THE NUMERICAL AND MATHEMATICAL MODELLING OF ELASTIC PLASTIC TRANSVERSELY ISOTROPIC MATERIALS

Abduvaly.A.Khaldjigitov[1], Uchqun E.Adambaev[1], Ravshan Khudazarov[2]

[1] Department of Computer Science, National University of Uzbekistan, Vuzgorodok, Tashkent-174, Uzbekistan
[2] Department of Mathematics, Tashkent State pedagogical Institute, Angren, Uzbekistan

ABSTRACT

A new strain space and stress space plasticity theories for a fiber reinforced composite materials (transversely isotropic) are proposed. On the basis of comparison of experimental and analytical curves is shown the validity of the offered theories for description of plastic deformations of fiber reinforced composite materials. It is shown that the offered strain space theory is valid not only for hardening, but also softening materials and is suitable for numerical realization of elastic-plastic problems in displacements. For the numerical solution of plasticity problems initial stress and initial strain and different modifications of iterative methods are applied. New variant of the initial stress method is proposed. 2D and 3D plasticity problems on equilibrium of transversely isotropic materials under action of different boundary conditions are solved.

1. INTRODUCTION

Development of science and engineering is connected with the wide application of composite materials having brightly expressed anisotropic properties, which cannot be neglected at structure calculation. The strength calculation problem for such structures becomes more complicate if a material is nonlinear or elastic-plastic composite one. In this connection there is an actual problem of construction of nonlinear relations between a stress and strain tensor for composite materials. The anisotropic nature of the mechanical response of fibre reinforced composites is now well understood. It is also generally recognized that a composite consisting of an isotropic matrix reinforced in one or two directions by families of continuous fibres is not just anisotropic but highly anisotropic, with the extensional modulus in the fibre-directions being much greate than any other extensional or shear module. Elastic-plastic behavior of fibrous composite materials has been investigated in many theoretical [23,25, 24,22, 17,4,6], experimental [5,14,30, 31,32] and numerical [9,26,27,10,8] studies in recent years. Most recent developments in the constitutive theory of the nonlinear response of fibre-reinforced composites is based on the micromechanical approach including the models proposed by Dvorak and Bahei-El-Din [23], Aboudi [25], Vanin [17,4]. Usually composite material on the basis of any averaging method may be considered as a homogeneous anisotropic material[4,15]. We shall remind, that fibrous and layered composite materials at averaging may be replaced with transversely isotropic or orthotropic materials [16,4,18]. By Hill[19], Bakhvalov [15], Pobedria[4] and others are proposed plasticity theories for anisotropic materials.

The given work is devoted to the development of the adequate modelling equations and effective numerical methods of elastic plastic problems for transversely isotropic materials. The strain space and stress space plasticity theories for transversely isotropic materials are offered. On the basis of comparison of experimental and theoretical curves are shown the validity of constitutive relationships for transversely isotropic materials. Experimental curves [5] received at extension (compression) of samples cut out from fiber reinforced composites under various angles concerning a direction of fibres are compared with theoretical one and is shown a good agreement. The stain space theory has a number of theoretical and numerical advantages as against stress space theory, and allows unequivocally to describe a softening materials.

Constitutive relations of strain space plasticity theory is based on the Ilyushin's [1] plasticity postulate and allows to express an increment of stress tensor through strain tensor and its increment. Such relationships is very convenient for the formulation of plasticity problems, and allows to study softening materials [2]. Strain space theory is very convenient for the numerical solution of elastic plastic problems on FEM, as there is no necessity for calculation of stress tensor at each increment of the external loading forces. In connection with stain space theory become possible to modify a initial stress method and to classify other methods from the point of view of convergence. Numerical test examples on equilibrium of 2D and 3D plasticity problems for transversely isotropic rectangle and a parallelepiped under various boundary conditions are solved.

2. STRAIN-SPACE PLASTICITY THEORY FOR TRANSVERSELY ISOTROPIC MATERIALS

Study of elastic plastic deformations process of fibrous composite materials can be reduced to the construction of constitutive relationships for transversely isotropic materials. A constructed in this section a plasticity theory for transversely isotropic materials is based on the Ilyushin's plasticity postulate [1]. This theory is valid not only for hardening and ideal plasticity materials, but also for softening ones too. For the construction of constitutive relationships at first the alternative rule of normality is determined using the Ilyushin's plasticity postulate. Let us represent a constitutive relation in the following form[3]

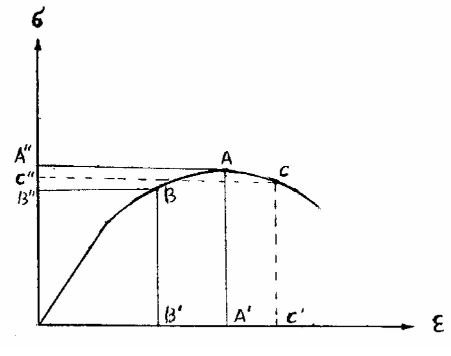
$$d\sigma_{ij} = C_{ijkl} d\varepsilon_{kl} - d\sigma_{ij}^{p} \tag{1}$$

where

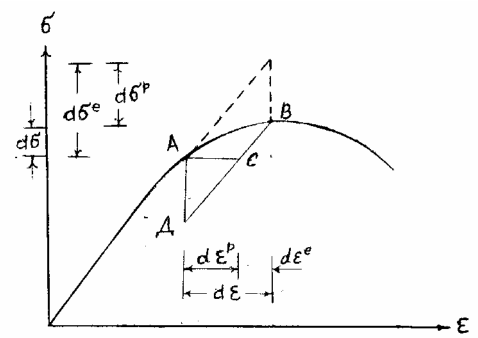
$$d\sigma_{ij}^{p} = C_{ijkl} d\varepsilon_{kl}^{p} \tag{2}$$

and $d\sigma_{ij}^{p}$ - stress tensor relaxation, C_{ijkl} - tensor of elastic module, ε_{ij}^{p}-plasticity deformation.

There are materials as concrete, rocks, a ground and some composite materials in a nature ,for which the diagram of deformation has a descending part AC. (Fig 1). It is known, that existing theories of plasticity do not allow unequivocally to describe the deformed condition of such materials. The diagram given in fig. 2 corresponds to geometrical interpretation of relation (1) in one-dimensional case



"Fig.1 Diagram of deformation for softening materials"



"Fig.2 Geometrical interpretations of rucker's and Ilyushin's plasticity postulates"

In case of the one-dimensional diagram of deformation, to the work done by external forces in a closed cycle of stress (Drucker's postulate) corresponds the area of a curvilinear triangle ABC (Fig.2) i.e.

$$W_D = \frac{1}{2} d\sigma d\varepsilon \succ 0 \tag{3}$$

The work done in a closed A- B-Д-A cycle of deformation (Ilyushin's postulate) can be found adding the area of the triangle ACД- $d\sigma^p \cdot d\varepsilon^p / 2$ to W_D i. e.

$$W_u = W_D + \frac{1}{2} d\sigma^p \cdot d\varepsilon^p = \frac{1}{2} d\varepsilon \cdot d\sigma^p > 0 \tag{4}$$

Thus, the work done by external forces on Drucker's and Ilyushin's postulates differ from each other for work of the relaxation tensor in plastic deformations. Notice that, the Ilyushin's plasticity postulate as against the Drucker's one is valid not only for hardening but also for softening materials. The last expression can be generalized and is written in the following form

$$\oint_{\sigma} d\sigma_{ij} d\varepsilon_{ij}^{p} + \int d\sigma_{ij}^{p} d\varepsilon_{ij}^{p} = \oint_{\varepsilon} d\varepsilon_{ij} d\sigma_{ij}^{p} \geq 0 \tag{5}$$

Let us assume that there exists a loading surface in the strain-space

$$F\left(\varepsilon_{ij}, \sigma_{ij}^{p}, K\right) = 0 \tag{6}$$

where K is the parameter characterizing the history of loading. From (5) we find the following normality rule

$$d\sigma_{ij}^{p} = d\tilde{\lambda} \frac{\partial F}{\partial \varepsilon_{ij}} \tag{7}$$

where $d\tilde{\lambda}$ is the differential parameter.

A strain tensor which is invariant concerning a rotations around of axis OX_3 may be presented as the following orthogonal decomposition

$$\varepsilon_{ij} = \frac{\tilde{\theta}}{2}\left(\delta_{ij} - \delta_{i3}\delta_{j3}\right) + \varepsilon_{33}\delta_{i3}\delta_{j3} + p_{ij} + q_{ij} \tag{8}$$

where

$$p_{ij} = \varepsilon_{ij} + \frac{\tilde{\theta}}{2}\left(\delta_{i3}\delta_{j3} - \delta_{ij}\right) + \varepsilon_{33}\delta_{i3}\delta_{j3} - \left(\varepsilon_{i3}\delta_{j3} + \varepsilon_{3j}\delta_{i3}\right) \tag{9}$$

$$q_{ij} = \varepsilon_{i3}\delta_{3j} + \varepsilon_{3j}\delta_{3i} - 2\varepsilon_{33}\delta_{i3}\delta_{j3}, \tag{10}$$

$$\tilde{\theta} = \varepsilon_{11} + \varepsilon_{22}, \quad \varepsilon_{33}, \quad p^2 = \frac{1}{2} p_{ij} p_{ij}, \quad q^2 = q_{ij} q_{ij} \tag{11}$$

In case of transversely isotropy , there are two loading surfaces in the strain space. They are connected with the quadratic invariants of the strain tensor about rotation round OX_3 axis

$$F_p = \frac{1}{2} p_{ij} p_{ij} - R_3(P) = 0, \quad F_q = \frac{1}{2} q_{ij} q_{ij} - R_4(Q) = 0 \tag{12}$$

where tensors p_{ij} and q_{ij} are determined with the use of equations (9), (10); P, Q are the functions characterizing the history of loading.

According to the equation (9-10) P and Q have the following forms:

$$P = \int p_{ij} dP_{ij}^{p}, \qquad Q = \int q_{ij} dQ_{ij}^{p} \tag{13}$$

In case of transversely isotropy we derive from the normality rule (7)

$$dP_{ij}^{p} = d\lambda_3 \frac{\partial F_p}{\partial p_{ij}}, \qquad dQ_{ij}^{p} = d\lambda_4 \frac{\partial F_q}{\partial q_{ij}},$$

$$d\sigma_{ij}^{p} = dP_{ij}^{p} + dQ_{ij}^{p} \tag{14}$$

Differentiating eq.(12) and taking into consideration eq.(13) we obtain

$$d\tilde{\lambda}_3 = H_3 \frac{\partial F_p}{\partial p_{ij}} dp_{ij}, \quad d\tilde{\lambda}_4 = H_4 \frac{\partial F_q}{\partial q_{ij}} dq_{ij} \tag{15}$$

where H_3, H_4 are the experimentally determined functions.

In case of transversely isotropy taking into consideration relationships (14-15) and (8-11) one can represent relationship (1) in the following form

$$d\sigma_{ij} = d\tilde{\sigma}\left(\delta_{ij} - \delta_{i3}\delta_{j3}\right) + d\sigma_{33}\delta_{i3}\delta_{j3} + dP_{ij} + dQ_{ij}, \tag{16}$$

where

$$d\tilde{s} = (\lambda_1 + \lambda_4)d\tilde{\theta} + \lambda_2 d\varepsilon_{33}$$
$$d\sigma_{33} = \lambda_2 d\tilde{\theta} + d\varepsilon_{33}$$
$$dP_{ij} = 2\lambda_4 dp_{ij} - H_3(p_{kl}dp_{kl})p_{ij} \tag{17}$$
$$dQ_{ij} = 2\lambda_5 dq_{ij} - H_4(q_{kl}dq_{kl})q_{ij}$$

loading conditions in that case are following

$$F_p = 0, \ \frac{\partial F_p}{\partial p_{ij}} dp_{ij} > 0$$
$$F_q = 0, \ \frac{\partial F_q}{\partial q_{ij}} dq_{ij} > 0 \tag{18}$$

Note that the relationships (16)-(18) allows us to investigate not only the monotonically increasing brunch (hardening) but also the decreasing brunch (softening) the deformation diagram. It is known that stress space plasticity are unacceptable to softening materials because Drucker's postulate become uncorrected in the decreasing brunch and the loading conditions becomes not unique. Another advantage of strain space plasticity theory (16)-(18) consists in their independence of stress tensor and gives a considerably facilitates in numerical solution of elastic-plastic problems in displacements because one can avoids the calculation of the stress tensor increment at every step of external load.

3. STRESS-SPACE PLASTICITY THEORY FOR TRANSVERSELY ISOTROPIC MATERIALS

A transversely isotropic tensor invariant concerning rotations around of axis OX_3 can be written down as the sum of orthogonal terms

$$\sigma_{ij} = \tilde{\sigma}(\delta_{ij} - \delta_{i3}\delta_{j3}) + \sigma_{33}\delta_{i3}\delta_{j3} + P_{ij} + Q_{ij} \tag{19}$$

where $P_{ij} = \sigma_{ij} + \tilde{\sigma}(\delta_{i3}\delta_{j3} - \delta_{ij}) + \sigma_{i3}\delta_{i3}\delta_{j3} - (\sigma_{i3}\delta_{j3} + \sigma_{3j}\delta_{i3})$ (20)

$$Q_{ij} = \sigma_{i3}\delta_{j3} + \sigma_{3j}\delta_{i3} - 2\sigma_{33}\delta_{i3}\delta_{j3}, \quad \tilde{\sigma} = (\sigma_{11} + \sigma_{22})/2 \tag{21}$$

Let us accept two yield conditions of Mises type connected with the quadratic invariants of stress tensor (19) for construction of plastic theory for transversely isotropic materials, i.e.

$$f_p = \frac{1}{2}P_{ij}P_{ij} - K_3(p) = 0,$$
$$f_q = \frac{1}{2}Q_{ij}Q_{ij} - K_4(q) = 0 \tag{22}$$

where K_3, K_4 are the hardening functions;

$$p = \int P_{ij}dp_{ij}^p, \qquad q = \int Q_{ij}dp_{ij}^p \tag{23}$$

Taking into consideration the associated flow rule of plasticity one can derives from (22)

$$dp_{ij}^p = d\lambda_3 \frac{\partial f_p}{\partial P_{ij}}, \qquad dq_{ij}^p = d\lambda_4 \frac{\partial f_q}{\partial Q_{kl}} \tag{24}$$

where $d\lambda_3$, $d\lambda_4$ are the differential parameters of proportionality.

Let us represent $d\lambda_3$, $d\lambda_4$ in the following form using the consistency condition for loading surface (22) and equations (23-24):

$$d\lambda_3 = h_3 \frac{\partial f_p}{\partial P_{kl}} dP_{kl}, \qquad d\lambda_4 = h_4 \frac{\partial f_q}{\partial Q_{kl}} dQ_{kl} \tag{25}$$

where h_3, h_4 are the scalar multipliers. Representing the general increment of deformations as a sum of elastic and plastic parts

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \tag{26}$$

where

$$d\varepsilon_{ij}^{e} = C_{ijkl}^{-1} dP_{kl}, \qquad d\varepsilon_{ij}^{p} = dp_{ij}^{p} + dq_{ij}^{p} \tag{27}$$

$$\begin{aligned} C_{ijkl} = {} & \lambda_1 \delta_{ij}\delta_{kl} + \lambda_4 \left(\delta_{ik}\delta_{jl} + \delta_{jl}\delta_{jk}\right) + \lambda_3 \delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3} + \\ & + \lambda_2 \left(\delta_{i3}\delta_{j3}\delta_{kl} + \delta_{k3}\delta_{l3}\delta_{ij}\right) + \lambda_5 \left(\delta_{il}\delta_{k3}\delta_{j3} + \delta_{jk}\delta_{l3}\delta_{i3} + \delta_{ik}\delta_{l3}\delta_{j3} + \delta_{jl}\delta_{k3}\delta_{i3}\right) \end{aligned} \tag{28}$$

multiplying relationship (26) by $\dfrac{\partial f_p}{\partial P_{mn}} C_{mnij}$ and taking into consideration equation (24) one can obtain

$$d\lambda_3 = h_3 \frac{\partial f_p}{\partial P_{mn}} C_{mnkl} d\varepsilon_{kl} \Bigg/ \left(1 + h_3 \frac{\partial f_p}{\partial P_{ij}} C_{ijkl} \frac{\partial f_p}{\partial P_{kl}}\right) \tag{29}$$

$$d\lambda_4 = h_4 \frac{\partial f_q}{\partial Q_{mn}} C_{mnkl} d\varepsilon_{kl} \Bigg/ \left(1 + h_4 \frac{\partial f_q}{\partial Q_{ij}} C_{ijkl} \frac{\partial f_q}{\partial Q_{kl}}\right) \tag{30}$$

Then using the Hook's law and equations (26)-(28), (22), (24) one can obtain the basic relationships of plasticity theory for transversely isotropic solids in the stress space

$$d\sigma_{ij} = d\tilde{\sigma}\left(\delta_{ij} - \delta_{i3}\delta_{j3}\right) + d\sigma_{33}\delta_{i3}\delta_{j3} + dP_{ij} + dQ_{ij} \tag{31}$$

where

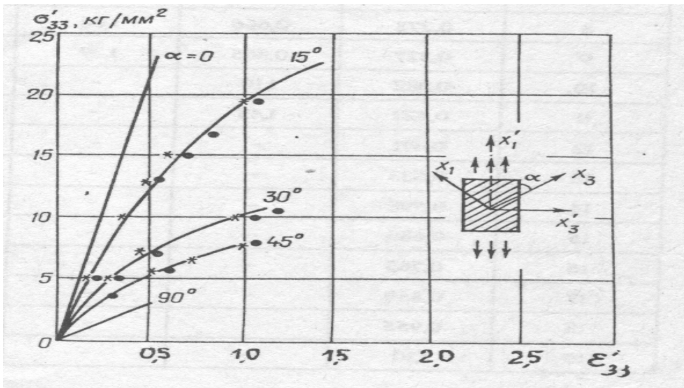
$$\begin{aligned} d\tilde{\sigma} &= \left(\lambda_1 + \lambda_4\right) d\tilde{\theta} + \lambda_2 d\varepsilon_{33} \\ d\sigma_{33} &= \lambda_2 d\tilde{\theta} + \lambda_3 d\varepsilon_{33} \\ dP_{ij} &= 2\lambda_4 dp_{ij} - \frac{4\lambda_4^2 P_{kl} dp_{kl}}{1/h_3 + 4\lambda_4 P^2} P_{ij} \\ dQ_{ij} &= 2\lambda_5 dq_{ij} - \frac{4\lambda_5^2 Q_{kl} dq_{kl}}{1/h_4 + 4\lambda_5 Q^2} Q_{ij} \end{aligned} \tag{32}$$

in that case the loading conditions have the following form

$$\begin{aligned} f_p &= 0, \qquad \frac{\partial f_p}{\partial P_{ij}} dP_{ij} > 0 \\ f_q &= 0, \qquad \frac{\partial f_q}{\partial Q_{ij}} dQ_{ij} > 0 \end{aligned} \tag{33}$$

For change to the plastic state it is sufficient to satisfy one of the loading conditions.

Fig. 3 shows the comparison of theoretical and experimental curves obtained in tension of a plane specimens cut out from unidirectional composite material(Glass/Epoxy) at different angles to fibers [5]. Theoretical curves were constructed with the use of strain space($\bullet$) and stress space ($*$) theories.



"Fig.3. Experimental and analytical stress-strain curves of unidirectional lamina under tension."

4. NUMERICAL METHODS OF PLASTICITY PROBLEMS

Usually, for numerical solution of elastic - plastic (isotropic and anisotropic) boundary problems an elastic solutions method, an initial stress method, an initial strain method, a variable elasticity parameters method and a different iterative methods[9, 10, 26, 27] are used. In connection with the proposed strain space theory became possible to modify an initial stress method. Thus the initial stress method converges not only at small hardenings and ideal plasticity, but also for softening materials.

The elastic solution method has received its substantiation and developing in the following works [4,8] and allows to reduce a nonlinear problem of the theory of plasticity to a sequence of elastic problems with a variable right part in the equations. The elastic solution method is usually applied to the solution of elastic plastic problems formulated on the basis of deformation theories.[4]

Usually, in a case of flow theories the initial strain method and initial stress method in combination with elastic solution method are used ,accounting at each stage of increment of the external force the initial stress and strains accordingly.

The initial strain method is based on the associated flow rule and strain is represented as the sum of elastic and plastic parts. The plastic strain is considered as an initial strain and displacements is found from the previous approach. We shall notice, that on each increment of external loading it is necessary to check up loading and flow conditions i.e. it is necessary to keep up the end of the stress "vector" was on a loading surface. In Zienkiewicz work [26] the approached method for satisfaction of these conditions, with introduction of auxiliary parameter is offered, but this parameter imposes restriction on value of an increment of external loading. On the other hand the convergence of the initial strain method at small hardenings and ideal plasticity is worsened.

If a constitutive relation of the flow theory to resolve concerning an stress increment tensor it is possible to find more convenient relation for the formulation of elastic plastic problems. The right part of which depends from stress tensor and an increment of strain tensor and it is convenient for application of initial stress method. In this case initial stress method it is possible to apply for hardening, ideal plasticity and softening materials. But, solving the elastic plastic problems in displacement, necessity of calculation of stress tensor and its increments at continually loading, and restriction on value of an increment of external loading remains. In connection with strain space plasticity theory it is possible to offer the modified variant of the initial stress method, without above stated lacks. The right part of constitutive relationships of strain space theory depends only from strain tensor and its increment and it is convenient for numerical realization of plasticity problems in displacement on FEM. Satisfying the loading and flow conditions also are easy and no required an introduction of additional parameters, as against the stress space plasticity theory .

Elastic plastic boundary problem consists of the balance equation, constitutive relationship of plasticity theory, Cauchy relation and corresponding boundary conditions. Constitutive relationship for plasticity theories may be write down in a such form

$$d\sigma_{ij} = C_{ijkl}\, d\varepsilon_{ij} - d\sigma^{o}_{ij} \tag{34}$$

where, $d\sigma^{o}_{ij}$ - an initial stress. The initial stress of stain space and stress space theories for isotropic materials has such forms, correspondingly:

$$d\sigma^{o}_{ij} = \frac{C_{ijtu}\dfrac{\partial f}{\partial\sigma_{tu}} C_{klpq}\dfrac{\partial f}{\partial\sigma_{pq}}}{\dfrac{1}{h} + \dfrac{\partial f}{\partial\sigma_{mn}} C_{mnpq}\dfrac{\partial f}{\partial\sigma_{pq}}} d\varepsilon_{kl} \tag{35}$$

$$d\sigma^{o}_{ij} = H\frac{\partial F}{\partial\varepsilon_{ij}}\frac{\partial F}{\partial\varepsilon_{kl}} d\varepsilon_{kl} \tag{36}$$

Last relations in case of bilinear stress-strain diagram correspondingly become

$$d\sigma^{o}_{ij} = \frac{\mu-\mu'}{\sigma_u^2}\left(s_{kl}de_{kl}\right)s_{ij}$$

$$d\sigma^{o}_{ij} = \frac{\mu-\mu'}{\varepsilon_u^2}\left(e_{kl}de_{kl}\right)e_{ij}$$

The right part of the first equation of (37) depends from stress tensor and an increment of strain tensor, and the second relation depends only from strain tensor and its increment. As it was already spoken the dependence of the initial stress from strain tensor and of its increment is convenient in numerical realization of elastic plastic problems on FEM in displacement. Thus when a loading surface considered in strain space, the loading and flow conditions too depends only of strains. The loading and flow conditions concerning strains are satisfied with unequivocal image not only for hardening and ideally plastic materials, but also for softening materials.

5. RESULTS & DISCUSSION

In this section, the above stated strain space and stress space constitutive equations are used for numerical solution of plasticity problems on equilibrium of transversely isotropic parallelepipeds and rectangulars.

The discrete analogy of plasticity problems for numerical integration are constructed by the variational-difference method. For the solving of obtained discrete non-linear equations corresponding to different constitutive relationships an initial stress method, an iterative method [5], two-step iterative method [14] are used. In section 4 also was mentioned, that the use of the strain space plasticity theory overcomes many difficulties encountered by application of stress-space plasticity theories for numerical integration.

Test examples on equilibrium of a rectangular and a parallelepiped under various boundary conditions are solved. The problem on equilibrium of a rectangular under action of regular distributed forces on opposite sides of a rectangular is solved. The problem, when on opposite sides of a parallelepiped under action of domes similar forces also is solved.

In table 1 are compared the numerical results of elastic-plastic problems of equilibrium of transversely isotropic parallelepipeds solving by aforementioned iterative methods.

	U(3,4,3)	V(3,2,1)
Iterative method	0,0479	-0,0080
Two step iterative method	0,0480	-0,0081
Initial stress method	0,0479	-0,0080

$$P^* = 0.1, \quad q^* = 0.1, \quad \lambda_7' = 1.6, \quad \lambda_9' = 0.7$$

$$C_{ijkl} = \begin{pmatrix} 5.68 & 0.21 & 0.19 & & & \\ 0.21 & 5.68 & 0.19 & & & \\ 0.19 & 0.19 & 5.35 & & & \\ & & & 2.39 & & \\ & & & & 2.39 & \\ & & & & & 2.735 \end{pmatrix} \cdot 10^5 \frac{кг}{см^2}$$

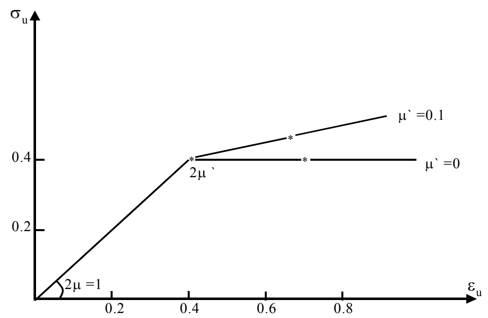
The problem on equilibrium of isotropic rectangular (Fig.5, $l_1=l_2=1$, $\lambda = 1$, $\mu = 0.5$) under the regular distributed loading (S=1) is solved. The diagram of deformation is approximated by bilinear function (Fig.4.). In tables 2-3 is shown the displacement and stress defined on initial stress method.. External loading was given in two increments. You can see from tables 2-3 that the given and calculated values σ_{22} are approximately equal.

"Table 2. u,v and σ_{22} accounting in two increments for hardening material (μ`=0.1)"

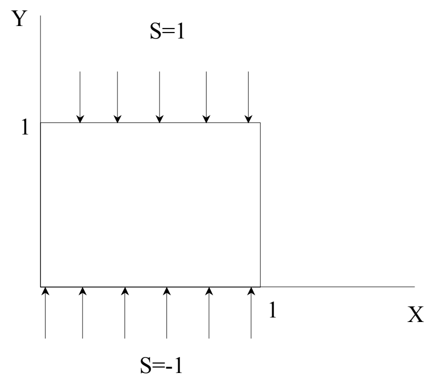
IS		u	v	σ_{22}	σ_u	ε_u	2_μ
I	I_K	-0.125	0.250	-0.750	0.375	0.377	0.994
II	1	-0.167	0.333	-0.889	0.400	0.503	0.199
	2	-0.220	0.395	-0.928	0.424	0.618	0.205
	3	-0.254	0.467	-1.050	0.445	0.724	0.197

"Table 3. u,v and σ_{22} accounting in two increments in case of ideal plasticity (μ'=0) "

IS	u	v	σ_{22}	σ_u	ε_u	2_μ
1	-0.125	0.250	-0.750	0.375	0.377	0.994
2	-0.280	0.495	-0.981	0.376	0.778	$0.434 \cdot 10^{-3}$



"Fig.4.Bilinear deformation diagram for hardening (μ' =0.1) and ideal plasticity (μ' =0) cases"



"Fig.5. Equilibrium of rectangular'

A fiber reinforced composite materials using any averaging method is replaced by transversely isotropic materials. The strain space and stress space plasticity theories for transversely isotropic materials are proposed. On the basis of comparison of experimental and analytical curves is shown the validity of the offered theories for description of plastic deformations of fiber reinforced composite materials. For the numerical solution of plasticity problems initial stress and initial strain methods and different modifications of iterative methods are applied. New variant of the initial stress method is proposed. 2D and 3D plasticity problems on equilibrium of transversely isotropic materials under action of different boundary conditions are solved.

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