BUCKLING OF FRP BEAMS AND COLUMNS

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ABSTRACT

The paper presents the stability analysis of thin walled open and closed cross section orthotropic composite beams subjected to axial or transverse loads. In the analysis both the transverse shear and the restrained warping induced shear deformations are taken into account. Explicit expressions are presented for the buckling load of axially loaded composite columns and for the lateral torsional buckling load of transversely loaded composite beams. Simple expressions are also presented to determine the approximate reduction in the buckling load due to shear deformations. The local buckling analysis of composite beams is also discussed.

1. INTRODUCTION

Pultruded fiber reinforced plastic (FRP) thin-walled members are important structural elements. Since the stiffness of the composite is relatively low, buckling is a major consideration in design. Slender members usually buckle globally [13]. The shorter the member and the thinner the walls, the more likely that local buckling will occur first. For intermediate-span members interaction between local and global buckling modes is an important phenomenon [2].

In a beam or column the axial stresses are the highest, hence most of the fibers are oriented in the axial direction. Consequently, the stiffness of the walls perpendicular to the axis of the beam and the shear stiffness are significantly lower then the axial stiffness. This has two consequences: the local buckling of the segments is very important in the design of composite members and the shear deformation plays an important role in the global buckling analysis [1, 2, 5].

2. PROBLEM STATEMENT

We consider open and closed thin-walled section prismatic members consisting of flat rectangular wall segments. The layup of each wall segment is orthotropic (which is the case of pultruded members). The material and the layup may be different in each wall segment, but must be uniform along the width of each wall segment of the member. The cross section of the beam has one axis of symmetry.

The ends of the composite member may be simply supported, built in or free.

The composite beam may be subjected to an axial load (Fig. 1a) or to a transverse load shown in Fig. 1b to e.

We assume that the material behaves in a linearly elastic manner and the deformations are small.

We will determine the loads which result in global or local buckling of the member.

When a column is subjected to an axial load it may result in global flexural-torsional buckling, which, for doubly symmetrical cross sections simplify to two in-plane buckling and to a pure torsional buckling (Fig. 2a). This will be treated in Section 3.

When the beam is subjected to a transverse load it may result in a lateral-torsional buckling (Fig. 2b), which will be treated in Section 4.

Both axial load and transverse load (i.e. bending) introduce axial compressive stresses, which may cause local buckling in thin-walled members. The local buckling will be discussed in Section 5.

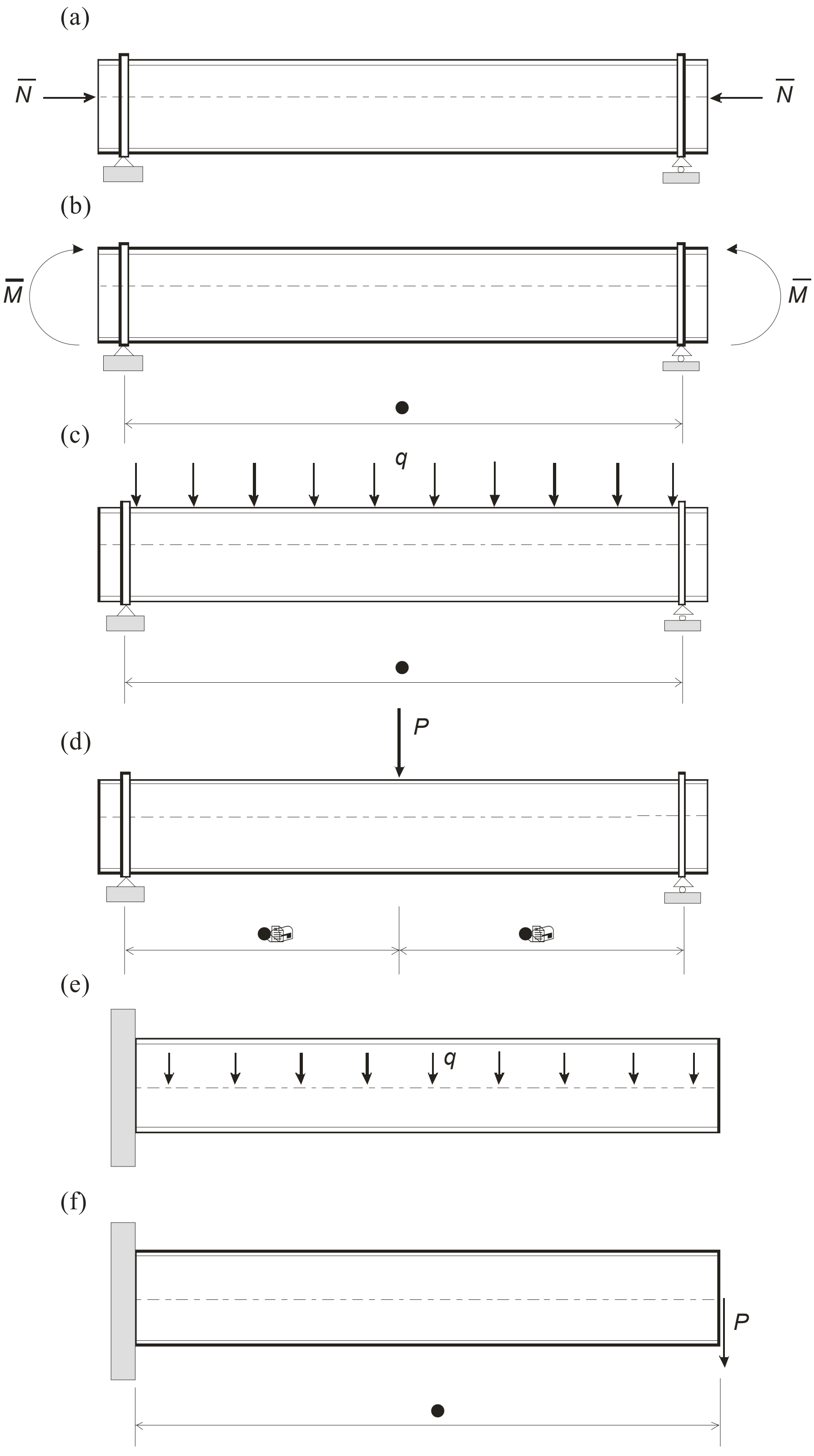


Fig. 1: The different load cases and support conditions

3. BUCKLING OF COMPOSITE COLUMNS SUBJECTED TO AXIAL LOADS

Buckling of *open section* beams was treated in detail in [6], and only the final results are presented below. For an open section composite beam the bending stiffnesses ($\overline{EI}_{zz}$ and $\overline{EI}_{yy}$), the torsional stiffness ($\overline{GI}_{t}$), and the warping stiffness ($\overline{EI}_{\omega}$) can be defined as given in [5].

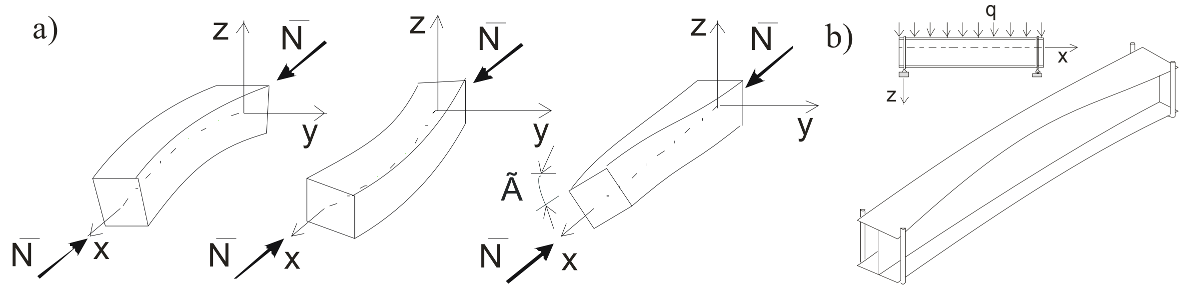


Fig. 2: Buckling modes to axial load (a), and lateral-torsional buckling (b)

One of the major differences between an isotropic and a composite beam is that in the latter case the effect of shear deformations must be taken into account. The shear stiffnesses ($\overline{S}_{ij}; i, j = y, z, \omega$) of different cross section beams are also given in [5]. Note that in torsion the shear deformations may play an important role, which is represented by the warping shear stiffness ($\overline{S}_{\omega\omega}$).
When buckling occurs in the *x-y* or *x-z* planes the buckling loads are

$$\overline{N}_{crz} = \left(\left(\frac{\pi^2}{(kl)^2} \overline{EI}_{zz} \right)^{-1} + \overline{S}_{yy}^{-1} \right)^{-1}, \qquad \overline{N}_{cry} = \left(\left(\frac{\pi^2}{(kl)^2} \overline{EI}_{yy} \right)^{-1} + \overline{S}_{zz}^{-1} \right)^{-1} \tag{1}$$

When pure torsional buckling occurs about the shear center the buckling load is

$$\overline{N}_{cr\psi} = \left(\left(\frac{1}{i_\omega^2} \frac{\pi^2}{(kl)^2} \overline{EI}_\omega \right)^{-1} + \left(\frac{1}{i_\omega^2} \overline{S}_{\omega\omega} \right)^{-1} \right)^{-1} + \frac{1}{i_\omega^2} \overline{GI}_t \tag{2}$$

where i_ω is the polar radius of gyration, and kl is the effective length, where $k = 1, 0.5, 2$ for simply supported, built in and for cantilever columns, respectively.
These loads are identical to the buckling loads of beams with doubly symmetrical cross sections. When the section is monosymmetrical and *z* is the axis of symmetry the buckling load can be calculated by the expressions given in [6].
As an approximation we may neglect the effect of the cross term ($\overline{S}_{y\omega}$) in the analysis and obtain an expression and a second order equation for the buckling load $\overline{N}_{cr}$:

$$\overline{N}_{cr1} = \overline{N}_{cry}, \quad \overline{N}_{cr}^2 \left(i_\omega^2 - z_{sc}^2 \right) - \overline{N}_{cr} \left(\overline{N}_{crz} + \overline{N}_{cr\psi} \right) i_\omega^2 + \overline{N}_{crz} \overline{N}_{cr\psi} i_\omega^2 = 0, \tag{3}$$

where $\overline{N}_{cry}$, $\overline{N}_{crz}$ and $\overline{N}_{cr\psi}$ are defined by Eqs. (1) and (2).
In [5] only open section beams were considered. For isotropic *closed section* beams, as a rule, the effect of restrained warping is negligible. However, for composite beams, when the stiffnesses of the adjacent walls are significantly different, the effect of warping shear deformations cannot be neglected. Here we present an engineering approach to obtain approximately the buckling load. We calculate first the buckling load by two conservative methods, Method A and Method B (Fig. 3).
In Method A we neglect the warping stiffness of the cross section thus we have

$$\overline{GI}_t^{\text{closed}}, \qquad \overline{EI}_\omega = 0, \overline{S}_\omega = 0. \tag{4}$$

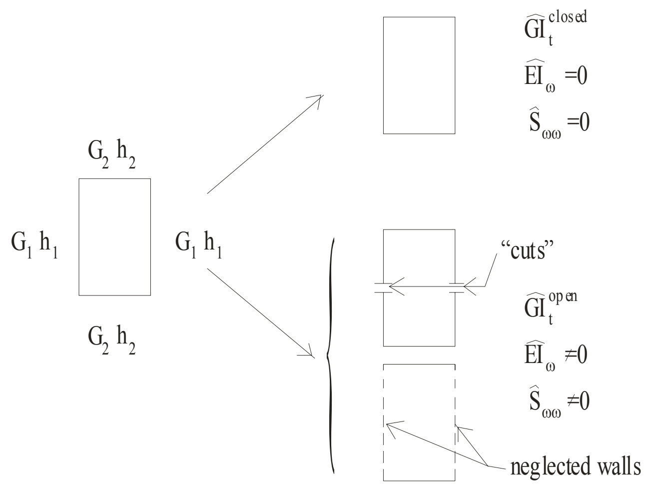


Fig. 3: Proposed approximate calculation method for closed cross sections

In Method B we assume one or more "cuts" in the longitudinal direction in the wall(s) (or even we neglect one or more walls, Fig. 3) and hence we obtain an open section beam with stiffnesses

$$\overline{GI}_t^{\text{open}}, \qquad \overline{EI}_\omega^{\text{open}} \neq 0, \overline{S}_\omega^{\text{open}} \neq 0, \tag{5}$$

where $\overline{GI}_t^{\text{closed}}, \overline{GI}_t^{\text{open}}$, $\overline{EI}_\omega^{\text{open}}$ and $\overline{S}_\omega^{\text{open}}$ are determined in Appendix A.
First we calculate the critical load with stiffnesses given by Eq.(4) then with the stiffnesses given by Eq.(5). The critical loads are denoted by $\overline{N}_{cr}^{A}$ and $\overline{N}_{cr}^{B}$, respectively. Both are conservative estimates, hence we may approximate the buckling load by

$$\overline{N}_{cr} = \max\left(\overline{N}_{cr}^{\text{A}}, \overline{N}_{cr}^{\text{B}}\right). \tag{6}$$

4. BUCKLING OF COMPOSITE BEAMS SUBJECTED TO TRANSVERSE LOADS

When monosymmetrical *open section* beams are loaded in the plane of symmetry at a certain level of the applied load the beam may buckle laterally, while the cross sections of the beam rotate simultaneously about the beam's axis. This phenomenon is called lateral-torsional buckling. If we neglect the effect of the cross term ($\overline{S}_{y\omega}$) in the analysis, the buckling load of simply supported or cantilever composite beams can be calculated in the form of [11]

$$\overline{M}_{cr} = C_1 \overline{N}_{crz} \left(C_2 f + C_3 \beta_1 + \sqrt{\left(C_2 f + C_3 \beta_1\right)^2 + i_\omega^2 \frac{\overline{N}_{cr\psi}}{\overline{N}_{crz}}} \right), \tag{7}$$

where C_1, C_2, C_3 are constants, and $\overline{M}_{cr}$ is the critical value of the maximum bending moment, which is related is to the loads (see Table 1). $\overline{N}_{crz}$ and $\overline{N}_{cr\psi}$ are calculated according to Eqs. (1) and (2). The value of parameter *k* depends on the end conditions of the beam and it is given also in Table 1. The detailed analysis can be found in [11].
In case of *closed section* beams the same solution method can be applied to obtain the buckling load ($\overline{M}_{cr}$) as it was introduced for the axially loaded columns: for Method A the

Table 1: Parameters in Eq.(7)

	C_1	C_2	C_3
Simply supported beam (k=1)			
End moments (Fig. 1b)	1	0	0.5
Uniformly distributed load $\overline{M}_{cr} = q_{cr} l^2 / 8$ (Fig. 1c)	1.13	0.45	0.267
Concentrated load at the midspan $\overline{M}_{cr} = P_{cr} l / 4$ (Fig. 1d)	1.35	0.55	0.212
Cantilever beam (k=1)			
Distributed load $\overline{M}_{cr} = q_{cr} l^2 / 2$ (Fig. 1e)	2.05	n.a	n.a
Concentrated force at the end $\overline{M}_{cr} = P_{cr} l$ (Fig. 1f)	1.28	n.a	n.a

warping stiffness is neglected ($\overline{EI}_\omega = 0$), and we calculate $\overline{M}_{cr}^A$, while in Method B the torsional stiffness ($\overline{GI}_t^{\text{open}}$) of an opened section is taken into account, which results in $\overline{M}_{cr}^B$. The greater of these is the buckling load

$$\overline{M}_{cr} = \max\left(\overline{M}_{cr}^A, \overline{M}_{cr}^B\right). \tag{8}$$

5. LOCAL BUCKLING

Local buckling analysis of thin-walled FRP composite members is of great practical importance and is a well explored area [3, 7, 8, 10, 12, 14]. In most of the cases the formulations are rather complicated; the solutions are not usually in a simple form.

Explicit expressions can be derived by modelling the wall segments as orthotropic plates and by assuming that edges common to two or more plates remain straight. Then the buckling load is determined, by considering the wall segments as individual plates, which are elastically restrained by the adjacent walls [4].

Here we do not present the local buckling analysis of composite members, rather we refer to [7, 5], where explicit expressions are presented for the buckling load.

6. NUMERICAL EXAMPLES

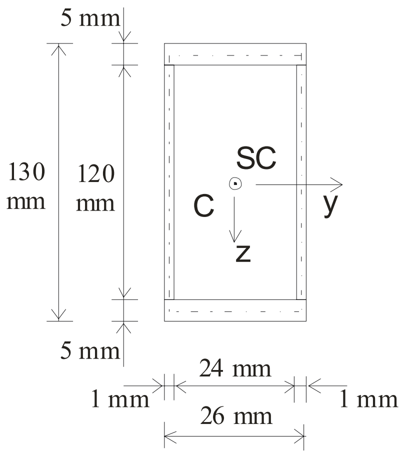
Numerical examples for buckling of open section composite columns and beams can be found in [5] and [11] therefore we only present examples for closed section members.

We consider a beam made of unidirectional carbon fiber reinforced epoxy. The Young modulus in the fiber direction is $E_1 = 148$ kN/mm^2 and the shear modulus is $G_{12} = 4.55$ kN/mm^2. The cross section of the beam is given in Fig. 4. The thickness of the top and the bottom walls is 5 mm, while the thickness of the vertical walls is 1 mm. The beam is simply supported at the ends, its length is $l = 1600$ mm.

The beam subjected to axial forces at the ends. We determine the critical load ($\overline{N}_{cr\psi}$) below.

The beam is doubly symmetric. The nonzero stiffnesses were calculated according to the expressions given in Appendix A. The cross-sectional properties are presented in Fig. 4.

First we calculate the torsional buckling load according to Method A. In this case we consider a closed-section and neglect the warping stiffness ($\overline{EI}_\omega$) and the warping shear stiffness ($\overline{S}_{\omega\omega}$) of the beam. The torsional stiffness ($\overline{GI}_t^{\text{closed}}$) is calculated by Eq.(A.6) and is given in



$\overline{EI}_{zz}$	7.72 x 10^6
$\overline{EI}_{\omega}^{open}$	15.06 x 10^9
$\overline{GI}_{t}^{open}$	9.86 x 10^3
$\overline{GI}_{t}^{closed}$	6.84 x 10^5
$\overline{S}_{yy}$	947.9
$\overline{S}_{\omega\omega}^{open}$	3.85 x 10^3
i_{ω}^{2}	2713

Fig. 4: Cross section and stiffnesses of the closed section beam

Fig. 4. With the above described stifnesses Eq.(2) gives

$$\overline{N}_{cr\psi}^{A} = 252.00 \text{ kN}.$$

Next the torsional buckling load according to Method B is calculated. In this case the originally closed-section is opened with four "cuts" at the edges. The warping stiffness ($\overline{EI}_{\omega}^{open}$) and the warping shear stiffness ($\overline{S}_{\omega\omega}^{open}$) of the open cross section are calculated according to Eqs.(A.8) and (A.9), and the torsional stiffness ($\overline{GI}_{t}^{open}$) is calculated by Eq.(A.7). With these stiffnesses Eq. (2) gives

$$\overline{N}_{cr\psi}^{B} = 24.71 \text{ kN}.$$

The calculated analytical buckling load is the higher value

$$\overline{N}_{cr\psi} = \max\left(\overline{N}_{cr\psi}^{A}, \overline{N}_{cr\psi}^{B}\right) = 252.00 \text{ kN}.$$

The same beam was also investigated with the aid if a finite element program ANSYS. Four node elastic shell elements were used with the maximum element size of 12.5 mm.
The result is

$$\overline{N}_{cr\psi}^{ANSYS} = 298.00 \text{ kN}.$$

We also investigated the effect of the length of the beam; the buckling load versus the beam length is shown in Fig. 5, top.
We also present the results for beams subjected to transverse loads (Fig. 5, bottom).
The calculated buckling loads agree surprisingly well with the results of numerical calculations performed by the FE program.

7. CONCLUSION

In the paper we presented simple expressions for the global buckling load of composite members, taking the effect of shear deformation into account. The shorter the beam the more important is the shear effect.

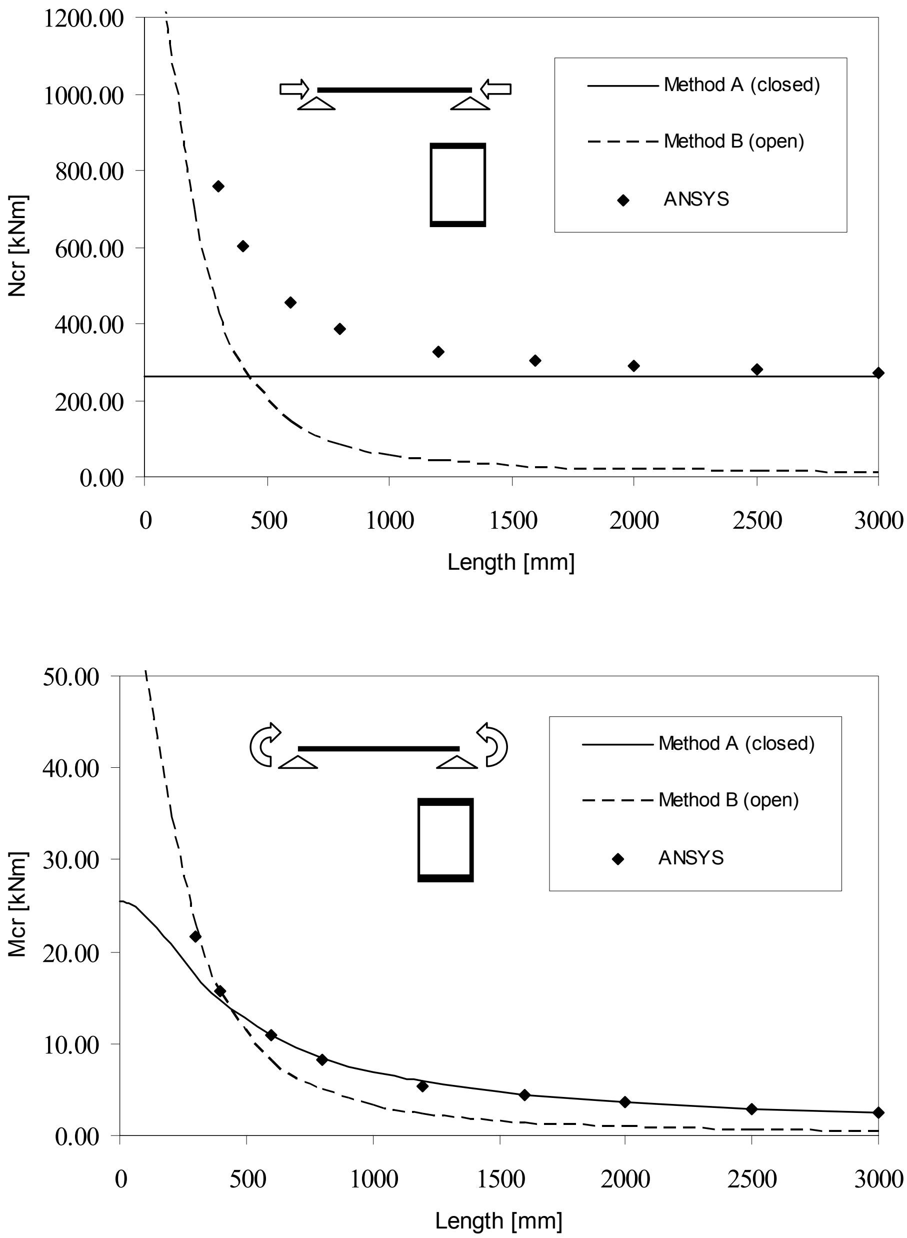


Fig. 5: Critical loads of doubly symmetric simply supported closed section beams

The question arises, when the effect of shear deformations must be taken into account. We introduce a parameter α which is the reduction in the buckling load due to the shear deformation.

For an I or a box beam, which buckles about the *y* (horizontal) axis, α can be approximated by [9] (see Fig. 6a)

$$\alpha = \frac{1}{1 + \dfrac{l^2}{\pi^2} \dfrac{\overline{S}_{zz}}{\overline{EI}_{yy}}} = \frac{1}{1 + \dfrac{l^2}{\pi^2} \dfrac{2(a_{11})_f}{b_f d (a_{66})_w}}, \tag{9}$$

where a_{11} and a_{66} are defined by Eq.(A.3), subscript *f* and *w* refer to the flange and the web, respectively.

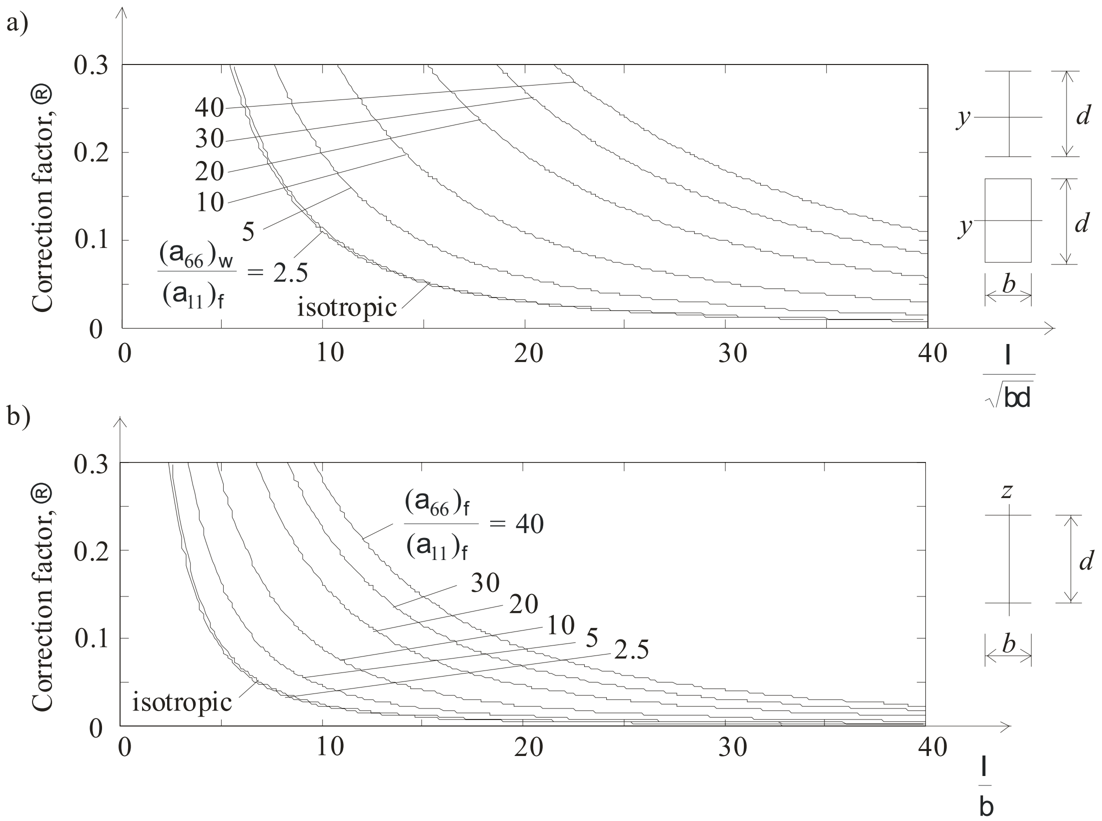


Fig. 6: The correction factors in Eqs.(9) and (10)

For an I-beam, which buckles about the z (vertical) axis or torsionally [11] it can be approximated by (see Fig. 6b)

$$\alpha = \frac{1}{1+\dfrac{l^2}{\pi^2}\dfrac{\overline{S}_{yy}}{\overline{EI}_{zz}}} = \frac{1}{1+\dfrac{l^2}{\pi^2}\dfrac{10(a_{11})_f}{b_f^2(a_{66})_w}}. \tag{10}$$

Note that even for slender composite beams the effect of shear deformations is significant.

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APPENDIX A. ELASTIC PROPERTIES OF A BOX BEAM

In the following we present the elastic properties of a thin-walled rectangular closed-section beam illustrated in Fig. 7. We assume that the layups of the wall are symmetrical and orthotropic.

The tensile stiffness ($\overline{EA}$) and the bending stiffnesses ($\overline{EI}_{zz}$ and $\overline{EI}_{yy}$) are

$$\overline{EA} = \frac{2b_f}{(a_{11})_f} + \frac{2b_w}{(a_{11})_w}, \qquad \overline{EI}_{zz} = \frac{b_w}{(a_{11})_w}\frac{d_f^2}{2} + \frac{2b_w}{(d_{11})_w} + \frac{2b_f^3}{12(a_{11})_f}, \tag{A.1}$$

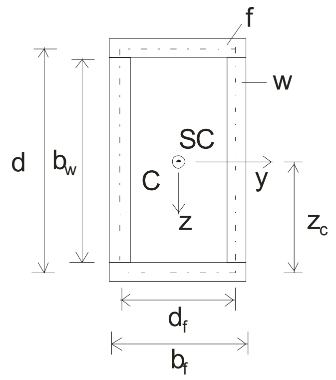

Fig. 7: Cross section of a thin-walled rectangular closed section beam

$$\overline{EI}_{yy} = \frac{b_f}{(a_{11})_f}\frac{d^2}{2} + \frac{2b_f}{(d_{11})_f} + \frac{2b_w^3}{12(a_{11})_w}, \tag{A.2}$$

where subscripts f and w refer to the flanges and the webs, respectively. a_{ij} and d_{ij} are the elements of the compliance matrices of the laminate, and are calculated as

$$\begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}^{-1}, \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{12} & d_{22} & 0 \\ 0 & 0 & d_{66} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}^{-1} \tag{A.3}$$

where A_{ij} and D_{ij} are the elements of the stiffness matrices of a laminate and must be calculated for the top flanges (f_1), and for the web (w).
The lateral shear stiffness is

$$\overline{S}_{yy} = 2\frac{d_f}{1.2(a_{66})_f}, \tag{A.4}$$

and the polar radius of gyration of the cross section about the shear center is calculated as

$$i_\omega^2 = \frac{EI_{yy} + \overline{EI}_{zz}}{\overline{EA}}. \tag{A.5}$$

The torsional stiffness in Method A for a closed section beam is as follows

$$\overline{GI}_t^{\text{closed}} = \frac{2d_f^2 d^2}{(a_{66})_f d_f + (a_{66})_w d}. \tag{A.6}$$

In Method B we cut the closed section into two parts (to “webs” and to “flanges”) at the corners (see Fig. 3). The separated parts are two doubly symmetric I-sections which webs are missing. The torsional stiffness is calculated as

$$\overline{GI}_t^{\text{open}} = 8\left(\frac{d_f}{(d_{66})_f} + \frac{d}{(d_{66})_w}\right), \tag{A.7}$$

while the warping stiffness is

$$\overline{EI}_{\omega}^{\text{open}} = \frac{d_f^3}{24(a_{11})_f} d^2 + \frac{d^3}{24(a_{11})_w} d_f^2 . \qquad \text{(A.8)}$$

Both the "webs" and the "flanges" are doubly symmetric, hence $\overline{S}_{y\omega}$ and

$$\overline{S}_{\omega\omega}^{\text{open}} = 2\left(\frac{d^2 d_f}{1.2(a_{66})_f} + \frac{d d_f^2}{1.2(a_{66})_w} \right). \qquad \text{(A.9)}$$

When the flanges and the web are made of a single orthotropic layer the expressions of a_{11}, a_{66}, d_{11} and d_{66} (Eq. B.3) simplify to

$$a_{11} = \frac{1}{E_1 h}, \qquad a_{66} = \frac{1}{G_{12} h}, \qquad d_{11} = \frac{12}{E_1 h^3}, \qquad d_{66} = \frac{12}{G_{12} h^3},$$

where E_1 is the Young modulus in the direction of the beam's axis, G_{12} is the in-plane shear modulus, and h is the thickness of the laminate.

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