IDENTIFICATION OF ELASTIC STIFFNESS AND DAMAGE MODEL FOR COMPOSITE MATERIALS BY THE VIRTUAL FIELDS METHOD

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ABSTRACT: The objective of the present work is to identify nonlinear constitutive equations of composite materials from heterogeneous full-field strain measurements. The nonlinearity considered here is induced by the damage inherent to the in-plane shear response. In the first part of the paper, strain fields are simulated by finite element computations and processed by the VFM for validating the procedure. As measured strain fields are expected to be noisy due to error sources of optical techniques, an appropriate strategy aimed at minimising the effect of noise is introduced. In the second part of the paper, heterogeneous strain fields are measured by the grid method. They are processed by the Virtual Fields Method. Parameters governing the nonlinear behaviour law of a glass/epoxy 0° unidirectional composite are identified and compared to the values obtained by standard tests.

1. INTRODUCTION

The identification of parameters governing the mechanical behaviour of composite materials usually requires the use of several test configurations because of the anisotropy of these materials. These test configurations are generally simple enough to produce a uniform or nearly uniform state of stress. Therefore, it is possible to obtain the stress distribution and so, local strain measurements can provide stiffness values. This classical procedure suffers however a number of drawbacks. First, several tests are needed to obtain the full orthotropic set of parameters. Then, parasitic effects affecting boundary conditions will have a great impact on the measured stiffnesses ([1, 2] among others). Finally, the study of the coupling between different stress components will require the use of biaxial test machines which is expensive.

An alternative strategy, called the Virtual Fields Method and noted hereafter (VFM), developed by Grédiac [3], has been successfully applied to determine the in-plane [4] and through-thickness [5] mechanical stiffness of anisotropic composite materials. The identification technique relies on the processing of the strain fields when expressing the global equilibrium of a structure through the principle of virtual work expressed with specific virtual displacement fields.

In the present work, the identification procedure is applied for the direct and simultaneous determination of five material parameters characterising an orthotropic E-glass/M10-epoxy composite. These constants are the in-plane elastic stiffness and the parameter governing the non-linearity due to the damage inherent to the in-plane shear response. The sensitivity of the procedure when processing noisy data has been investigated. Then, the present paper exhibits experimental results of the identification of the in-plane orthotropic stiffnesses of a unidirectional glass/epoxy specimen tested with the Iosipescu fixture (Fig. 1)

2. MATERIAL MODEL

The material model considered in the present work is a linear elastic orthotropic behaviour coupled to a damage law affecting only the in-plane shear behaviour. For unidirectional composite materials, the major source of non-linearity is the in-plane shear response.

The damage model is the one proposed by Ladevèze [6] restricted to the in-plane shear behaviour. It states that the shear stiffness for any stress increment can be written as:

$$Q_{66} = Q_{66}^{0}(1 - d_{66}) \tag{1}$$

where d_{66} is the damage parameter. As a consequence, the global stress-strain relationship writes:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66}^{0}(1 - d_{66}) \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_6 \end{pmatrix} \tag{2}$$

where **Q** is the in-plane stiffness matrix, $\boldsymbol{\sigma}$ the stress tensor and $\boldsymbol{\varepsilon}$ the train tensor. In [6], the evolution of shear damage was modelled as a linear function of shear strain with a threshold. Here, it was chosen to represent it as a quadratic polynomial functions of the shear strain:

$$d_{66} = \alpha \gamma_6^2 \tag{3}$$

Therefore, the stress-strain relationship writes:

$$\sigma_6 = Q_{66}^{0} \gamma_6 - K \gamma_6^3 \tag{4}$$

with $K = \alpha Q_{66}^{0}$. It must be noted that other more sophisticated polynomial forms could have been proposed. This would increase the number of parameters to be identified. If more sophisticated models were used (taking into account the effects of loading history, for instance), then the problem may become non-linear and other numerical strategies would have to be sought.

Now the objective of the next section is to illustrate how to extract simultaneously the five parameters involved in Eq. (2) (Q_{11}, Q_{22}, Q_{12}, Q_{66}^{0} and K) from the knowledge of the strain field over the surface of the tested specimen.

3 THE VIRTUAL FIELDS METHOD
3.1 THEORTICAL BACKGROUND

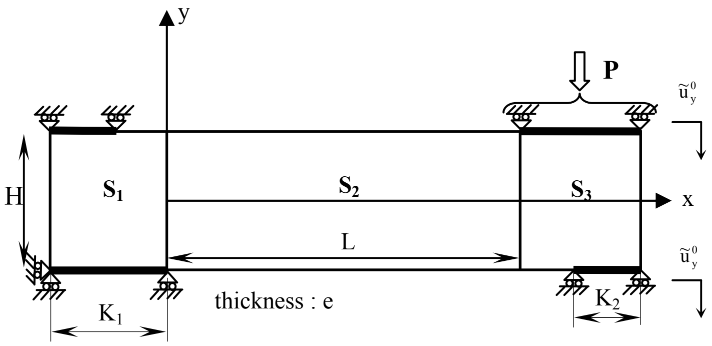
The VFM is an identification procedure based on the Principle of the Virtual Works (PVW). Assuming in-plane stresses, a quasi-static loading and in absence of volume forces, the PVW can be written as, for any Kinematically Admissible (KA) virtual displacement field:

$$-\int_V \sigma_{ij}\ \varepsilon_{ij}^{*}\ dV + \int_{\partial V} T_i\ u_i^{*}\ dS = 0 \tag{5}$$

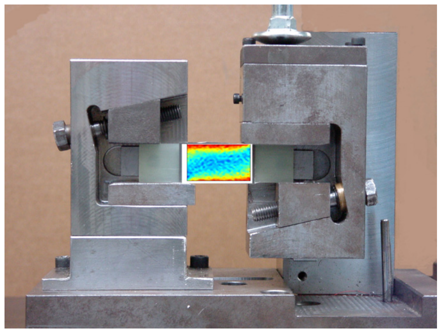
where V is the volume of the solid, ∂V the boundary surface of the solid, dS the elementary boundary surface of the specimen, σ_{ij} is the stress field, ε_{ij} is the virtual strain field, T the external surface load density applied over ∂V and u^{*} is the virtual displacement field associated to ε^{*}.
Considering the constitutive equation in Eq. (2), and supposing that the stiffness parameters are constant, Eq. (5) can be written as:

$$Q_{11}\int_{S_2}\varepsilon_1\varepsilon_1^* dS + Q_{22}\int_{S_2}\varepsilon_2\varepsilon_2^* dS + Q_{12}\int_{S_2}(\varepsilon_1\varepsilon_2^* + \varepsilon_2\varepsilon_1^*)dS + Q_{66}^0\int_{S_2}\gamma_6\gamma_6^* dS - K\int_{S_2}\gamma_6^3\gamma_6^* dS = -\frac{P\tilde{u}_y^*(L)}{e} \quad (6)$$

where P is the global shear load applied to the specimen, L is the length of the central part of the specimen sketched in Fig. 1(a), e its thickness and $\tilde{u}_y^*$ the virtual displacement of the right hand-side of the fixture.
The principle of the VFM consists in writing Eq. (6) with as many virtual fields as there are unknown parameters (here, five). Thus, one can build a linear system to determine the unknown parameters. It can be seen here that full-field measurements are necessary to compute the integrals in Eq. (6), where the virtual fields act as numerical filters. The choice of the virtual fields is decisive for the stability of the method. Here, the so-called "special" virtual fields have been used [7, 8, 9].



(a) Schematic of bending/shear test



(b) Iosipescu fixture with straight composite coupon

Fig. 1: Bending/shear test based on the Iosipescu fixture

3.2 NUMERICAL SIMULATION

The application of the procedure is carried out on an orthotropic 0° glass/epoxy composite material. In practice, the strain field measurements can be obtained experimentally by some full-field optical technique, such as moiré or speckle interferometry, digital image correlation, grid method etc. The objective of this section being the validation of the identification procedure, the strain fields are obtained with finite element simulations. These numerical calculations require the implementation of the constitutive model into implicit code ABAQUS by means of the User MATterial subroutine (UMAT) programmed in the Fortran 90 language [10].

The stiffness constants are: Q_{11}=25.93 GPa, Q_{22}=10.37 GPa, Q_{12}=3.112 GPa, Q_{66}^0 = 4.00 GPa. The material constant governing the nonlinearity, noted K, is K=4420 GPa. These values represent a generic material. The specimen represented in Fig. 1(a) was modelled. Its thickness and width are respectively 2 mm and 20 mm. The calculations were performed using the in-plane stress assumption and four-noded quadrilateral elements (CPS4). The imposed vertical displacement on the right-hand side (respectively left-hand side) of the specimen is constant and equal to $\tilde{u}_y^0$ = 1.3 mm (respectively 0 mm). As result, the central area (S_2) of the specimen tested has been meshed with 15000 elements (with a total of 19142 elements for the whole specimen, using an adapted mesh) for the non-linear calculations.

The five material parameters given above are the input values introduced in the finite element model to simulate the strain fields. These computed strain fields are then used as inputs to the VFM procedure and the objective is to check that the VFM retrieves the reference parameters input in the FE code.

3.3 IDENTIFICATION RESULTS

The aim of this section is to validate the present identification procedure of orthotropic composite material and to examine the sensitivity of the identified parameters on errors in the strain field. Indeed, experimental data are always subjected to noise. However, to simulate experimental errors, a gaussian white noise was added to strain values provided by the finite element computations. The average of the gaussian white noise is equal to zero (white noise). Two sets of noise were simulated. These two sets correspond respectively to standard deviation of strain values of 5.10^{-4} and 10^{-3}.
For each set of noisy strains, an identification is performed. The process is repeated 30 times with another copy of gaussian white noise yielding to a distribution of identified parameters.

Table 1: Identified stiffness and damage parameters for glass/epoxy(M10) composite from unnoisy and noisy simulated strain fields

$\tilde{u}_y^*$=1.52 mm, θ = 15°	Q_{11}	Q_{22}	Q_{12}	Q_{66}^0	K
Reference (GPa)	25.93	10.37	3.112	4.000	4420
Identified (GPa)	25.94	10.35	3.112	3.990	4347
Relative difference (%)	-0.03	0.19	0.0	0.12	1.65
Noise amplitude: 5.10^{-4}					
Identified (GPa)	25.94	9.400	3.430	3.990	4306
Relative difference (%)	-0.03	9.35	-9.72	0.25	2.58
Coefficient of variation (%)	1.02	35.9	23.9	0.70	5.28
Noise amplitude: 10^{-3}					
Identified (GPa)	26.02	11.48	4.070	4.090	5077
Relative difference (%)	-0.34	-10.7	30.78	-2.25	-1
Coefficient of variation (%)	1.08	43.0	37.2	7.46	40.9

When the identification is performed onto unnoisy strains, the identified damage parameter ($K=\alpha.Q_{66}^0$) and the four stiffness constants (Q_{ij}) are in agreement with the reference values as reported in table 1.
The coefficient of variation represents the scatter of the results extracted from noisy data. It can be seen that Q_{11}, Q_{66}^0 and the damage parameter K are the most stable. The sensitivity of the other parameter is greater. As expected, the results show that the relative difference and the scatter increase with the noise standard deviation. These results confirm the feasibility of the VFM when applied to such a nonlinear constitutive model.

The next part of the paper is devoted to the experimental identification of the full set of parameters for glass/epoxy composite material. The strain maps onto the surface S_2 are measured by the grid method and then processed by the VFM to extract the five parameters driving the law behaviour.

4 EXPERIMENTAL PROCEDURE

In order to measure the strain field at the surface of the specimen, a cross-grid of pitch 100 μm has been transferred onto the surface of the specimen (Fig. 2). The grid is printed onto a photosensitive film using a postscript file. Then, the photosensitive film is bonded onto the specimen while the polymer support film is removed. This enables to have only a very thin layer at the surface of the specimen [11]. Moreover, the use of a white glue ensures good contrast of light intensity on the surface.

Images of the underformed and deformed grid are digitized through 1280 × 1024 pixel CCD camera connected to a PC equipped with the Frangyne 2000 software. The phase maps of the initial and deformed grid are computed by spatial phase-stepping. The difference on these phase maps is related to the in-plane displacement in the direction perpendicular to the lines. A cross-grid ensures the measurement of both components of the in-plane displacement field by the use of an algorithm to separate the information in both directions [12, 13]. Here, 4 pixels have been used for the spatial phase-stepping procedure. The size of the interest area is 30×20 mm^2.

The spatial resolution of the displacement measurement is defined by the period of the grid. Two periods are used in the windowed discrete fourier transform algorithm to eliminate the effect of sampling mismatch [14]. Therefore, the spatial resolution in displacement Δx_d is 200 μm. The displacement resolution σ_d is defined as the smallest detectable value and depends strongly on the signal-to-noise ratio. It is determined experimentally by subtracting two different phase maps of the underformed grid and computing the standard deviation of the resulting phase map. Here, the standard deviation is about 2° of phase. Therefore, σ_d amounts to about 0.60 μm. From the displacement field, the strains are computed by local differentiation. First, two 33 × 33 average spatial smoothing filters are applied and then, the best linear least-square fit to 33 pixels is computed to obtain the strains. The spatial resolution in terms of strain Δx_ε is therefore equal to $\sqrt{\Delta x_d^2 + 3M^2}$ [12] where M is the size of the spatial filters, here the equivalent of 33 pixels. Finally, $\Delta x_\varepsilon = 1.44$ mm. The strain resolution σ_ε is obtained experimentally by applied a rigid body motion to the specimen and computing the apparent strain. Here, σ_ε is about 120 μdef. It is also important to note that for this technique, there is a parasitic effect due to out-of-plane displacements. Indeed, an out-of-plane displacement will be interpreted by the camera as a change in the pitch of the grid, resulting in apparent in-plane displacements. This sensitivity has been measured experimentally. It results in an apparent strain of 4 μdef for 1 μm of out-of-plan displacement. One has to ensure that out-of-plane displacements are negligible in order to ensure reliable strain measurements.

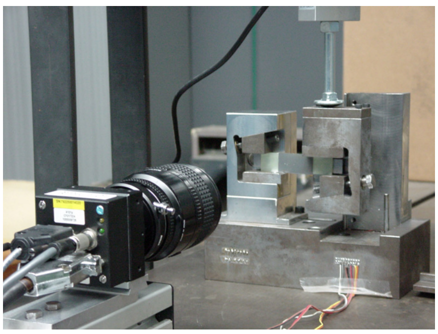


Fig. 2: Experimental set-up using the Iosipescu fixture and the grid method

5. IDENTIFICATION FROM FULL-FIELD MEASUREMENTS

Fig. 3 shows experimental strain fields obtained on a glass/epoxy 0° unidirectional composite loaded up to 30 MPa. One can see the bending effects on the longitudinal strain, one can guess the transverse strain close to inner loading points on the transverse strain, which is about zero in most of the interest area. Finally, the shear strain map is typical with maximum strain in the centerline and only a very small dependance on x. From these data, the first remark is that the shear strain is predominant and that the two other components are much smaller. Therefore, one can expect to have some difficulties retrieving Q_{11}, Q_{22} and Q_{12}.

The identification using the special virtual fields are reported in Table 2. The reference values have been obtained using uniaxial tension tests for Q_{11}, Q_{22} and Q_{12}. Q_{66}^{0} and K have been determined using classical Iosipescu specimen with notches [15, 16].
As expected by numerical simulation, although the identified values for Q_{11}, Q_{22} are of the same range of order of the reference, one can notice a slight discrepancy. The value of Q_{12} is not correctly identified. These difficulties are probably induced from:

1- longitudinal and transverse strain levels are too small.
2- the noise effects on the experimental maps have not the same distribution that those considered for numerical simulations. This may explain in a part the larger scatter than the one characterized with finite element method results, the noise was badly modelled.
3- it has been noticed that as applied load increases, out of plane displacements occur, causing parasitic strain in the central area (S_2).

From the shear strain map, Q_{66}^{0} is correctly identified with a relative difference less than the coefficient of variation of the reference value. K is obtained with some 25% difference. It is worth noting however that this is within the range of the coefficient of variation of the reference value. One can therefore conclude that the shear response is successfully retrieved from this test. In order to improve the results, it is possible to perform off-axis bending/shear tests, with the fibres at an angle with regard to the longitudinal axis of the specimen. In fact, recent results based on the procedure described in [17] have shown that 45° is the optimal angle for this test [18]. This configuration is currently being tested to check whether improved identification can be achieved.

Table 2: Reference and identified material stiffnesses for a glass/epoxy 0° specimen

	Q_{11}	Q_{22}	Q_{12}	Q_{66}^{0}	K
Reference Form standard tests (GPa)	44.9	12.2	3.86	3.78	1990
Coefficient of variation (%)	0.7	2.8	2.4	7.3	31
Identified values(GPa) (1 test)	56	27	-	3.6	1500
Relative difference (%)	25	130	-	-4.8	-25

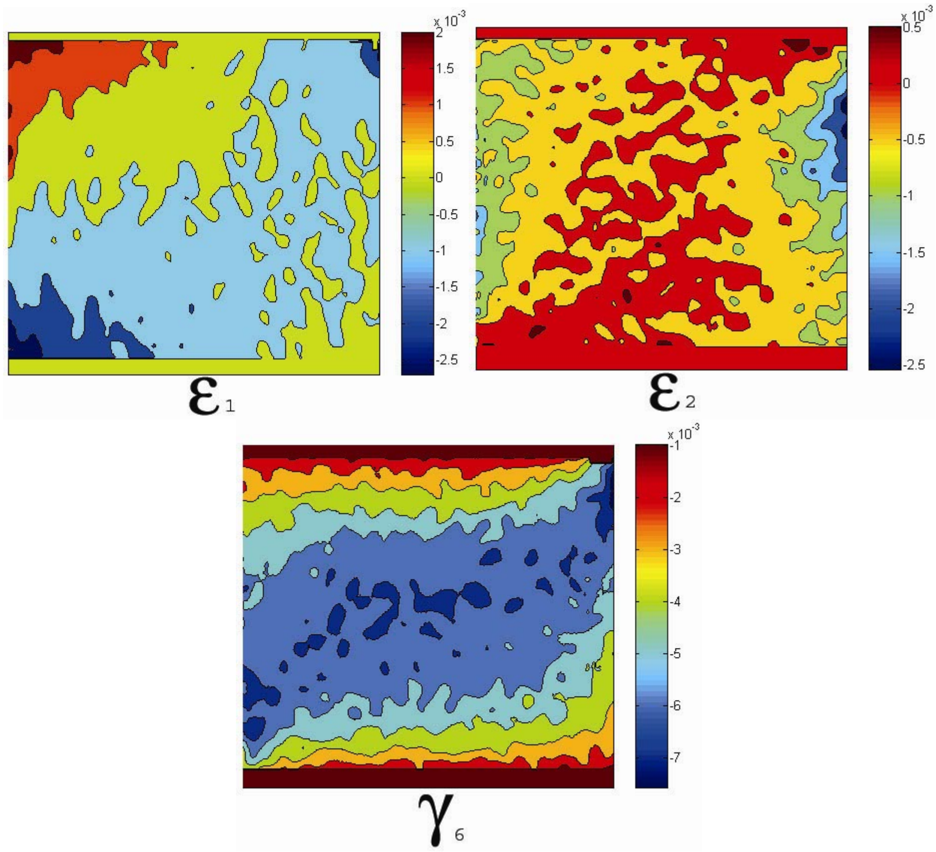


Fig. 3: Strain fields obtained on a 0° glass/epoxy specimen

6 CONCLUSION

The first part of the present paper has addressed the identification of nonlinear constitutive equations of orthotropic composite materials from heterogeneous strain field provided by finite element simulation. The identification of the elastic stiffnesses as well as the damage parameter driving the nonlinearity of the shear response are generally satisfactory. The accuracy and the stability of the procedure have been investigated with respect to noise in the simulated data.

The second part of the paper is devoted to analyse the experimental identification of the full set of parameters driving the elastic orthotropic damage behaviour of a glass/epoxy 0° unidirectional composite. The strain maps at the surface of a rectangular coupon submitted to a shear/bending loading is measured by the grid method. The strain maps are than processed by the Virtual Fields Method to retrieve the five parameters driving the damage law: the linear elastic stiffnesses Q_{11}, Q_{22} and Q_{12} and Q_{66}^{0} and a softening parameter K. The main conclusions and perspectives of this part are:

- the in-plane shear response Q_{66}^{0} has been well identified by the present procedure. The values of the shear modulus and the softening parameter (K) lie within the range of the reference values obtained from standard tests;
- the other stiffness components Q_{11}, Q_{22} and Q_{12} are rather badly estimated. This rises from the fact that the normal strains ε_1 and ε_2 are much smaller than γ_6 and therefore, much more sensitive to error sources;
- in order to improve the results, one can try to improve the strain resolution of the optical method without compromising too much on the spatial resolution. This could be achieved by using a better CCD camera and/or switching to another full-field

measurements technique. For instance, tests are currently underway using speckle interferometry in the LMPF research group;

- a second possibility to improve the results would be to change the loading configuration. However, there is an infinity of possibility to redesign a suitable test adapted to full-field measurement and the VFM. Tests are currently underway to check if better results could be obtained with an off-axis specimen, as predicted by recent theoretical results [15].
- An important field of work is now open in order to produce suitable test designs adapted to such inverse procedures and non-uniform strain fields.

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