NONLINEAR MECHANICAL RESPONSE OF CFRP LAMINATES UNDER OFF-AXIS TENSILE LOADING

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Abstract
Effects of tensile loading angle on the stress-strain behavior in CFRP laminates are investigated experimentally. A material system used is T300/2500 carbon/epoxy system. Both a cross-ply and a woven laminates are used. The nonlinear behavior in the stress-strain curve is focused. To divide the total strain into the elastic and inelastic ones, loading-unloading tests are performed. Plasticity models are made to characterize the nonlinearity in the stress-strain curves.

1. INTRODUCTION

It is known that fiber composites exhibit a nonlinear stress-strain response under off-axis loading. Many mechanical models have been proposed to model the nonlinear behavior of composites, using two approaches, one macroscopic and the other microscopic. In the macroscopic approach, composites are treated as a nonlinear elastic or plastic body. In the microscopic approach, attempts are made to describe the effective composite response using the properties of the fiber and matrix.

Hahn and Tsai [1] employed a complementary elastic energy density function which contained a biquadratic term for in-plane shear stress. The nonlinear stress-strain relation of unidirectional laminae under off-axis loading was predicted. Sun and Chen [2] developed the one-parameter plasticity model to describe the nonlinear behavior of unidirectional composites based on a more general approach [3], based on a quadratic plastic potential and the assumption that there is no plastic deformation in the fiber direction. Ogi and Takeda [4] proposed a model based on a fourth-order complementary elastic energy function and the one-parameter plastic potential in which an anisotropy parameter changes with plastic deformation. Tamuzs et al. [5] applied the Sun and Chen's one-parameter plasticity model to the deformation of a composite with complex microstructure and found that the nonlinear response of the composite under creep and cyclic loading follows the associated flow rule with the one-parameter plastic potential function.

Aboudi [6, 7] used a cell model in which the fibers have a rectangular cross section. He introduced material plastic behavior into the cell model by using the unified theory of plasticity proposed by Bodner [8]. For resin matrix composites, he employed a Ramberg-Osgood representation for matrix nonlinear elasticity. Dvorak and Bahei-El-Din developed a method to describe the elastic-plastic behavior of unidirectional composites [9] and multidirectional composites [10] consisting of aligned, continuous elastic fiber and elastic-plastic matrix. The composite is modeled as a continuum reinforced by cylindrical fibers of vanishingly small diameter which occupy a finite volume fraction of the aggregate. Sun and Chen [11, 12] developed a simple micromechanical model of elastic-plastic behavior of fibrous composites. In the model, the fiber is assumed to be linearly elastic and the matrix elastic-plastic following the J_2-flow rule. The micromechanical model is used to calculate stress-strain curves for off-axis boron/aluminum composites. These curves are used, in turn, to model a macromechanical orthotropic plasticity response [13].

The models above are basically unidirectional composites. Compared to the models for unidirectional composites, there are few models of the plastic behavior of multidirectional and woven composites. Varzi et al. [14] suggested a plasticity model for bidirectional composite laminates that requires knowledge of the axial ad shear yield strength, which can be difficult to define and to obtain experimentally for composite materials. Naik [15] suggested using an empirical relationship between shear stress and shear strain. Odegard et al. [16] suggested a very simple plastic potential function for woven graphite/PMR-15 composite.

This paper describes the nonlinear behavior of a cross-ply and a woven carbon/epoxy composite laminates under off-axis tension. Tensile tests were performed at off-axis angles, 15°, 30°, and 45°, as well as 0° and 90°. We take the starting point the general 2-D quadratic plastic potential function. To divide the total strain into the elastic and inelastic ones, loading-unloading tests were also performed. A macroscopic modeling was attempted to the inelastic strains by using the plasticity approach.

2. ANALYTICAL PROCEDURE

General 2-D Plasticity Model

A yield function that is quadratic in stress is assumed for the general 3-D fiber composite as

$$\begin{aligned} 2f\left(\sigma_{ij}\right) = {} & a_{11}\sigma_{11}{}^2 + a_{22}\sigma_{22}{}^2 + a_{33}\sigma_{33}{}^2 \\ & + 2a_{12}\sigma_{11}\sigma_{22} + 2a_{13}\sigma_{11}\sigma_{33} + 2a_{23}\sigma_{22}\sigma_{33} \\ & + 2a_{44}\sigma_{23}{}^2 + 2a_{55}\sigma_{13}{}^2 + 2a_{66}\sigma_{12}{}^2 = k \end{aligned} \tag{1}$$

where k is a state variable and the stresses σ_{ij} refer to the stresses in the principal material directions.

By using the associated flow rule, the yield function is taken as the plastic potential function from which the incremental plastic strain can be derived as

$$d\varepsilon_{ij}^{p} = \frac{\partial f}{\partial \sigma_{ij}} d\lambda \tag{2}$$

where superscript p denotes plasticity, and $d\lambda$ is a proportionality factor. The increment of plastic work per unit volume is given by

$$dW^{p} = \sigma_{ij} d\varepsilon_{ij}^{p} = 2 f d\lambda \tag{3}$$

Let the effective stress be defined as

$$\bar{\sigma} = \sqrt{3f} \tag{4}$$

The effective plastic strain increment can be defined such that

$$dW^{p} = \sigma_{ij} d\varepsilon_{ij}^{p} = \bar{\sigma} d\bar{\varepsilon}^{p} \tag{5}$$

Substitution of (3) and (4) into (5) yields

$$d\bar{\varepsilon}^{p} = \frac{2}{3} \bar{\sigma} d\lambda \tag{6}$$

and

$$d\lambda = \frac{3}{2}\left(\frac{d\bar{\varepsilon}^{p}}{d\bar{\sigma}}\right)\left(\frac{d\bar{\sigma}}{\bar{\sigma}}\right) \tag{7}$$

Consider a state of plane stress parallel to the x_1-x_2 plane. The plastic potential function reduces to

$$2f = a_{11}\sigma_{11}^{\ 2} + a_{22}\sigma_{22}^{\ 2} + 2a_{12}\sigma_{11}\sigma_{22} + 2a_{66}\sigma_{12}^{\ 2} \tag{8}$$

From (2) and (8), the plastic strain increments are obtained as

$$\begin{Bmatrix} d\varepsilon_{11}^{p} \\ d\varepsilon_{22}^{p} \\ d\gamma_{12}^{p} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & 2a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} d\lambda \tag{9}$$

Off-Axis Tension Test

The complete orthotropic plastic flow rule is defined if the parameters a_{11}, a_{22}, a_{12}, a_{66} and $d\lambda$ are determined. To determine $d\lambda$, the effective stress-effective plastic strain relation must be established. This can be accomplished from the results of tension tests on off-axis specimens.

Let the x-axis be the uniaxial loading direction which makes an angle θ with the fiber direction x_1-axis. This stresses referring to the material principal directions (x_1 and x_2) are related to the applied uniaxial stress σ_x as

$$\left.\begin{aligned}\sigma_{11} &= \sigma_x \cos^2\theta \\ \sigma_{22} &= \sigma_x \sin^2\theta \\ \sigma_{12} &= -\sigma_x \sin\theta\cos\theta\end{aligned}\right\} \tag{10}$$

Substitution of (10) into (8) and (4) yields

$$\bar{\sigma} = h(\theta)\sigma_x \tag{11}$$

where

$$h(\theta) = \sqrt{\frac{3}{2}\left\{a_{11}\cos^4\theta + a_{22}\sin^4\theta + 2\left(a_{12} + a_{66}\right)\sin^2\theta\cos^2\theta\right\}} \tag{12}$$

Using the coordinate transformation law for strains, the plastic strain increment in the loading direction is given by

$$d\varepsilon_x^p = d\varepsilon_{11}^p \cos^2\theta + d\varepsilon_{22}^p \sin^2\theta - d\gamma_{12}^p \sin\theta\cos\theta \tag{13}$$

Substituting (9) into (13) and using the relations (10), (11) and (6), the following relation can be derived.

$$d\bar{\varepsilon}^p = \frac{d\varepsilon_x^p}{h(\theta)} \tag{14}$$

For monotonic loading, the above equation is integrable. Thus,

$$\bar{\varepsilon}^p = \frac{\varepsilon_x^p}{h(\theta)} \tag{15}$$

The relation between the effective stress and effective plastic strain can now be obtained by the experimentally obtained uniaxial stress and plastic strain. Since the effective stress-effective plastic strain curve should be unique in monotonic loading for a given material, the parameters a_{11}, a_{22}, a_{12} and a_{66} must be chosen so that the resulting effective stress-effective plastic strain relations is independent of θ. It can also be shown that the plastic Poisson's ratio defined by the following equation can be expressed by the plasticity parameters and loading angles as

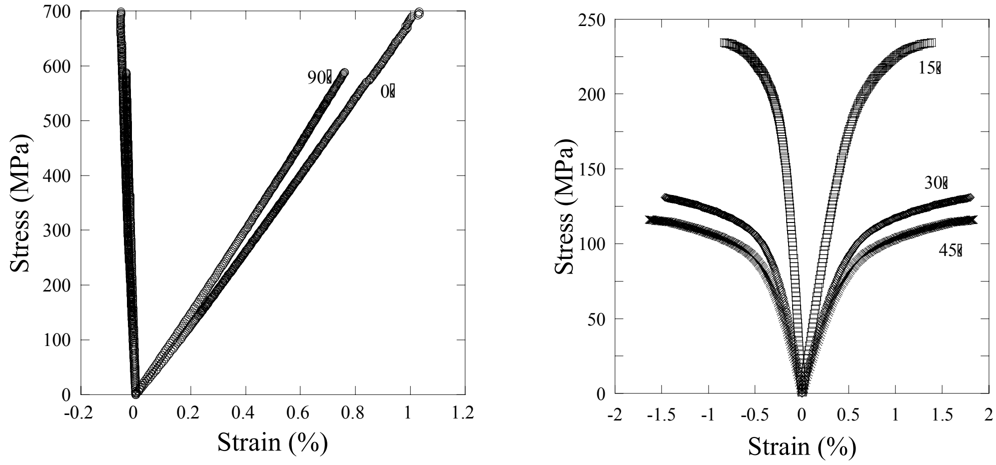
$$\nu_\theta^p \equiv -\frac{d\varepsilon_y^p}{d\varepsilon_x^p} = -\frac{\left(a_{11} + a_{22} - 2a_{66}\right)\sin^2\theta\cos^2\theta + a_{12}\left(\sin^4\theta + \cos^4\theta\right)}{a_{11}\cos^4\theta + a_{22}\sin^4\theta + 2\left(a_{12} + a_{66}\right)\sin^2\theta\cos^2\theta} \tag{16}$$

3. EXPERIMENTAL PROCEDURE

A material system used was T300/2500 carbon/epoxy system. Laminate configurations were cross-ply $(0/90)_s$ and 8-harness satin weave 5-ply laminates. Specimen size was 100mm long and 10mm wide. The thicknesses of the specimens were 0.50mm for cross-ply laminates and 2.0mm for woven laminates. Monotonic tension tests were conducted first for on- and off-axis specimens. The off-axis angles were 15°, 30°, and 45°. To divide the total strains into elastic and inelastic ones, loading-unloading tests were also performed. A biaxial strain gage was mounted at the center of each specimen to measure both the longitudinal and transverse strains. The crosshead speed was 0.5mm/min.

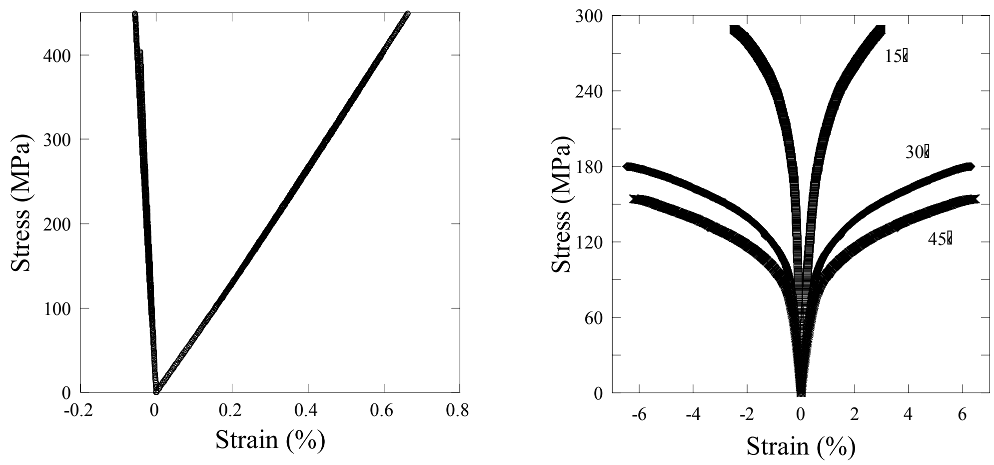
4. EXPERIMENTAL RESULTS AND DISCUSSION

Figure 1 and 2 show stress-strain curves for T300/2500 laminates under various loading directions for cross-ply laminates and woven laminates, respectively. It is seen that the on-axis specimens exhibit hardening nonlinearity which is considered to be the characteristics of the carbon fibers [17]. The off-axis specimens show apparent softening nonlinearity.



(a) On-axis specimens (b) Off-axis specimens

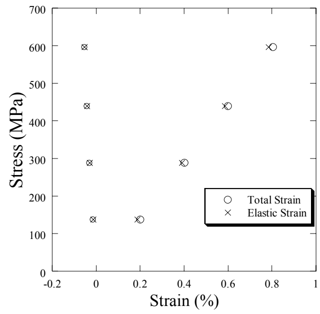
Figure 1. Stress-strain curves for T300/2500 cross-ply laminates under various loading angle ((a) on-axis specimens, (b) off-axis specimens).



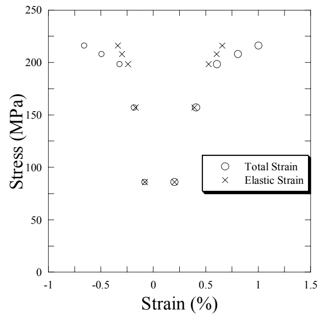
(a) On-axis specimens (b) Off-axis specimens

Figure 2. Stress-strain curves for T300/2500 woven laminates under various loading angle ((a) on-axis specimens, (b) off-axis specimens).

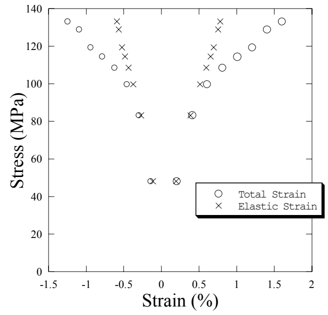
Figures 3 and 4 show the relation between the total stress and elastic stress at some stress levels obtained by the loading-unloading tests for cross-ply laminates and woven laminates, respectively. The stress shown is the maximum stress during a loading cycle and the total strain is the strain at the maximum strain. The residual strain just after the complete unloading was considered to be the inelastic strain. The elastic strain was obtained by subtracting the inelastic strain from the total strain. Inelastic strains are very small in the on-axis specimens. We can see large inelastic strains in off-axis specimens. It should also be noted that the relation between the elastic strain and stress is also nonlinear.



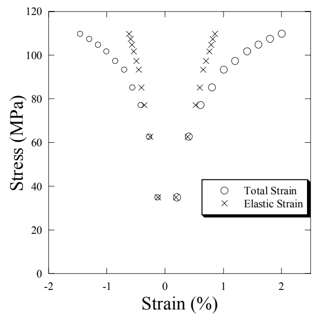
(a) 0°



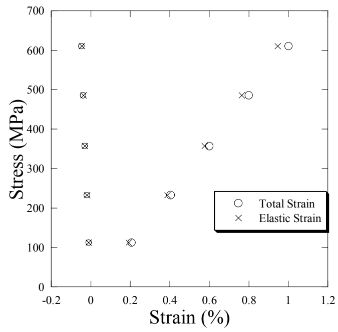
(b) 15°



(c) 30°,

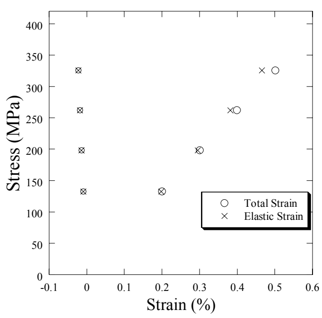


(d) 45°

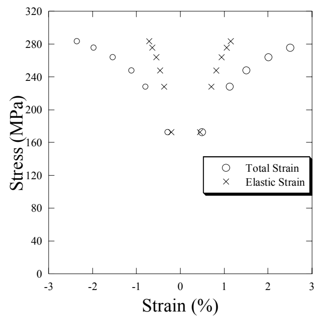


(e) 90°

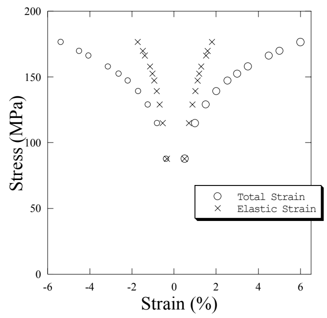
Figure 3. Relation between the total strain and the elastic strain obtained by the loading-unloading tests for T300/2500 cross-ply laminates ((a) 0°, (b) 15°, (c) 30°, (d) 45° and (e) 90°).



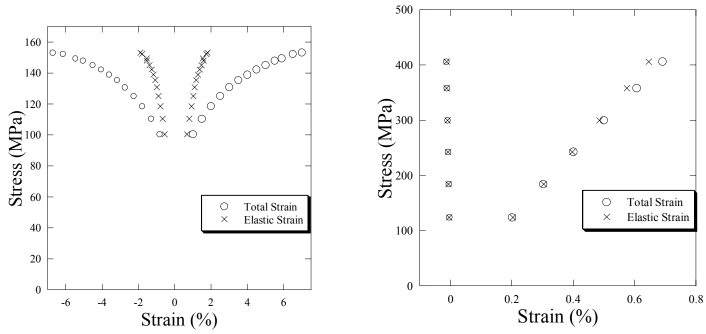
(a) 0°



(b) 15°



(c) 30°

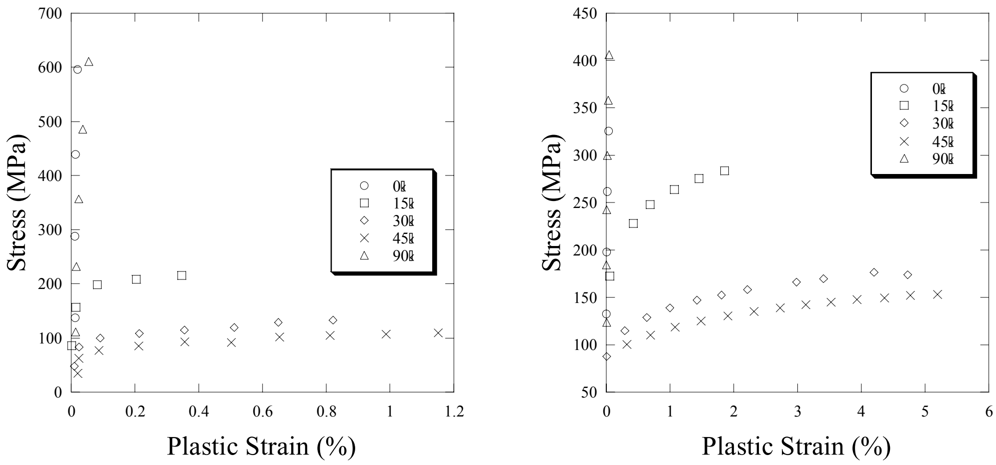


(d) 45° (e) 90°

Figure 4. Relation between the total strain and the elastic strain obtained by the loading-unloading tests for T300/2500 woven laminates ((a) 0°, (b) 15°, (c) 30°, (d) 45° and (e) 90°).

Figures 5 shows the relation between the inelastic strain and the stress ((a) cross-ply laminates, (b) woven laminates). In the figure, we call the inelastic strain the plastic strain. Large plastic stains are measured in off-axis specimens. Based on this experimental data, an attempt is made to characterize the inelastic part of strain using the plasticity approach outlined in the previous section.

If the plasticity model is valid for the laminates considered here, we can have a unique effective stress-effective plastic strain curve for each laminates using proper values for the plasticity parameters, a_{11}, a_{22}, a_{12} and a_{66}. In this model, we have four parameters, however, we can set a_{11}=1 without loss of generality, which results in the number of parameters to determine is three.



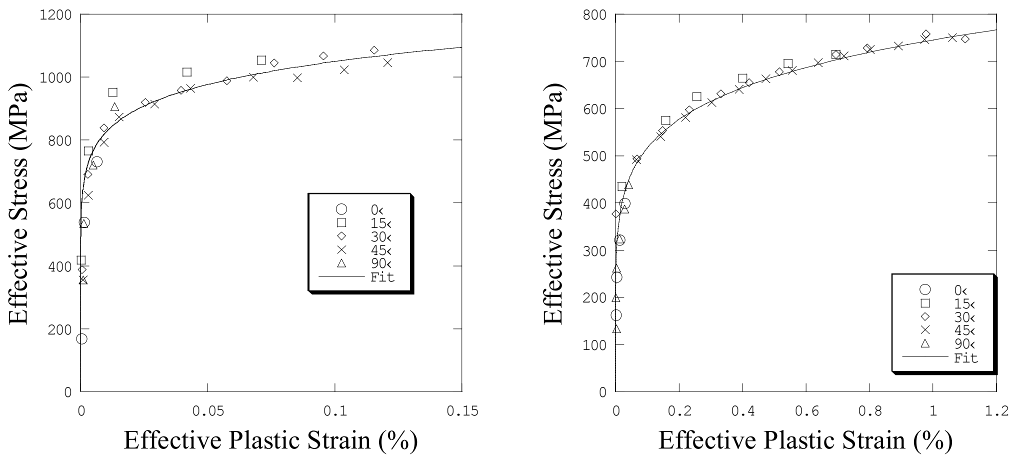
(a) Cross-ply laminates (b) Woven laminates

Figure 5 Relation between the uniaxial stress and the plastic strain ((a) cross-ply laminates, (b) woven laminates).

In this study, we also assume that a_{22}=1, because the laminates considered here have very similar property in x_1 and x_2 directions. We attempt to find a_{12} and a_{66} which give a master effective stress-effective plastic strain curve for cross-ply and wove laminates. We call this model A in this paper. The resulting effective stress-effective plastic strain curves are shown in Fig.6. The plasticity parameters obtained are listed in Table 1. The effective stress-effective plastic strain curves are fitted in the form

$$\bar{\varepsilon} = A\bar{\sigma}^n \tag{17}$$

The fitted curves are also shown in Fig.6.



(a) Cross-ply laminates (b) Woven laminates

Figure 6 Effective stress-effective plastic strain curves based on the plasticity model A ((a) cross-ply laminates, (b) woven laminates).

Table 1 Plasticity parameters for model A

	a_{11}	a_{12}	a_{22}	a_{66}	A (MPa^{-n})	n
Cross-ply laminate	1.0	0	1.0	120	1.0×10^{-30}	9.6
Woven laminates	1.0	0	1.0	31	9.0×10^{-19}	6.3

In this study, another model is also applied. If we assume that there is no plastic deformation in x_1 and x_2 direction, we can set a_{11}=a_{12}=a_{22}=0. The remaining parameter is only a66. Because we can set a_{66}=1 without loss of generality, we have not parameter to determine to have effective stress-effective plastic strain curves. We call this model B. This results in the same potential function used in Ref.[16]. The resulting effective stress-effective plastic strain curves and its fitted curves are shown in Fig.7. It can be seen that the model works well despite of its simplicity. The parameters are listed in Table 2.

Table 2 Plasticity parameters for model B

	a_{11}	a_{12}	a_{22}	a_{66}	A (MPa^{-n})	n
Cross-ply laminate	0	0	0	1	1.0×10^{-14}	7.1
Woven laminates	0	0	0	1	3.0×10^{-13}	6.3

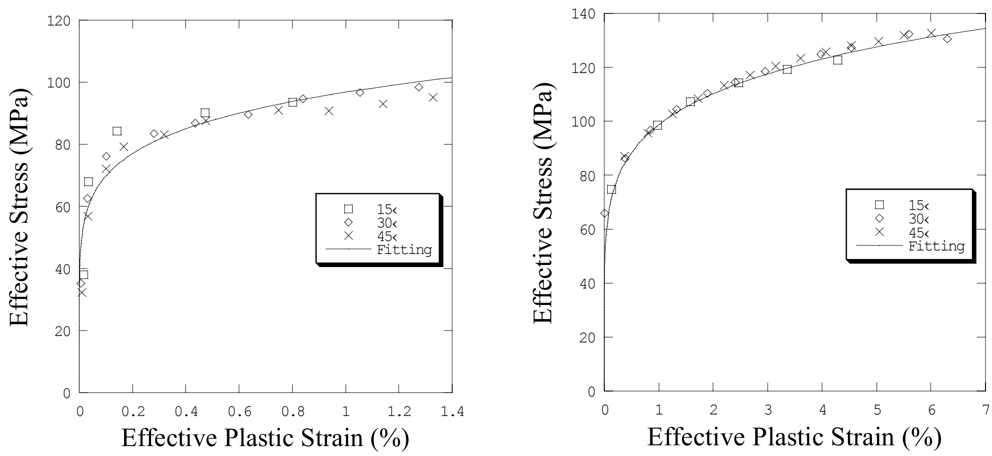


Figure 7 Effective stress-effective plastic strain curves based on the plasticity model B ((a) cross-ply laminates, (b) woven laminates).

5. CONCLUSIONS

The nonlinear stress-strain behavior in CFRP cross-ply and woven laminates under off-axis tension is investigated experimentally. The nonlinear behavior in the stress-strain curve is focused. To divide the total strain into the elastic and inelastic ones, loading-unloading tests are performed. The inelastic part of strain is characterized by using plasticity models.

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