A CRITERION BASED ON STRESS AND ENERGY FOR THE ONSET OF TRANSVERSE CRACKING

C. Huchette, D. Leveque, N. Carrere

ONERA, BP 72, 92322 Châtillon Cedex, France

ABSTRACT

In order to predict the first ply failure in composite laminates, fracture criteria for initiation and propagation of the transverse crack are proposed in this paper. These criteria are based on a multiscale approach. At the scale of the fibre, micro damage define initiation criterion. These micro damage are due to matrix microcracks or interfacial fibre/matrix debonding. Two models are proposed to define the damage threshold of the matrix and the interface at this scale. At the ply scale, the propagation criterion is defined by a criterion based on a fracture mechanics approach. In this way, the influence of the 90° plies thickness in $[0/90_n]$ can be taken into account on the onset of the transverse crack. By this model, this paper deals with the influence of the stacking on the onset of transverse crack for θ° plies in $[0/\pm\theta/90]_s$ laminates observed by [1]. In accord with the Leguillon's approach [2], the initiation of the transverse crack is a stress criterion and the propagation, an energy criterion and the two criteria are required to the transverse crack apparition.

1 INTRODUCTION

Laminate composite materials are a very useful answer for aerospace and automotive industrial issues. Laminate structures develop damage for very low loadings. This degradation increases progressively until the final rupture. Nevertheless, composite structures could succeed to achieve their function in presence of an important damage level. The failure process in Carbon fibre reinforced plastics (CFRP) could be described by the following scenario:

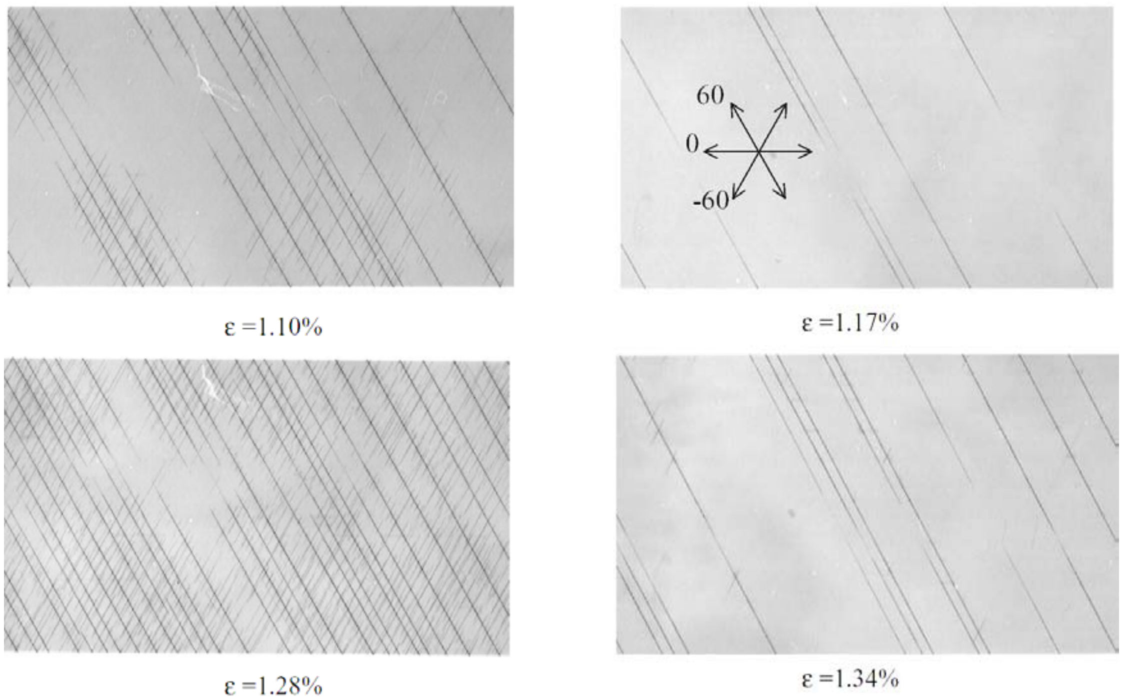
- microscopic damage such matrix microcracks and interfacial fibre/matrix debonding,
- coalescence of the previous micro damage,
- transverse cracks at the mesoscopic scale and inter ply debonding (delamination),
- fibres failure

The deterioration process must be fully understand and describe in order to improve the design of composite structure. In this frame, the objectives of this work are to propose a model with (i) a better description of the first mesoscopic crack and (ii) the interaction between intra and interlaminar damage.

Lot of authors [3-6] have developed mesoscopic models in the continuum damage mechanics frameworks. Moreover, the majority of them are not able to simulate two main problems which are the ply thickness and the stacking sequence influences. Another approach to study the degradation of laminated composites consists of using models base on micromechanics framework (see Berthelot [7] for a review); nevertheless this kind of models which take into account the reality of the presence of the transverse crack and its influences can not be used in a finite element structure analysis. In this approach, some authors [8] suggest criterion based on a critical stress or strain in the ply and other ones [9,10] criterion based on a critical strain energy release rate. Leguillon [2] shows after the experiences of Parvizy [10] that energy criterion and stress criterion must both be satisfied to create a meso crack. Boniface *et al.* [11] show in cross-ply laminates notched specimens, that mesoscopic cracks appear sooner in thick 90° plies than in the other case (thin plies, notched and/or unnotched coupons). These authors demonstrate that the limiting criterion for the first ply failure is the crack initiation for thick plies and on the opposite, the limiting criterion is the propagation of the crack initiated for thin plies laminate.

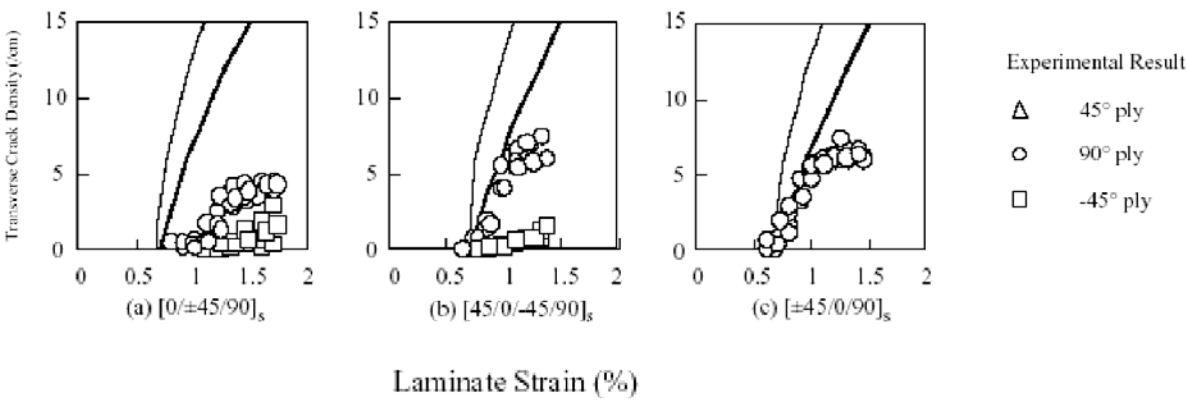
Recently, Kobayashi [1] and Yokoseki [12] show the influence of the stacking sequence on the onset of transverse cracking. Fig. 1. shows the influence of the relative position of the ±60° ply on the onset of transverse crack in this ply. When the −60° plies are close to the

+60° plies which are the first damaged due to their thickness (case a/), the onset of transverse crack is earlier than where the 0° ply separates the 60° plies to the -60° ones (case b/). The same conclusion can be made with quasi-isotropic laminates composed of 0°, ±45° and 90° plies [1] (see Fig. 2.). The onset of transverse crack in the -45° ply depend on its position: close to the 90° ply, the onset is sooner than separated of the 90° ply by a 0° one.
After the presentation of the methodology considered to realize the simulation (section 2), the present paper proposes to deal with the initiation and the propagation of the mesoscopic crack in section 3. To explain this initiation, numerical simulation at the microscopic scale have been performed. At the mesoscopic scale, the propagation is studied using a mechanical fracture approach. Results to explain ply thickness and stacking sequence effects are presented in this section. Finally section 4 is devoted to the discussion and the conclusion.



a/ $[0_2/-60_2/60_2]_s$ b/ $[0/-60_2/0/60_2]_s$

“**Fig. 1**.Influence of the stacking sequence in a carbon/epoxy laminate [12]”



“**Fig. 2.**Influence of the stacking sequence in a quasi-isotrope (carbone époxy) [1]”

2 METHODOLOGY

2.1 Micro scale

A periodic representative volume element is considered to compute the local stress and strain fields (see Fig. 3.). The fibre volume ratio is fixed to 0.6. The generalized plane strain assumption is made in the fibre's direction. A perfectly bonded interface or a perfectly debonded interface (no penetration and no friction) is used. The unit cell is loaded using the three traction components of the stress tensor and the in plane shear. A linear isotropic elasticity behaviour is considered for the matrix and a linear transverse isotropic elasticity behaviour for the fibre. In the case of the perfectly bonded interface, by the superposition theorem, only the four elementary loads are needed to know the local fields. On the opposite, in the case of a perfectly debonded interface, due to the non linearity induce by the fibre debounding, an extrapolation method have been used in order to approximate the response of the unit cell under a random loading. A polynomial approximation of degre n of a function of the four stress tensor components could be written as follow:

$$P(x_1, x_2, x_3, x_4) = \sum_{p+q+r+s \leq n} a^{pqrs} x_1^p x_2^q x_3^r x_4^s \quad (1)$$

The identification of the coefficients a^{pqrs} is performed either by interpolation or by approximation in a square sense. This identification depends on the number of numerical simulations. Let A denote the coefficients vector a^{pqrs} of size p, X denote a matrix of size $n \times p$ with n the number of simulation, X_{ij} is the value of the monome j of the simulation i and Y the solution vector of the n simulations, we have:

$$Y_i = X_{ij}.A_j \quad (2)$$

with implicit summation on repeated indices. A can be expressed in the following form:

$$A = ({}^t XX)^{-1}\, {}^t X.Y \quad (3)$$

a/ Perfectly bonded interface b/ Perfectly debonded interface

"**Fig. 3.** Unit cell used for the microscale computation"

2.1.1 Interface

A very simple failure criterion has been considered for the interface. It is a linear combination of the normal and tangential component of the stress tensor at the interface.

$$\frac{\sigma_n}{\sigma_c} + \frac{\tau_n}{\tau_c} = 1 \quad (4)$$

With σ_n et σ_t the normal and tangential stress at the interface. This present study has only used the normal component. Figs. 5-7. show the failure envelopes for different critical stress in function of the meso stresses.

2.1.2 Matrix

The matrix behaviour is defined by a model developed in the framework of continuum damage mechanics. The goal of this work is not to define the behaviour of the matrix but its failure criterion. This criterion is defined by the damage threshold of the behaviour law. This formulation is a simplification of the tensorial damage model developed by Maire [6] and Pottier [13]. In isotropic material, like polymer matrix, there is no preferential damage direction and, in this case, the damage is oriented by the load. To deal with this type of behaviour, a solution is a tensorial damage formulation. Let σ and ε denote the local stress and strain in the matrix, the thermodynamic force ***Y*** associated with the damage variable is defined by:

$$Y = \sigma . \varepsilon^{+} \tag{5}$$

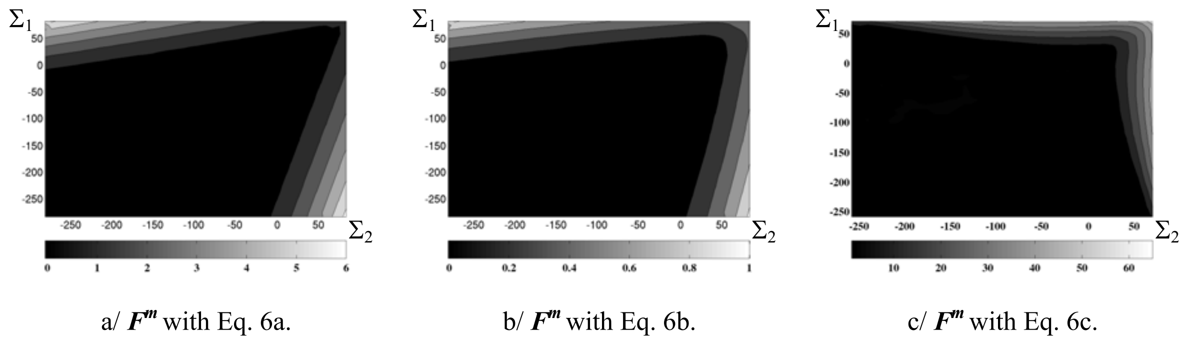
where ε^{+} is the positive part of the strain tensor defined by the usual spectral decomposition of Ju [14]. The initiation of the damage is commonly defined by a threshold value ($\boldsymbol{Y_c}$) of a norm of this thermodynamic force. It is important to note that in the present paper the goal is to obtain a criterion for the initiation of the transverse crack, which is only a function of the meso stress $\boldsymbol{\Sigma}$. Hence, an average of the thermodynamic force ***Y*** over the volume of matrix must be defined. Several choices could be taken:

$$F^m(\Sigma) = \max_{\text{Matrix}} \left\| Y^+(\sigma, \varepsilon) \right\| \tag{6a}$$

$$F^m(\Sigma) = \left\langle \left\| Y^+(\sigma, \varepsilon) \right\| \right\rangle_{\text{Matrix}} \tag{6b}$$

$$F^m(\Sigma) = \left\| Y^+\left(\langle \sigma \rangle_{\text{Matrix}}, \langle \varepsilon \rangle_{\text{Matrix}} \right) \right\| \tag{6c}$$

where $< >_{\text{Matrix}}$ defined the mean value over the matrix volume and || || the quadratic norm.
Fig. 4. show the value of $\boldsymbol{F^m}$ for a perfect bonded interface in function of the definition of $\boldsymbol{F^m}$. It is interesting to note that the shape of the damage envelope is a function of the chosen norm (see Fig. 4. a/ and Fig. 4. c/). For the case of a debonded interface, 625 simulations have been performed to identify a polynomial approximation (see section 2.1). The number of identification points can be quadrupled because of the symmetry of the two transverse axes and because the thermodynamic force doesn't depend on the shear stress sign. This approximation is used to determine the initiation of damage in the matrix. When the matrix failure criteria are defined, the initiation of the transverse crack depend on the state of the interface.



a/ $\boldsymbol{F^m}$ with Eq. 6a. b/ $\boldsymbol{F^m}$ with Eq. 6b. c/ $\boldsymbol{F^m}$ with Eq. 6c.

"**Fig. 4.** Influence on the matrix criterion of the norm choice"

2.2 Meso scale

At the meso scale, the linear fracture mechanics assumption is used to describe the apparition of the first meso crack. The aim of these computations is to obtain the variation of the potential energy in the presence of a transverse crack. The energical balance gives, in absence of kinetic dissipation:

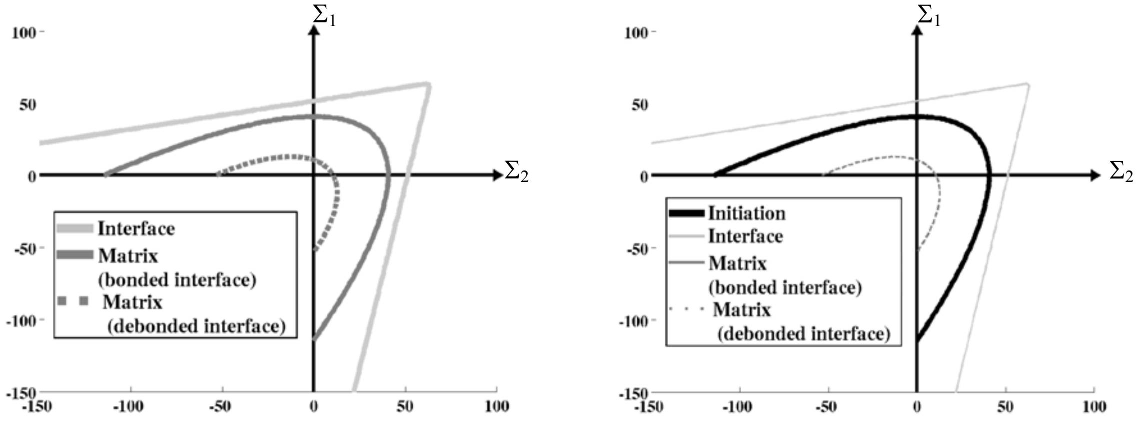
$$G_c S + \Delta W = 0 \tag{7}$$

with $\boldsymbol{G_c}$ the toughness of the ply (critical energy release rate needed to create crack inside the ply), $\boldsymbol{S}$ the free surface area due to the transverse crack and $\boldsymbol{\Delta W}$ the variation of the potential energy. It could be interesting to know in which zone and how the energical balance is achieved. The variation of the potential energy is determined by the difference of the potential energy contained in the laminate without and with a transverse crack. The studied sequences are $[0/90_n]_s$, $[0/\pm\theta/90]_s$ and $[\pm\theta/0/90]_s$ laminates where θ takes 30°, 45° and 60° and the transverse crack is located in the 90° ply. These computations have been made with a 3D finite elements model. These laminates are submitted to a tensile loading. Each ply have a linear transverse isotropic elastic behaviour (fibre's direction as the transverse isotropic axis). The loading is applied using nodal forces far from the zone of interest to retrieve the classical laminate theory. In each case, the average macroscopic stress witch permits to satisfy the energical balance, has been computed.

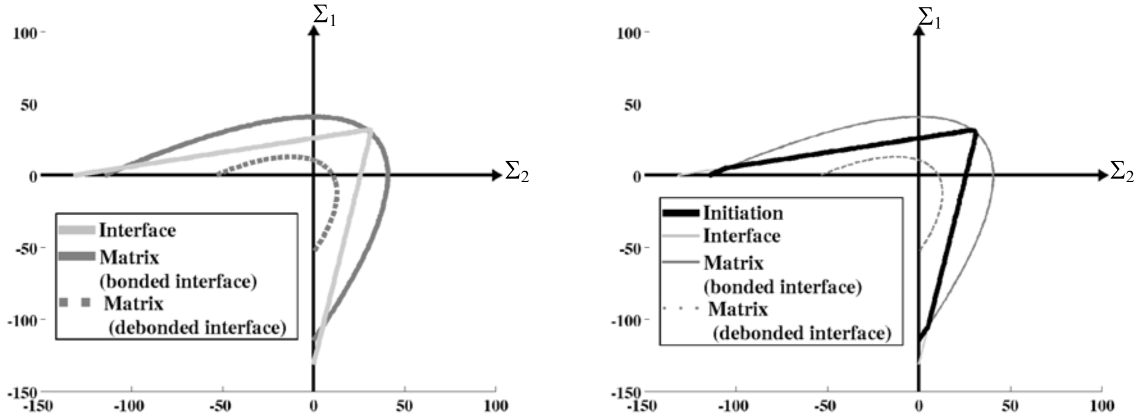
3 NUMERICAL RESULTS

3.1 Initiation

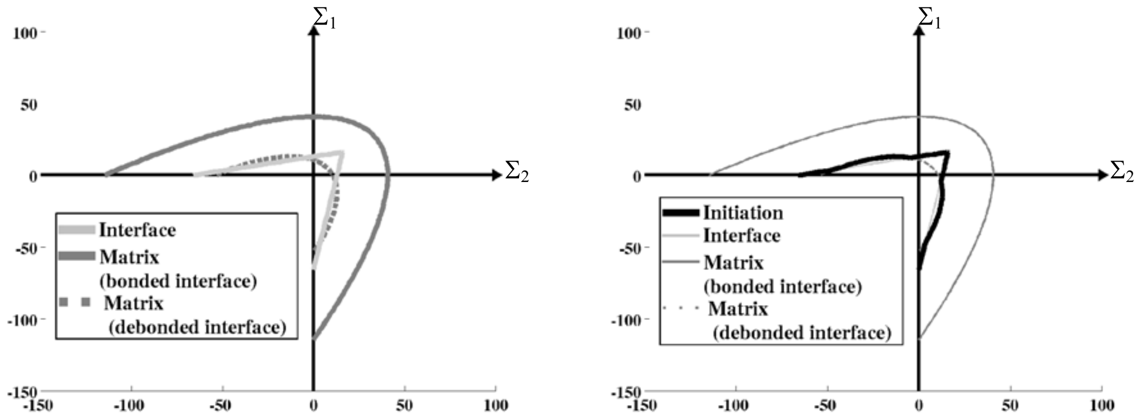
The initiation of the transverse crack corresponds to the initiation of micro damage in the matrix. In order to show the influence of the material parameters and of the loading, three cases were under investigation. For the three cases the damage threshold in the matrix is constant (Y_c=0.5 J/m^3), the loading is composed with the two transverse tensile components of the meso stress tensor (the others are null). $\boldsymbol{\sigma_c}$ of Eq. 4. is set to 20 , 40 and 80 MPa (see Figs. 5-7.). Fig. 5. shows that the matrix envelope, with debonded interface, is included in the matrix with bonded interface envelope. It is clear that, in absence of binding with the fibre, the transverse loading is fully support by the matrix and the damage threshold is achieved sooner in the matrix with a debonded interface. In consequence, the initiation of the transverse crack is hardly dependant of the state of the interface. There are two possibilities. The matrix damage appears either before the interface rupture or after the interface rupture. In the first case (see Fig. 5.), the limiting criterion is the matrix criterion with a perfect interface because it is included in the fracture envelope of the interface. In the other cases (see Fig. 6. and Fig. 7.), the interface is weaker than the matrix with a perfect interface. The limiting criterion for the initiation of the transverse crack is either the matrix with debonded interface or the interface criteria (it depends on their relative positions). In Fig. 6., the matrix (with debonded interface) criterion is included in the interface envelope and so, the interface fracture always induces a matrix fracture. In Fig. 7., the initiation of the transverse crack is defined in function of the meso stress either by the matrix or by the interface criterion. It is important to note that the development of micro damage is not only dependent of the material parameter but also of the loading path. The damage scenario generally admitted in the literature, and presented in the introduction, is, in fact, a function of the loading induced by the ply (due to the macroscopic loading and the stacking sequence).



“**Fig. 5.**Initiation Criterion for a strong interface (σ_c =80 MPa, Y_c =0.5 J/m^3)”



“**Fig. 6.**Initiation Criterion for a intermediate strength interface (σ_c =40 MPa, Y_c =0.5 J/m^3)”



“**Fig. 7.**Initiation Criterion for a weak interface (σ_c =20 MPa, Y_c =0.5 J/m^3)”

3.2 Propagation

3.2.1 Ply thickness influence

The development of transverse cracking during static loading is hardly dependent of the ply thickness. Fig. 8. shows results of the prediction of the first ply failure for $[0/90_n]_s$ laminates using a fracture mechanics approach (see section 2.2). The obtained tendencies are very close to the literature simulations [9,15] or experimental results[10]. The average stress applied to the laminate to create the first ply failure increases as the thickness of the 90°plies decreases.

In the Fig. 8., the stress criterion used by [16] for example has been reported. The first comment is the difference of the two criteria. Recently, Leguillon [2] have suggested to use a mixed criterion based on the stress and energy criterion to forecast the onset of meso crack. The both criteria must be satisfied to allow the creation of a transverse crack. In the present approach, the stress criterion is the initiation of the transverse crack (see section 3.1) and the energy criterion describes the propagation of microdomage through the ply thickness. For thin 90° plies, the limiting criterion is the propagation criterion and, in the opposite, the initiation criterion is the limiting criterion for thick plies. Our goal is to develop a damage behaviour model able to take into account these two phenomena. The stress criterion is an intrinsic property of the ply but the energy criterion is not, *a priori*, independent of the stacking sequence.

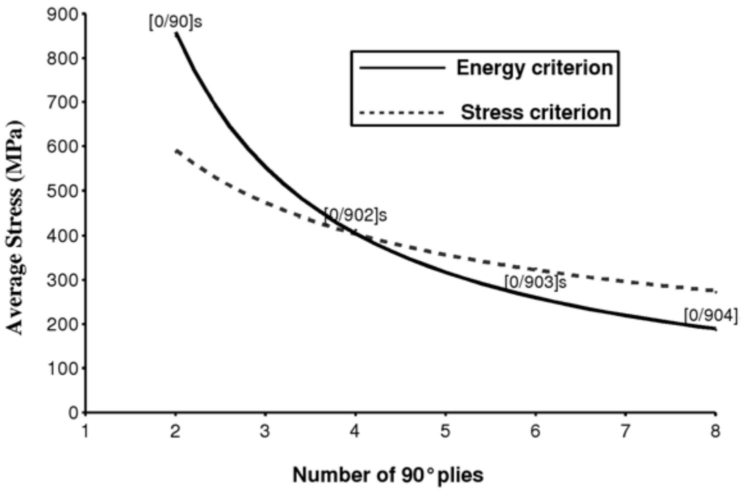
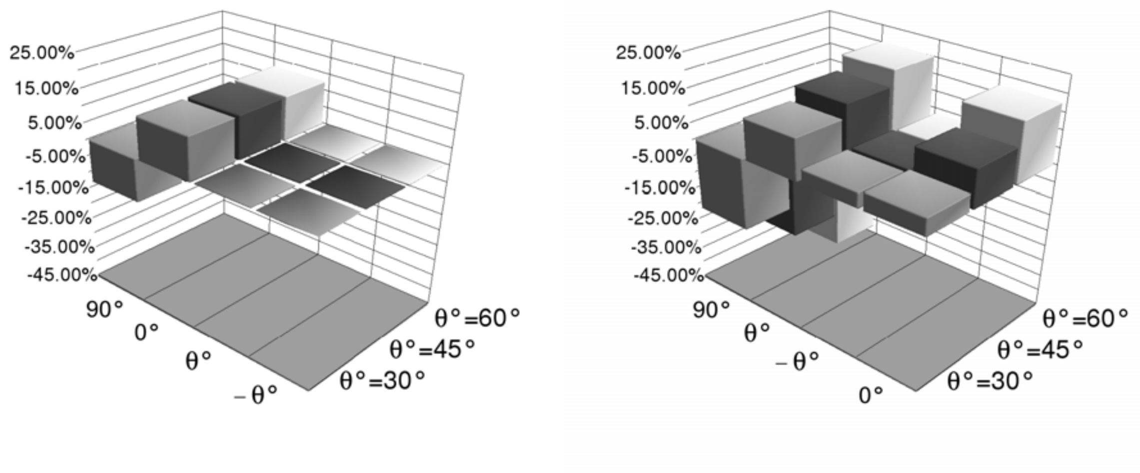


Fig. 8. Influence of the 90° ply thickness for a carbon/epoxy material (T700/M21)

3.2.2 Stacking sequence influence

In order to explain the experience of Kobayashi [1] and Yokoseki [12], the method described in the section 2.2 was applied. To create a new transverse crack in the laminate, a potential energy variation of $\boldsymbol{G_c S}$ is needed between the laminate with a crack and the undamaged laminate. Fig. 9. shows in each ply this variation of potential energy as a function of ply orientations. The value of $\boldsymbol{G_c S}$ was removed to underline the fact that the 90° ply is more unloaded than the unloading needed to create a transverse crack. This unloading implies an overloading in the adjacent plies. In the case a/, this overloading is integrally balanced by the 0° plies. In the case b/, the overloading is for a major part balanced by the θ° plies, in a minor part by the -θ° and 0° plies. It is important to note the fact that the more oriented the ply is the more important is the overloading for $[0/\pm\theta/90]_s$ laminates and so the unloading in the damaged ply. In the first case a/, the overloading is independent of the value of θ and the variation of the potential energy in the ± θ° plies is nearly null. These results can explain why the ±θ° plies is damaged sooner in the case b/ than in the case a/. In the case a/, the presence of the 90° ply mesoscopic crack has no influence on the ±θ° plies behaviour. The onset of damage in these plies could be predict by a standard model witch does not take into account the stacking sequence effect. On the opposite (case b/), the overloading of the ±θ° plies explain the sooner onset of transverse crack in these plies and underline the necessity of taking into consideration this effect.



a/ $[\pm\theta/0/90]_s$ b/ $[0/\pm\theta/90]_s$

Fig. 9. Influence of the stacking sequences on potential energy variation

4 DISCUSSION AND CONCLUSION

The first ply failure in a laminate is a consequence of the presence of micro damage in the matrix and the propagation of this micro damage through the ply thickness. The initiation can be relied on the onset of damage in the matrix. This onset is hardly dependent of the state of the interface. Results presented in section 3.1 show that the initiation of transverse crack could be due to matrix damage or due to interface debonding in function of the properties of the constituents. This section also demonstrates the importance of the loading on the succession and the type of micro damage. By the present approach, any assumption is made on the limiting and on the sequence of the damage. The only hypothesis is made on the necessity of damage onset in the matrix but an interfacial debonding could be sufficient to initiate a transverse crack in the case it induces a matrix damage.

Section 3.2 shows the influence of the thickness of the ply and of the stacking sequence on the propagation of these micro damage. The overloading of the θ° plies of the Fig.9.b/ due to the closeness of damaged 90°-ply can explain the experiments of Kobayashi [1] and Yokoseki.[12]. In order to develop a damage model able to forecast the onset of transverse crack, the section 3.2 underlines the importance of the stacking sequence of the laminate on the kinetic of the damage in each ply. The stress criterion could be easily considered as an intrinsic property of the ply but the propagation criterion seems depend on the adjacent plies orientation. The reality of the meso crack (discrete nature) is not taken into account by a damage model. This limitation induces on error on the real stress field in each ply due to the presence of the crack and only represent the effect of the transverse crack on the loss of stiffness. These simulations show that a correction on the kinetic of the damage must be done to simulate the damage process. A mesoscopic model, witch would permit to take into account the stacking sequence and the thick ply effects in a FE structure calculation, is now under development at ONERA.

ACKNOWLEDGEMENTS

This work was carried out under the AMERICO project (Multiscale Analyses: Innovating Research for Composites) directed by ONERA (French Aeronautics and Space Research Center) and funded by the DGA/STTC (French Ministry of Defence) which is gratefully acknowledged.

References

1. **Kobayashi, S.;Takeda, T.;Ogihara, S. ;Kobayashi, A.** "Effects of stacking sequence on the transverse cracking in quasi-isotropic interleaved CFRP laminates", *Proc of 12th International Conference on Composite Material*, (1999), ID 1111.
2. **Leguillon, D.** "Strength or toughness? A criterion for crack onset at a notch", *Eur. J. Mech. A-Solid*, **21** ER1 (2002), 61-72.
3. **Allen, D.H; Harris, C.E.; Groves, S.E** "A thermomechanical constitutive theory for elastic composites with distributed damage", *Int J. Solids Struct.*, **23**/9 (1987), 1401-1338.
4. **Perreux, D.; Oytana C.** "Continuum damage mechanics for microcracked composites", *Comp. Eng.*, (1993), 155-122.
5. **Ladeveze P.; Lubineau G.** "On a damage mesomodel for laminates: micromechanics basis and improvement", *Mech Mater.*, **35**/8 (2003), 763-775.
6. **Maire, J.-F; Chaboche, J.-L** "A new formulation of continuum damage mechanics (CDM) for composite materials", *Aero Sci Tech*, **4**/ (1997), 247-257.
7. **Berthelot, J.-M.** "Transverse cracking and delamination in cross-ply glass-fiber and carbon-fiber reinforced plastic laminates: Static and fatigue loading", *Appl Mech Rev*, **56**/1 (2003), 111-147.
8. **Berthelot, J.-M.; Le Corre, J.-F.** "Statistical analysis of the progression of transverse cracking and delaminage in cross ply laminates", *Compos Sci Technol*, **60**/14 (2000), 2659-2669.
9. **Nairn, JA** "The Strain Energy Release Rate of Composite Microcracking: A Variational Approach", *J. Comp. Mater.*, **23**/11 (1989), 1106.
10. **Parvizi, A;Garrett, K. W.; Bailey, J. E.** "Constrained cracking in glass fibre-reinforced epoxy cross-ply laminates", *J. Mater. Sci.*, **13** (1978), 195-201.
11. **Boniface, L.; Smith, P.A.; Bader, M.G.** "Transverse ply cracking in cross ply CFRP laminates - Initiation or propagation controled ?", *J. Comp. Mater.*, **31**/11 (1997), 1080-1112.
12. **Yokozeki,Tomohiro; Aoki,Takahira; Ishikawa,Takashi** "Transverse cracking in CFRP laminates with angled plies", *43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural*,(2002), AIAA 2002-1255.
13 **Pottier, T** "Modélisation multiéchelle du comportement et de l'endommagement de composite à matrice métallique", PhD thesis Ecole Nationale des Ponts et Chaussée (Paris) (1998)
14. **Ju J.W** "On energy-based coupled elastoplastic damage theories: constitutives modeling and computational Aspect", *Int J. Solids Struct.*, (1989), 803-833.
15. **Nairn, J.A.** "Exact and variational theorems for fracture mechanics of composites with residual stresses, traction-loaded cracks, and imperfect interfaces", *Int. J. Fracture*, **105**/3 (2000), 243-271.
16. **Berthelot, J.-M.; Le Corre, J.-F.** "A model for transverse cracking and delamination in cross-ply laminates", *Compos Sci Technol*, **60**/7 (2000), 1055-1066.