an exponential decay function for the core stress while Allen used Airy's stress function.

$$P_{Plantema} = \frac{3}{2}\sqrt[3]{2D_f E_c G_c} \tag{2}$$

$$P_{Allen} = \left(\frac{1}{2^{\frac{2}{3}}} + 2^{\frac{1}{3}} \right) \frac{1}{\pi^2} \sqrt[3]{D_f a^2} \approx 0.88 \sqrt[3]{D_f a^2} \qquad (3)$$

$$\text{where } a = \frac{2\pi E_c}{(3-\nu_c)(1+\nu_c)} \text{ and } l = \sqrt[3]{\frac{2\pi^4 D_f}{a}} \qquad (4)$$

It has previously been shown by the authors, [7] and [8], that the local bending stiffness should be included in the wrinkling analysis if anisotropic layered face sheets are used. The authors have also addressed the effect of multi-axial loading on wrinkling of sandwich panels [8] as well as the effect of imperfections [1]. Key points of this papers author's previous work on wrinkling are presented in the following sections.

2. MULTI AXIAL LOADING

In [8] it was shown that by using strip theory it was possible to predict at which angle and at which load level a multi-axially loaded sandwich panel would fail in wrinkling. The theory uses traditional composite laminate theory, see [9], to compute the in plane bending stiffness, D_f, for each strip direction, φ. This is used together with formulas like (2) or (3) to derive the critical compressive load in each direction $P_{\varphi,cr}$. The applied load can also be calculated for each direction, P_φ, and the ratio between the applied load is thereafter evaluated for each strip. At the angle, φ_{cr}, where the ratio $P_\varphi / P_{\varphi,cr} = \lambda$ has its maximum, λ_{cr}, wrinkling occurs. The result of an example analysis is shown in Figure 1.

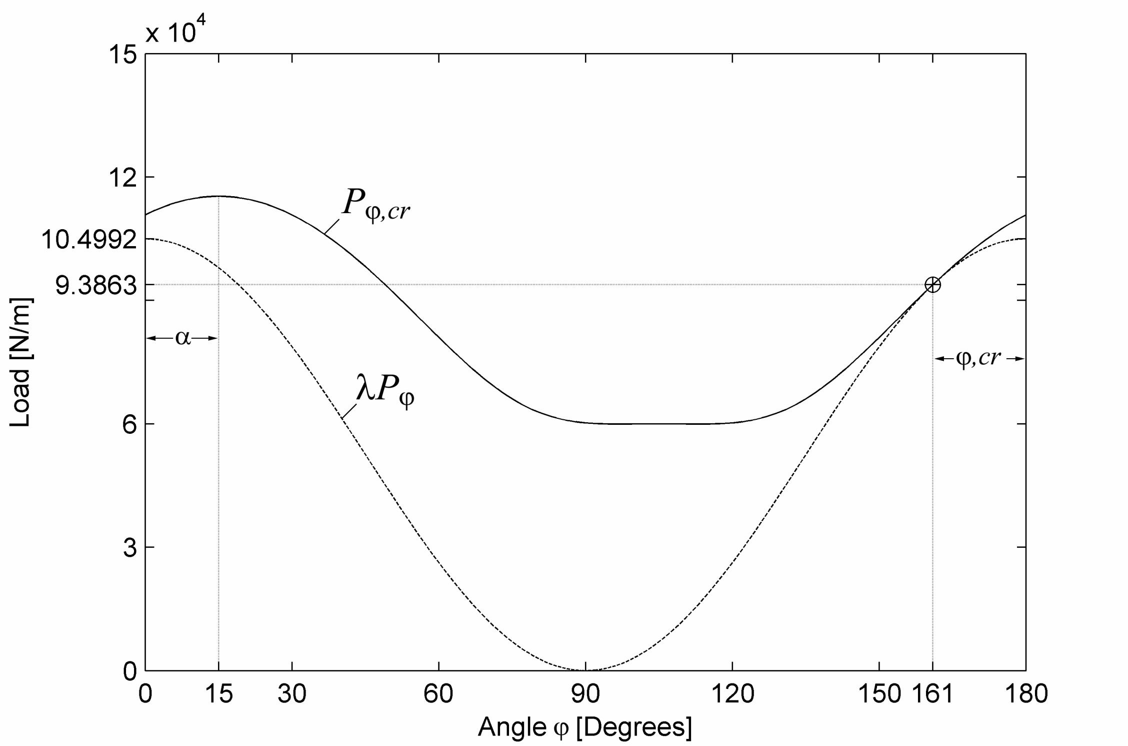

Figure 1. Analysis example of a uni-axially loaded sandwich plate with a compressive load applied with an angle of 15° (α in figure) compared to the fibres. $P\varphi,cr$ and $\lambda P\varphi$ is plotted versus φ and $\varphi_{,cr}$ is illustrated in the figure. Reproduced from [8].

3. EFFECT OF IMPERFECTION

In [1] the classical wrinkling problem is transformed from stability of a perfect sandwich structure to predicting actual failure strength of a sandwich structure with initial imperfections. This is achieved by combining classical wrinkling theories with first order extension of finite deformations. Basically this is performed assuming an imperfection with the same shape as the wrinkling wave but with small initial amplitude, see equation (5).

$$w_0 = W_0 \sin\left(\frac{\pi x}{l}\right) \tag{5}$$

This initial wrinkling wave is amplified by the applied load and corresponding strain as a function of load relations can be calculated, see equation (6) and (7). These formulas are derived using the same amplification formula as Timoshenko [10] and Brush and Almroth [11] used to predict instability of imperfect struts.

$$\varepsilon_f = \frac{\pi^2 t_f}{2l^2} w_0 \frac{P}{(P_{cr} - P)} + \frac{P}{t_f E_f} \tag{6}$$

$$\varepsilon_c = \frac{\sigma_z}{E_c} = \frac{a}{l\,E_c}(w_t - w_0) = \frac{a}{l\,E_c} W_0 \frac{P}{P_{cr} - P} \tag{7}$$

Thereafter the strength data of the constituent materials is used to evaluate at with load the sandwich fails and if it is the core or face sheet that fails first. Using this method it is possible to create both failure load charts for specific sandwich configurations as well as estimate the “efficiency” of the combination. In the rest of this paper Efficiency is defined as the ratio of maximum load carrying capacity of the imperfect structure compared to capacity predicted using traditional formulae for wrinkling or pure compression failure respectively, see equation (8).

$$Efficiency = \frac{P_{failure}}{P_{traditional}} \tag{8}$$

The result for an example analysis of a uni-axially loaded sandwich panel with initial imperfections is shown in Figure 2. The figure also includes results from actual testing of the analysed material combination.

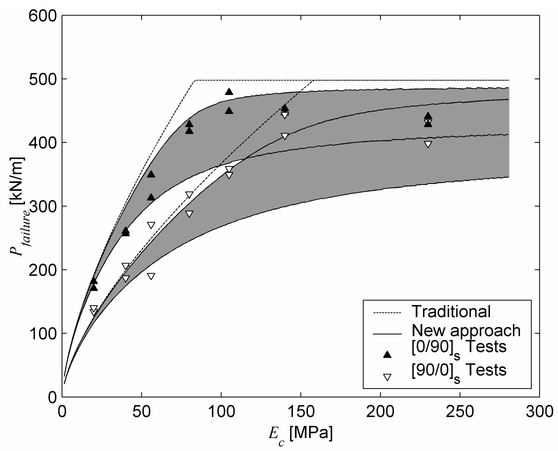

Figure 2. Developed theory compared to test results. [0/90]s and [90/0]s carbon fibre vinylester face sheet on Divinycell H-grade core material. Black triangles pointing up shows test results for [0/90]s specimens and white triangles pointing down show test results for [90/0]s specimens.

Dashed lines are traditional theory (wrinkling and compression failure) and solid lines is the developed theory. The upper solid line for each material configuration corresponds to an initial imperfection amplitude of 0.01 mm and the lower line in each pair to 0.25 mm. Reproduced from [1].

4. MULTI-AXIALLY LOADED SANDWICH PANEL WITH IMPERFECTIONS

It is possible to combine the previously presented methods by using the strip theory for multi-axial loading in conjunction with the theory for the effect of imperfections. This is achieved by assuming an imperfection shaped like a sinusoidal wrinkling wave with initial amplitude W_0, see equation (5), in the length direction of each φ direction strip.

By using the same amplification function for the imperfection amplitude it is possible to decide at which load the face sheet fails for each angle φ. To be able to compute the critical failure load for layered face sheets the theory presented in [1] is expanded with a first ply failure criterion and the maximum strain is hence evaluated per lamina. The maximum strain in each lamina can be expressed as in equation (9), where i is the laminae index, z_i the maximum distance to the neutral axis of the face sheet and κ the curvature of the face sheet, see equation (10).

$$\varepsilon_{f,\varphi,i} = \frac{P_\varphi}{A_{11,\varphi}} + z_i \kappa_\varphi \tag{9}$$

$$\kappa_\varphi = -\frac{\pi^2}{l^2} W_0 \frac{P_\varphi}{P_{Allen,\varphi} - P_\varphi} \tag{10}$$

Finding the actual critical load for the face sheet, $P_{\varphi,face}$, is solved numerically. The critical load for the core is also computed for each angle φ. This is accomplished in the same manner as presented in [1] but here expanded to strip theory and evaluated per angle φ.

$$P_{\varphi,core} = \frac{P_{Allen,\varphi}}{1 + \frac{a W_0}{l E_c \hat{\varepsilon}_c}} \tag{11}$$

The minimum critical load for face and core respectively is used as the critical load for the strip.

$$P_{\varphi,cr} = \min(P_{\varphi,face}, P_{\varphi,core}) \tag{12}$$

It should be noted that the presented approach does not include effects of non-zero ***B*** matrix (bending coupling matrix). Hence the theory is only valid for symmetric lay-up sequences. The theory also neglects the effect of stress perpendicular to each evaluated strip as well as shear stress on each strip.

5. PRACTICAL EXAMPLES

To show the effect of the assumed imperfections and proposed approach a uni-directional carbon fibre laminate on a ductile core can be used as an example. The same configuration has been used in a previous paper [8]. Figure 3 shows both the unreduced critical wrinkling load (topmost dashed line) and the reduced load (solid line) for each angle φ. It is apparent that the imperfection theory both reduces the maximum load carrying capacity and also affects the wrinkling angle.

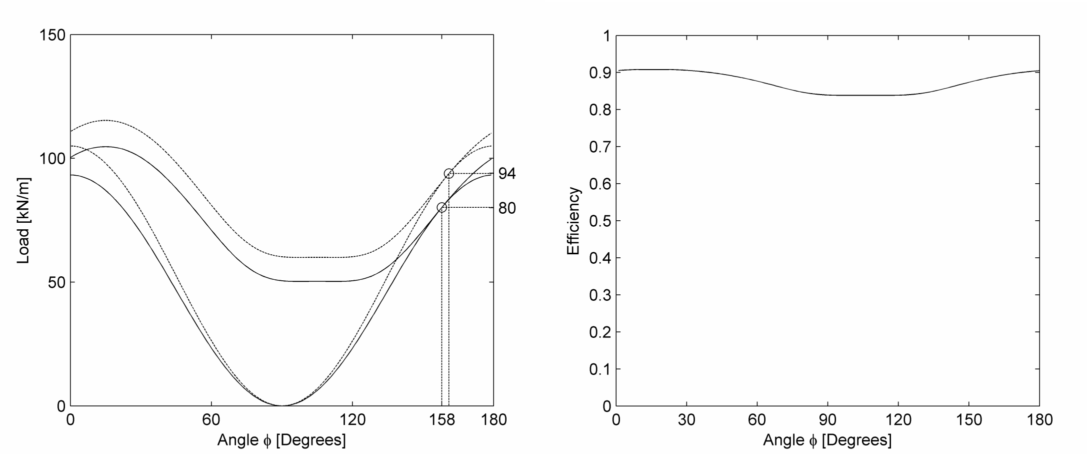

Figure 3. Analysis example of a uni-axially loaded sandwich plate with a compressive load applied with an angle of 15° compared to the fibres. Left figure show $P_{\varphi,cr}$ and λP_{φ} plotted versus φ. φ_{cr} is also illustrated in the figure. Right figure shows efficiency for the current configuration example.

The same material configuration was tested for different lay-up angles α and the results of these test has previously been presented in [8]. Using the here suggested approach the following result is obtained, see Figure 4.

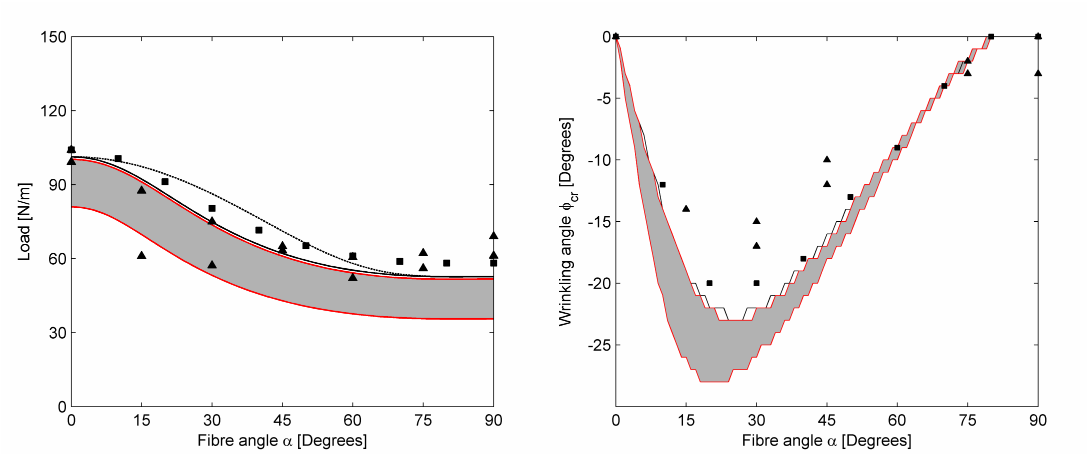

Figure 4. Example of a uni-axially loaded sandwich panel with thin carbon fibre face sheets. The left figure shows the maximum load as a function of lay-up angle and the right figure the angle between applied load and wrinkling failure (skew wrinkling). Triangles in the figure represent test results and squares FE calculations. Shaded sections are predicted failure region obtained by analysing the configuration for two different initial imperfection amplitudes, 0.01 mm and 0.15 mm.

From Figure 4 it is seen that by using the approach with initial imperfections it is possible to predict the maximum load carrying capacity and by assuming a reasonable imperfection amplitude it is possible to do this conservatively without resorting to coarse and drastic knockdown factors where they are not called for. The correlation between tests and the developed theory is better for the critical load than for wrinkling angle. This is possible due to that the presented theory does not include boundary effects apparent both in test and in FE calculations. The boundaries of the test specimens and FE-models make the wrinkling wave more parallel to the edges than the developed theory predicts. This in not surprising since the theory assumes periodic wrinkling waves without interfering boundaries (i.e. infinite plate).

Another test series from a previous publication [8] are compared with the results from the imperfection theory in Figure 5. Herein the failure load for a multi-axially loaded sandwich panel is plotted versus the load ratio, $r=N_y/N_x$. Examining the graphs show that the proposed theory captures the behaviour of both failure load and wrinkling angle very well.

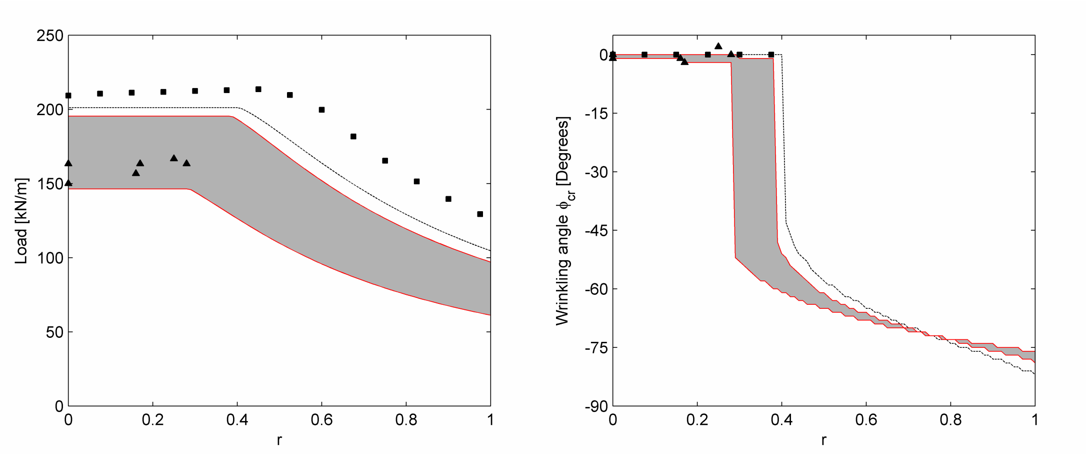

Figure 5. Example of a multi-axially loaded sandwich panel with thin carbon fibre face sheets. The left figure shows the maximum load as a function of relation between principal and perpendicular load. The right figure shows the wrinkling angle. Triangles in the figures represent test results and squares FE calculations. Shaded sections is predicted failure region obtained by analysing the configuration for two different initial imperfection amplitudes, 0.01 mm and 0.15 mm.

6. DISCUSSION AND FUTURE POSSIBILITIES

The developed approach is a powerful tool enabling studies of load carrying capacity of compressible loaded sandwich panels. Several phenomena not obvious while examining traditional wrinkling or compression failure formulae can be explored by straightforward parametric studies. For example efficiency plots or knockdown maps can easily be extracted for different material combinations and the sensitivity to imperfections can be explored. Figure 6 shows the efficiency (see equation (8)) for the previously studied thin, uni-directional, carbon fibre laminate on a low density brittle core. The figures show efficiency as function of face thickness and core modulus.

From both figures (3D surface and contour plot) the canyon of low efficiency around the traditional design point, where traditional formulae predict the same failure load for wrinkling and compression, are clearly visible. The low efficiency along this chasm also indicates a high sensitivity to imperfections in this material combination. From the plots it is also clearly visible that imperfection sensitivity decreases with thicker face sheets. The contour plot can be used as a knockdown map for the analysed material combination.

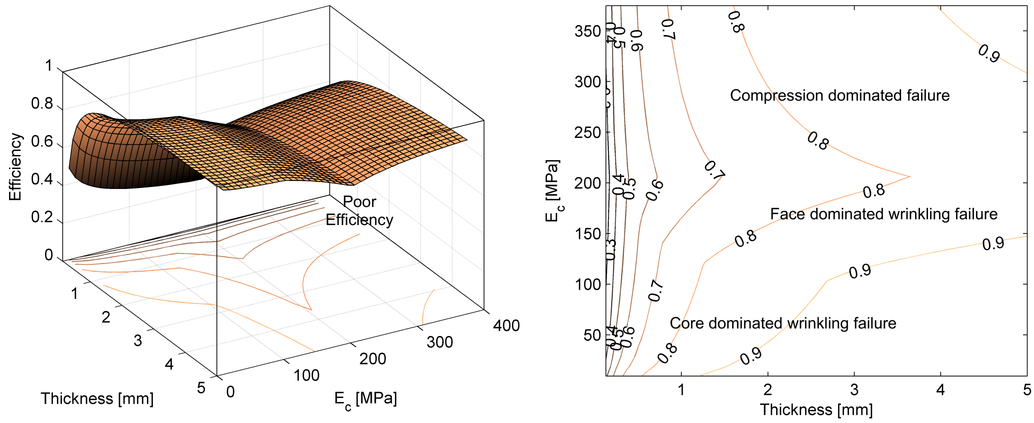

Figure 6. Efficiency plots for a uni-axially loaded sandwich panel with a single layer carbon fibre face sheet. Both plots show efficiency as a function of shell thickness and core modulus.

For example a suitable efficiency factor for a 3 mm thick face sheet on a 200 MPa core is close to 0.8, i.e. the wrinkling load according to Allen should be multiplied with 0.8 to achieve a reasonable failure load. It should be noted that the efficiency applies both to wrinkling failure as well as compression failure but only in the applicable region of the map. That means that (for this face sheet) at higher core modulus than 200 MPa the compression failure load should be multiplied with the efficiency factor and at lower core modulus the Allen wrinkling load should be multiplied with the predicted efficiency. By using the theory presented in this paper and a suitable analysis method the failure type prediction is included in the progress of calculating the failure load.

It is also possible to plot $P_{\varphi,cr}$ as function of φ for different material combinations showing the importance of stacking sequence as well as imperfection sensitivity, see Figure 7. For example, one can see that the imperfection sensitivity is larger for a thinner laminate (left in Figure 7).

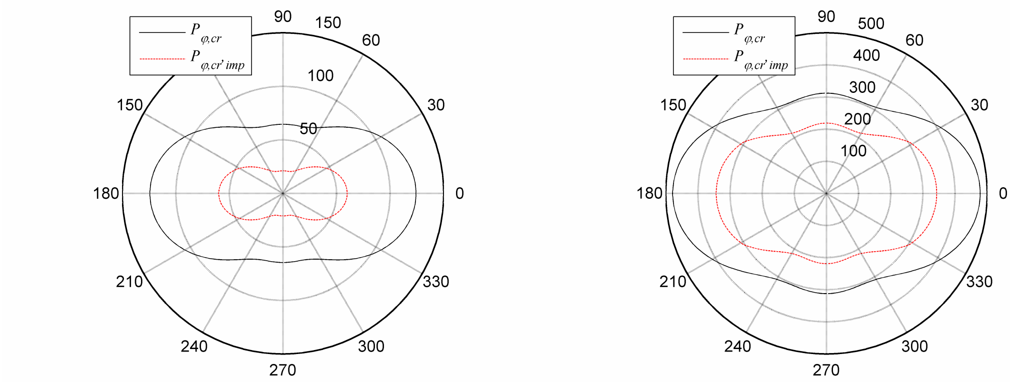

Figure 7. $P_{\varphi,cr}$ and $P_{\varphi,cr,imp}$ for two different sandwich configurations with 0.1 mm initial imperfection on a lightweight brittle core. The left figure shows critical load for [0] (single laminae with 0.25 mm thickness) and the right $[0\ 90]_s$ carbon fibre face sheets.

7. CONCLUSIONS

By using the presented method it is possible to predict a reasonable failure load for a compressible loaded sandwich structure. The method includes the effect of material strength and amplitude of initial imperfections.

It is also shown that the knockdown factor to be used when analysing wrinkling is not a constant but depending on material combination, strength of constituent materials, direction of load, stacking sequence etcetera. Using simplified analytical functions (i.e. Hoff's formula) in wrong situations can result in poor design in some cases and in premature failures in others.

By using the proposed theory it is possible to predict reasonable knockdown factors to be used for local compression failure of sandwich panels taking following into account; material combination (stiffness, strength etcetera), anisotropic layered face sheets, multi-axial load and initial imperfection amplitude.

8. ACKNOWLEDGEMENT

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