# OPTIMAL DESIGN OF THE SWEPT FORWARD WING IN COMPOSITE MATERIALS

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**Abstract**

Aeroelastic divergence is a critical problem that must be carefully treated by the wing designers. Advanced composite materials are effective to strengthen the wing structure and arrest the divergence of wing. In this paper, the treatment of aeroelastic divergence for moderate swept-forward wing using the state-of-the-art composite materials is explored. Combining the optimization of the composite structure design with the numerical solutions of the forward analysis developed by Prof. Hwu, the optimal design of the composite wing structure can be done to prevent the generation of the divergence effect in this paper. At the subsonic speed, the factors of aeroelastic divergence in the wing structure including swept-forward angle, aspect ratio, fiber orientation, and transverse shear strain are employed as the design variables. An optimal design by exploiting the initial design model and optimization method is proposed, and some useful insights for the optimal design of the composite wing structures are released.

**Keywords**: Composite Materials, Optimal Design, Swept-Forward Wing, Aeroelastic Divergence.

## 1. INTRODUCTION

In World War II, cases of the air accidents induced the German scientists to find the divergence phenomenon of the wing structure. In order to overcome the characteristic of the aeroelastic divergence, the wing structures were designed to be a non-straight wing by changing the swept angle of the wing forward or backward. Thus, there are three types of aircraft wings in the configuration design - the straight wing, the swept-forward wing and the swept-back wing. The swept-forward wing, compared to a swept-back wing of the same wing area, has a number of advantages such as higher lift to drag ratio, higher capacity in dogfight maneuvres, higher range at subsonic speed, improved stall resistance and anti-spin characteristics, improved stability at high angles of attack, a lower minimum flight speed, and a shorter take-off and landing distance.

Due to the limitation of the manufacture technology in the past, it is hard and expensive to build the huge wing structure using the composite materials. In 1980's, the advancement of new technology, for examples, cutting, weaving and modulus, has greatly improved the manufacture processes for the composite materials. Boeing company developed a three dimensional CAD/CAM software system to simulate the design procedures of the composite structure in 1999 that successfully reduced 25% weight and 20% cost in the later three years [1]. It is a new trend to design the aircraft wing structure using the composite materials.

Especially the anti-bending effect of the wing may be increased by the proper layout of the fiber orientation in the composite structure; and furthermore, the aeroelastic divergence of the wing structure can be effectively improved.

The structure design of the aircraft is a combination of structural mechanics, aerodynamics, vibrations and composite material mechanics [2]. Without the technology problems in the manufacture as the improvement described above and the motivation of the weight reduction in the aircraft design, the wing structure with the complete composite material is the popular event in the near future. Obviously, the designers should carefully deal with the divergence problems of the wing structures. Hwu and Tsai [3,4] presented the detailed theoretical analysis and numerical simulation for the divergence of the wing structures. They pointed out that the computational burden of FEM (finite element method) and CFD (computational fluid dynamics) is not indispensable for the optimal design. The analytical solution for the wing box structures made up of the composite sandwich plate greatly saves the time and computer memory. An optimal method is proposed to overcome the aeroelastic divergence of composite wing structure. The orientation angle of the fiber direction in the composite materials, the number of layers, plate thickness, material properties and the dimension are chosen to be the design variables of the optimization problems. For a detailed description of the optimal design for the composite laminate, beam, shell or sandwich, refer to Miravete [5].

Publications studied about the structural divergence characteristics are as follows. Tayler et. al [6] used a beam model with a series of elements, stepped in thickness at discrete nodes to calculate the natural frequency of the model. The model did not consider the aerofoil shape function to approach the aerodynamic and structural effect accurately. Standfast [7] studied the effects of fiber oriented composites on forward swept wing structural divergence by using three-layered Balsas wood to represent a fiber reinforced composite. The optimal layering orientation that prevented bending and twisting of the forward swept wing was obtained during wind tunnel test. Miyakawa et. al [8] presented the design study of forward swept wing for transonic transport looking for high drag divergence performance. The development of aero-structural integrated design code was mentioned and the performance is verified by transonic wind test.

## 2. THEORECTICAL FORMULATION

The wing of the aircraft consists of spars, ribs, stringer, and lower and upper skin. From the traditional method and the fundamental assumptions to deal with the composite sandwich plate [9,10], the corresponding composite wing structure can be simulated under the assumptions as follows [4]:

1. The spar and rib in the wing structure are assumed to be the core of the composite sandwich. The skin and stringer of the wing are assumed to be the faces of the composite sandwich.
2. The wing structure is an uniform wing cross-section.

3. The faces of the sandwich are subjected to the in-plane force and the in-plane bending moment only.
4. The effect of thickness in the sandwich plate is taken into consideration. The changes in the curvatures of the polynomial functions of the upper and lower wing surfaces subjected to the bending moment are neglected.
5. The core is the structure subjected to the shear stress.

According to the publications of Hwu and Tsai [3,4], the velocity of the aeroelastic divergence takes place when the displacement of the wing becomes infinite. The velocity or the dynamic pressure of the divergence can be obtained by solving the following equations [3]:

$$\left\| \mathbf{J}\,[\mathbf{K}_0\mathbf{K}_s(l)\mathbf{J}\mathbf{K}_1 + \mathbf{K}_1\mathbf{K}_s^{'}(l)\mathbf{J}\mathbf{K}_1] \right\| = 0 \tag{1}$$

where,

$$\mathrm{J} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathrm{K}_0 = \begin{bmatrix} 0 & 0 & \tilde{A}_{44} & \tilde{A}^{*}_{44} \\ 0 & 0 & 0 & \breve{D}_{26} \\ 0 & 0 & 0 & \breve{D}_{66} \\ 0 & 0 & \tilde{A}^{*}_{44} & \tilde{A}^{**}_{44} \\ 0 & 0 & 0 & \breve{D}^{*}_{26} \end{bmatrix}, \quad \mathrm{K}_1 = \begin{bmatrix} \tilde{A}_{44} & -\tilde{A}^{*}_{44} & 0 & 0 \\ 0 & \breve{D}_{26} & \breve{D}_{22} & \breve{D}^{*}_{22} \\ 0 & \breve{D}_{66} & \breve{D}_{26} & \breve{D}^{*}_{26} \\ \tilde{A}^{*}_{44} & -\tilde{A}^{**}_{44} & 0 & 0 \\ 0 & \breve{D}^{*}_{26} & \breve{D}^{*}_{22} & \breve{D}^{**}_{22} \end{bmatrix} \tag{2a}$$

$$\mathrm{K}_s(l) = L^{-1}([\mathrm{S}(\mathrm{K}_0 + s\mathrm{K}_1) - \Omega]^{-1}) \tag{2b}$$

$$\Omega = aq_n c \begin{bmatrix} s\tan\Lambda & -1 & 0 & 0 \\ se\tan\Lambda & -e & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} s & 0 & 0 & 0 & 0 \\ 0 & 0 & s & -s & 0 \\ -1 & s & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & s \end{bmatrix}. \tag{2c}$$

$\breve{D}_{ij}$ is the bending stiffness which depends on the profile of the upper and lower aerofoil shape function $f_u(x)$ and $f_l(x)$, $\tilde{A}_{ij}$ is the shear stiffness, and $L^{-1}$ is the inverse Laplace transformation, $a$ is the slope of the wing lift coefficient to the angle of attack, $q_n$ is the dynamic pressure normal to the front edge of the wing structure, $c$ is the length of chordline and $e$ is the distance between the aerodynamic center and the elastic center as shown in Fig. 1. Symbol ~ means the functional integration with respect to the variable $x$; symbol * means the integration of the product of the function and $x$ and symbol ** means the integration of the product of the function and $x^2$. $\breve{\ }$ is the combination of the ~, * and **. $c$ and $e$ are set to be constant in this paper. $\Lambda$ is the swept angle. If $\Lambda$ is positive, the wing is swept-forward, or else if $\Lambda$ is negative, the wing is swept-backward. The equation (1) is a complex equation including the inverse Laplace transformation, integration, and the differentiation. The detailed derivation can be found in Tsai [3] and Hwu and Tsai [4].

## 3. NUMERICAL SIMULATION OF THE FORWARD PROBLEMS

At first, the variations of the divergence velocity with respect to the design variables are calculated. Based upon the results of Hwu and Tsai, the higher order of the polynomial function in the upper and lower surfaces of the aerofoil leads to the more accurate numerical results. For the NACA2412 aerofoil, there is not a significant improvement in the accuracy of dynamic pressure when the approximate polynomial function is higher than fourth-order. To save the time of the optimal design in each iteration, the order of the polynomial functions for the upper and lower surfaces of the aerofoil are chosen to be fourth.

The angle of the wing swept is the second factor to be considered. According to numerical results the divergence effect will not occur when the wing swept angle is backward. The displacement of the swept backward wing will not be infinite when the dynamic pressure is very large. It is the case of non-divergence. The situation agrees with the general idea that the swept backward wing is hard to diverge. Therefore, only the swept forward wing is considered in the paper.

The effect of the transverse shear strain is large when the aspect ratio $A_r$ of the wing is high. This is due to the flexibility is higher when the span of the wing is longer and the effect of the shear strain proportional to the effect of the wing divergence increases. The variation of the dynamic pressure of the divergence with respect to the aspect ratio is shown here numerically. $q_n(A_r=4)/\ q_n(A_r=8)=9.16$ when the swept forward angle is $30^o$. Thus, the aspect ratio naturally becomes the design parameters in the paper. At last, the effect of the fiber orientation in the composite materials is considered. From the numerical results, it is observed that the dynamic pressure of the divergence is large when the fiber orientation is located in the range of $75^o \sim 135^o$ and the swept forward angle is big. The optimal orientation can be predicted at $\theta=105\,^o$. On the other hand, the transverse shear strain also affects the dynamic pressure of the divergence [4]. The selection of the design variables is very important that depends on the effect of the divergence velocity or dynamic pressure with respect to the selected parameters. The design parameters will be the same if the transverse shear strain is considered in the design problems.

## 4. OPTIMIZATION OF THE WING STRUCTURES

Specifically, the factors of aeroelastic divergence at subsonic speed include the swept forward angle $\Lambda$, the aspect ratio $A_r$, the orientation angle of the fiber in the composite materials $[\theta_{ui}]_n$, $[\theta_{li}]_n$ (i=1~n for n layers. $\theta=\theta_u$ for the upper skin and $\theta=\theta_l$ for the lower skin), the transverse shear strain $S_h$. The dynamic pressure of the front edge of the wing can be expressed as:

$$q_n = q_n(A_r, \Lambda, [\theta_{ui}]_n, [\theta_{li}]_n) \tag{3a}$$

$$q_{n\_S_h} = q_{n\_S_h}(A_r, \Lambda, [\theta_{ui}]_n, [\theta_{li}]_n) \tag{3b}$$

where $q_n$ is the dynamic pressure without considering the factors of transverse shear strain, and $q_{n\_S_h}$ is the dynamic pressure considering the factors of transverse shear strain $S_h$. Note that the design variables are $A_r$, $\Lambda$, $[\theta_{ui}]_n$ and $[\theta_{li}]_n$.

The formulation of nonlinear optimization [11] to deal with the sandwich wing structure is

Objective function : Max. $q_n = q_n(A_r, \Lambda, [\theta_{ui}]_n, [\theta_{li}]_n)$, (4*a*)

Subjected to : $\left\| \mathbf{J}\,[\mathbf{K_0 K_s}(l)\mathbf{J K_1} + \mathbf{K_1 K_s'}(l)\mathbf{J K_1}] \right\| = 0$ , (4*b*)

$$A_{r\_l} < A_r < A_{r\_u}, \Lambda_l < \Lambda < \Lambda_u,\ \theta_{l\_l} < \theta_{li} < \theta_{l\_u},\ \theta_{u\_l} < \theta_{ui} < \theta_{u\_u}. \tag{4c}$$

The dynamic pressure $q_n$ is replaced by $q_{n\_S_h}$ as the effects of the transverse shear strain are taken into account.

<u>Example 1:</u>

The aerofoil NACA2412 is used for illustration. The slope of the lift coefficient to the angle of attack is $5.73 rad^{-1}$, the pitch moment coefficient of the aerodynamic center is -0.04, and the distance ratio of the aerodynamic center to the torsional center is 0.257. The composite material is made up of the carbon epoxy with the thickness 2.5mm for single layer (n=1) and fiber orientation $\theta_{u1}= \theta_{l1}=\theta$ is the design variable. The materials properties are $E_{11}=200GPa$, $E_{22}=5Gpa$, $\nu_{12}=0.25$, $G_{12}=2.5GPa$. The shear modulus of the wing spar is $G=8GPa$. The designed model of the aircraft is shown in Figure 2. After 37 iterations of nonlinear optimization, the optimal fiber orientation is $\theta=103.6^{o}$ when the aspect ratio is 6 and the swept forward angle is $60^{o}$. The objective function during iterations is shown in Figure 3. The result can be used to test the accuracy of the program for the optimal problem. The numerical solution is identical to what we expect.

<u>Example 2:</u>

This example uses the same model as Example 1 except that the transverse shear strain is taken into consideration and the fiber orientation of the upper and lower skin is $[90/45/-45/0]_s$. There are eight layers in the upper and the lower surfaces of the skin and the ply thickness is

0.5mm. Let the design variable be the swept forward angle and the aspect ratio be 4. The upper and lower bounds of the swept forward angle are $45^o$ and $5^o$ respectively. After 52 iterations of the nonlinear optimization the final result of the maximum dynamic pressure is 436521*Pa* and the swept forward angle is $5^o$. If we change the upper and lower bounds of the swept forward angle to be $70^o$ and $25^o$ respectively, the optimal swept forward angle is $70^o$. The reason is that the variation of the dynamic pressure with respect to the swept forward angle is a concave curve. The dynamic pressure is small when the swept forward angle is in the middle region. Therefore, the aeroelastic divergence must be carefully examined when the swept forward angle is changed.

## 5. CONCLUSIONS

Composite material is a good choice of wing structure because the stiffness could be increased in designated direction by changing the fiber orientation in order to prevent the aeroelastic divergence. From the numerical results, three essentials are summarized. First, the specific fiber orientation can be optimized to maximize the dynamic pressure. Second, the transverse shear strain that decreases the dynamic pressure plays an important role in the aeroelastic divergence of wing structure. Third, there is a certain region of swept forward angle which results in the decrease of dynamic pressure; the aeroelastic divergence for a wing with a critical swept-forward angle must be taken good care of.

## ACKNOWLEDGEMENTS

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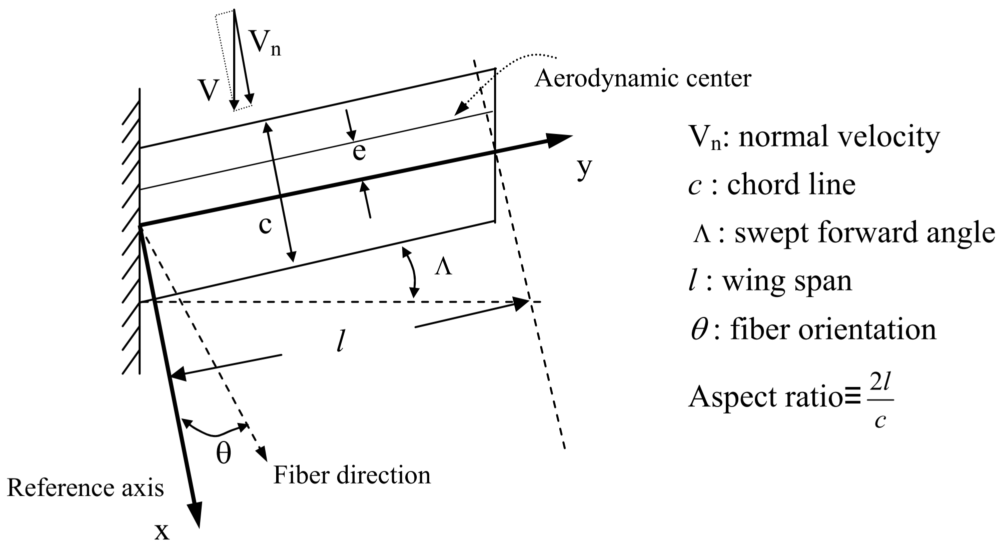
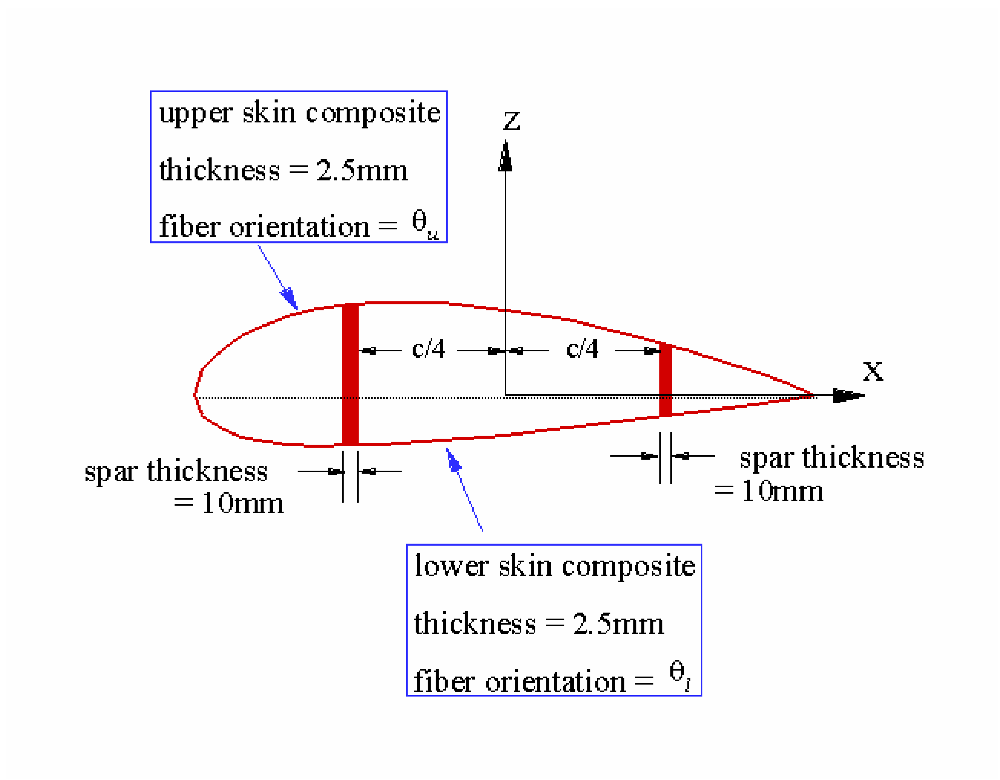
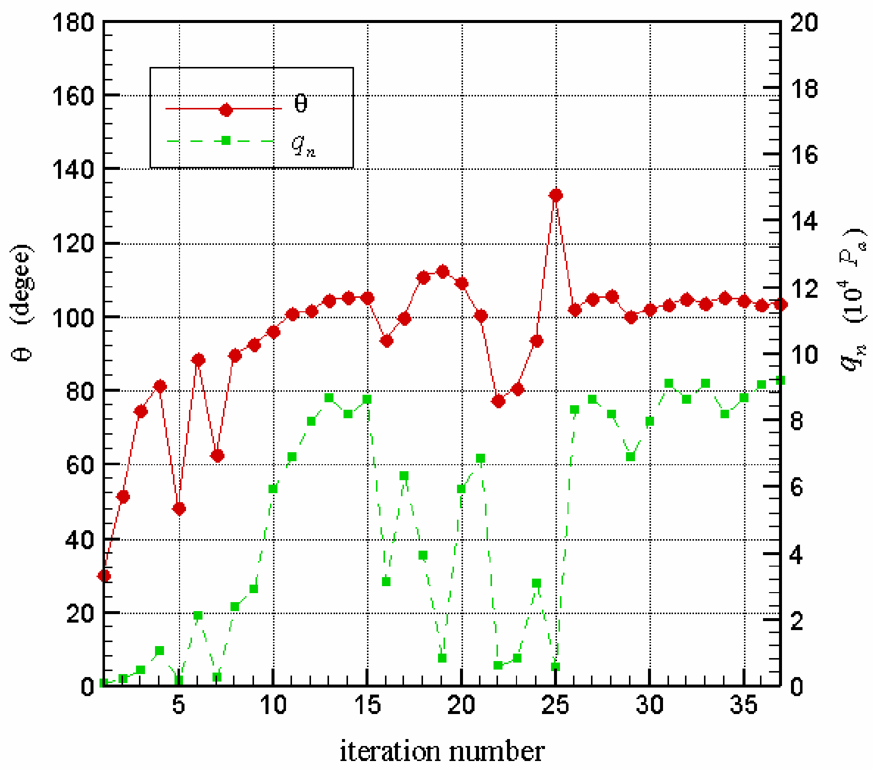



"Fig.1. The Diagram of the swept-forward wing. "




"Fig.2. Wing cross section model for NACA 2412 aerofoil. "




"Fig.3. The searching results of nonlinear optimization for example 1. "