

THERMO-MECHANICALLY INTERACTING COMPOSITE PIPES OPTIMISED BY GENETIC ALGORITHMS

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ABSTRACT

In many applications pipes made of composite materials offer properties that are superior to those of pipes made of other materials. A laminate setup involving continuously fibre reinforced laminae is often chosen. Although some design rules exist for continuously fibre reinforced composite components it is often difficult to find laminate setups that are optimal (at least in an approximate sense) owing to the large number of laminate parameters and their interdependence. A group of optimisation methods, which have the potential of finding near-optimum solutions in multi-dimensional parameter spaces, are Genetic Algorithms. These algorithms may provide suitable solutions even in the presence of non-differentiable target functions and highly nonlinear parameter constraints.

In this paper Genetic Algorithms are shown to provide reasonable solutions for a problem of unusual complexity, namely the simultaneous optimisation of two thin-walled composite pipes, which, being mechanically coupled at both ends by rigid flanges, are subjected to a combination of thermal and mechanical loads.

1. INTRODUCTION

Composite pipes are candidates for components that have to withstand high thermo-mechanical loads, and, at the same time, have to be of light weight. Continuously fibre reinforced composite components have particularly good properties when the fibres are oriented in the principal loading directions. By stacking unidirectionally reinforced laminae of varying thicknesses at varying fibre orientation angles a wide range of effective thermo-mechanical properties can be obtained. Ideally, the orientation angle and thickness of each individual ply contributes to the effective strength and the effective stiffness of the laminate in an optimised way depending on its location both in the laminate and in the component.

For setups of simple geometry and simple boundary conditions, for example rectangular plates or cylindrical pipes, it may be well possible to define close-form solutions of optimum ply thickness and fibre orientation angles, or at least to apply general design rules. With increasing complexity in terms of component geometry and loading as well as boundary conditions it becomes increasingly difficult to define optimal laminate lay-ups, since the mathematical cost of describing the problem becomes unfeasible, and the number of involved parameters increases with each ply that is allowed to have an individual angle and thickness.

In this paper we address the optimisation of a composite component that is simple enough to be described by closed-form mathematical solutions with regard to its thermo-mechanical behaviour, but already too involved for being treated with conventional, analytical optimisation methods. This component comprises two thin-walled composite pipes, which, being mechanically coupled at both ends by rigid flanges (see Fig. 1) are subjected to a combination of thermal and mechanical loads. Examples of such a component would be cryogenic pipelines that confine a pressurised fluid at cryogenic temperatures inside the inner pipe and an insulating vacuum between the inner and the outer pipe.

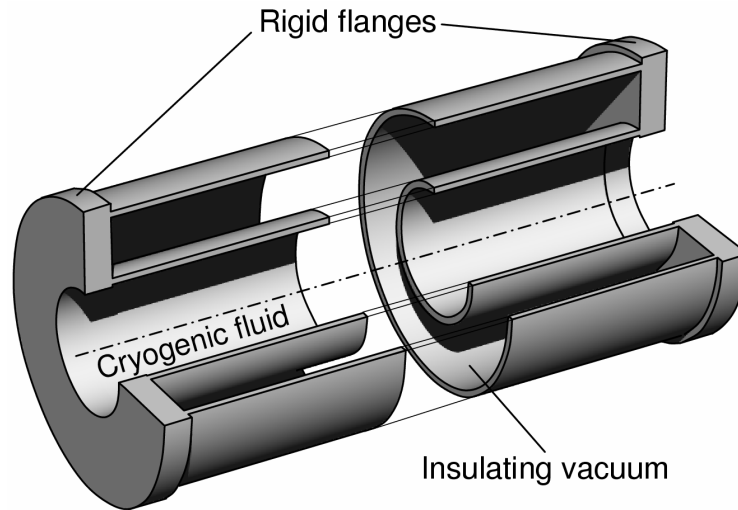


Fig. 1. Schematic cut-away view of the mechanical configuration: two co-axial composite pipes coupled at both ends by rigid flanges.

The peculiarity of this setup is the interaction of the two pipes, which is, of course, influenced by any change in the lay-up of either pipe. Furthermore, the imposed loads comprise not only mechanical loads such as axial forces, bending and torsion moments, as well as pressure loads, but also changes in temperature. For finding optimum or close-to-optimum solutions in this setting a genetic algorithm is employed. This algorithm, which is described in more detail in Section 3, allows for the simultaneous optimisation of the two coupled pipes without restricting the number laminate parameters unnecessarily.

The objective of the optimisation process is the determination of lay-up rules for double-walled composite pipes that are able to withstand the imposed loads while having the least possible weight. The target property for minimisation is, therefore, the total mass of the pipe arrangement while the optimisation is constrained by the load-bearing capacity of the pipes. Here, the failure of the pipes by first-ply failure, as well as by loss of stability due to local or global (Euler) buckling is taken into account.

2. MECHANICAL MODEL

2.1 Overview

In this section the applied analysis and optimisation methods are described briefly. The mathematical description of the thermo-mechanical behaviour of the interacting pipes is based on the assumptions of *Classical Lamination Theory* [1] for thin, shell-like structures. The kinematical coupling between the pipes is described by Bernoulli-Euler beam theory. First-ply failure is detected by means of the Tsai-Hill failure criterion. For assessing the critical loads with respect to elastic buckling analytical expressions by Vinson and Sierakowski [2] are used.

2.2 Constitutive Law

The effective thermo-mechanical behaviour of the continuously fibre reinforced lamina material can be described as being transversely elastic. Since thin-walled, shell-like structures are under consideration in this paper, the formulation of a constitutive relationship between the stress tensor and the strain tensor can be restricted to the plane stress case, for which out-of-plane stress components, for example, due to an applied pressure, are neglected.

In a reference system l - q which is aligned with the fibre direction (l) and the transverse direction (q) the thermo-elastic relationship between the vector of in-plane stress components $(\sigma_{ll}, \sigma_{qq}, \sigma_{lq})^T$ and the vector containing the in-plane normal strains as well as the shear angle $(\varepsilon_{ll}, \varepsilon_{qq}, \gamma_{lq})^T$ is given by

$$\begin{pmatrix} \sigma_{ll} \\ \sigma_{qq} \\ \sigma_{lq} \end{pmatrix} = \begin{pmatrix} E_{11} & E_{12} & 0 \\ & E_{22} & 0 \\ sym. & & E_{44} \end{pmatrix} \begin{pmatrix} \varepsilon_{ll} \\ \varepsilon_{qq} \\ \gamma_{lq} \end{pmatrix} - \Delta T \begin{pmatrix} \alpha_l \\ \alpha_q \\ 0 \end{pmatrix}, \quad (1)$$

where ΔT is the difference between the actual temperature and a stress-free reference temperature, α_l and α_q are the axial and the transversal coefficients of thermal expansion in the local coordinate system and the elements E_{ij} of the elasticity matrix can be calculated from the effective engineering constants of the composite material using the abbreviation

$$E_0 = 1 - \nu_{lq}^2 E_q / E_l \text{ as}$$

$$E_{11} = E_l / E_0, \quad E_{22} = E_q / E_0, \quad E_{12} = \nu_{lq} E_{22}, \quad E_{44} = G_{lq} \quad (2)$$

For a detailed description of the engineering constants and their actual values, see Table 1.

Table 1: Effective properties of the fibre reinforced material.

| | | | |
|---|--------------|------------------------|-----|
| Young's modulus in the fibre direction: | $E_l =$ | 135.70 | GPa |
| Young's modulus in the transverse direction: | $E_q =$ | 8.70 | GPa |
| In-plane shear modulus: | $G_{lq} =$ | 4.48 | GPa |
| In-plane Poisson's ratio: | $\nu_{lq} =$ | 0.34 | |
| Coefficient of thermal expansion in fibre direction: | $\alpha_l =$ | 0.11×10^{-6} | 1/K |
| Coefficient of thermal expansion in transverse direction: | $\alpha_q =$ | 44.80×10^{-6} | 1/K |
| Tensile strength in the fibre direction: | $X_t =$ | 2550 | MPa |
| Compressive strength in the fibre direction: | $X_c =$ | 1470 | MPa |
| Tensile strength in the transverse direction: | $Y_t =$ | 69 | MPa |
| Compressive strength in the transverse direction: | $Y_c =$ | 200 | MPa |
| In-plane shear strength: | $S =$ | 138 | MPa |

2.3 Lamination Theory

The analytical treatment of thin-walled composite structures is simplified significantly by the analytical framework of Lamination Theory, which combines Kirchhoff shell kinematics (shell cross sections remain plane and perpendicular to the middle surface of the laminate) and a mathematical description of the layered setup of laminate shells. Based on the thermo-elastic material law Eq. (1) and suitable transformations between the local lamina coordinate system and the global laminate coordinate system, a thermo-elastic relationship between the vector of sectional forces $\mathbf{N} = (N_x, N_y, N_{xy})^T$ and moments $\mathbf{M} = (M_x, M_y, M_{xy})^T$ and the vector of laminate mid-plane strains $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y, \gamma_{xy})^T$ and laminate curvatures $\boldsymbol{\kappa}$ can be obtained:

$$\begin{pmatrix} \mathbf{N} \\ \mathbf{M} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{pmatrix} - \Delta T \begin{pmatrix} \mathbf{A}_{th} \\ \mathbf{B}_{th} \end{pmatrix} \quad (3)$$

The matrices in Eq. (3) are defined as follows:

$$\mathbf{A} = \int_{-h/2}^{h/2} \mathbf{E}(z) dz = \sum_{i=1}^n \mathbf{E}_i h_i, \quad \mathbf{D} = \int_{-h/2}^{h/2} \mathbf{E}(z) z^2 dz = \sum_{i=1}^n \mathbf{E}_i \frac{z_i^3 - (z_{i-1})^3}{3} \quad (4)$$

$$\mathbf{A}_{th} = \int_{-h/2}^{h/2} \mathbf{E}(z) \boldsymbol{\alpha}(z) dz = \sum_{i=1}^n \mathbf{E}_i \boldsymbol{\alpha}_i (z_i - z_{i-1}), \quad \mathbf{B}_{th} = \int_{-h/2}^{h/2} \mathbf{E}(z) z dz = \sum_{i=1}^n \mathbf{E}_i \boldsymbol{\alpha}_i \frac{z_i^2 - (z_{i-1})^2}{2} \quad (5)$$

The index i refers to the position of the individual layers and runs from 1 to n , the total number of layers. The variable h_i contains the thickness of the layer (i), the total thickness of the laminate being h . The coordinate z is the out-of-plane position with respect to the mid-plane of the laminate.

The vector $\boldsymbol{\alpha}_i = (\alpha_x, \alpha_y, \alpha_{xy})_i^T$ in Eq. (5) holds the coefficients of thermal expansion for the laminate layer (i) expressed in the laminate reference system x - y . Also, the elasticity matrices \mathbf{E} , which are layer wise constant, have to be transformed to the laminate coordinate system. For symmetric laminates the coupling matrix \mathbf{B} vanishes identically. Since this paper deals only with symmetric laminates, the respective definition was omitted.

2.4 Effective Laminate Properties

In this study, the equations for the thermo-mechanical interaction of the two pipes are formulated in terms of effective laminate properties, see Section 2.5. These effective laminate properties comprise (among others) the effective elastic modulus E_x in the axial direction, the effective in-plane shear modulus G_{xy} and the Poisson's ratio ν_{yx} , which is necessary to calculate the longitudinal contraction of the pipes under an applied internal pressure. All these properties can be calculated from elements of the compliance matrix:

$$\mathbf{a} = \begin{pmatrix} a_{11} & a_{12} & a_{16} \\ & a_{22} & a_{26} \\ \text{symm.} & & a_{66} \end{pmatrix} = \mathbf{A}^{-1}, \quad (6)$$

which is the inverse of the sub-matrix \mathbf{A} of the laminate stiffness matrix by

$$E_x = \frac{1}{a_{11} h}, \quad G_{xy} = \frac{1}{a_{66} h}, \quad \nu_{yx} = -\frac{a_{12}}{a_{22}} \quad (7)$$

The effective in-plane thermal expansion coefficients can be calculated by inserting a temperature jump of unity, $\Delta T = 1\text{K}$, in Equation (3) and solving for the vector of in-plane strains while keeping all applied forces equal to zero. For the case of a symmetric laminate, this procedure gives the effective in-plane coefficients of thermal expansion as:

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{pmatrix} = \mathbf{A}^{-1} \mathbf{A}_{th}. \quad (8)$$

2.5 Mechanical Coupling between the Outer and the Inner Pipe

For the derivation of the following interaction equations, kinematical coupling by rigid end flanges was assumed between the two pipes. For an applied bending moment M_B , this leads to a radius of curvature ρ of the pipes' common middle axis of

$$\rho = \frac{1}{M_B} \sum_{i=1}^2 E_x^{(i)} I^{(i)}. \quad (9)$$

The superscript (i) is the index denoting the individual pipes and I is the second moment of inertia of the pipe cross-section. Knowing ρ it is straightforward to evaluate the extreme in-plane strain $\varepsilon_B^{(i)}$ due to bending for a pipe with the radius $R^{(i)}$ as $\varepsilon_B^{(i)} = \pm R^{(i)} / \rho$. Similar considerations hold for an applied torsion moment M_T .

The description of the interaction of the two coupled pipes with regard to axial thermal expansion and Poisson's effect is a little bit more involved; denoting the cross-sectional area of pipe (i) as $A^{(i)}$, the effective overall strain of the coupled pipes can be given by:

$$\varepsilon_{\text{total}} = \frac{\sum_{i=1}^2 A^{(i)} E_x^{(i)} \alpha_x^{(i)} \Delta T^{(i)} - \sum_{i=1}^2 A^{(i)} \nu_{yx}^{(i)} \sigma_{yy}^{(i)}}{\sum_{i=1}^2 A^{(i)} E_x^{(i)}}, \quad (10)$$

where $\sigma_{yy}^{(i)}$ is the effective circumferential stress due to the pressure difference on pipe (i). Corresponding sectional laminate forces can be calculated by application of Eq. (3).

2.6 First Ply Failure

By means of the compliance matrix \mathbf{a} , the laminate deformation vector can be calculated from the vector of the laminate sectional forces. By applying Kirchhoff shell kinematics it is then possible to calculate the strain state in each individual lamina. After the transformation of the lamina strain state into the local l - q coordinate system it is possible to apply Hooke's law, Eq. (1), for the transversely isotropic fibre reinforced material and to calculate the local stress state $\boldsymbol{\sigma} = (\sigma_{ll}, \sigma_{qq}, \sigma_{lq})^T$ at an arbitrary location in the lamina.

For the determination of first ply failure the Tsai-Hill failure criterion is used. In it first ply failure is assumed to occur when the criterion

$$\frac{\sigma_{ll}^2}{X^2} - \frac{\sigma_{ll} \sigma_{qq}}{X^2} + \frac{\sigma_{qq}^2}{Y^2} + \frac{\sigma_{lq}^2}{S^2} = 1 \quad (11)$$

is fulfilled. In Equation (11), X is the strength of the composite material in the fibre direction, Y is the strength of the material in the transverse direction, and S is the shear strength. According to the sign of σ_{ll} and σ_{qq} appropriate values X_t , X_c , Y_t , and Y_c have to be inserted for taking into account the difference of the respective laminate strength in tension and in compression. The effective strength parameters used in this study are summarised in Table 1.

2.7 Analytical Assessment of Elastic Buckling

An *efficient* assessment of the safety against elastic buckling is an important prerequisite for the application of a genetic algorithm for the presented problem, because these algorithms rely on the evaluation of thousands of different configurations for finding near-optimum solutions. With this fact in mind an analytical approach (as opposed to a repeated application of the Finite Element method) was chosen for the assessment of the buckling resistance of individual pipe configurations.

The required analytical relationships were taken from Vinson and Sierakowski [2], who propose expressions for the calculation of buckling loads for circular cylindrical composite shells. Their formulae cover the following load cases:

- (1) Buckling due to a critical axial compressive sectional force N_X^* ,
- (2) Buckling due to bending, causing a critical compressive sectional force N_B^* ,

- (3) Buckling of long cylinders due to a critical external pressure p^* ,
- (4) Buckling due to a critical torsion load M_T^* .

Most of these formulae are restricted to symmetric laminates, that is, laminates with vanishing coupling matrix $\mathbf{B} = \mathbf{0}$, which does not pose a constraint in the context of this study.

Presenting the formulae in their entirety would be beyond the scope of this paper. We refer to [2] for more details. It should be noted, however, that in [2] the critical (buckling) loads for axial compression and bending are expressed in terms of the number of half waves m in the axial and of waves n in the circumferential direction. Finding the critical loads involves the minimisation of the buckling loads with respect to the wave numbers m and n , which poses an optimisation problem of its own right!

In a parallel study [3], critical buckling loads obtained by the formulae mentioned above were checked against the results of supplementary Finite Element simulations, and were found to agree well.

As far as the interaction of the individual load contributions was concerned, the following heuristic stability condition was used for assessing the safety ξ against buckling:

$$\text{Max} \left(\frac{N_x^{(X)}}{N_x^*(m,n)} + \frac{N_x^{(B)}}{N_B^*(m,n)} \right) + \frac{M_T}{M_T^*} + \frac{p}{p^*} < \frac{1}{\xi} \quad (12)$$

Comparison against results of linear stability analyses carried out with Finite Element models showed this criterion to be conservative enough for buckle-safe design.

2.8 Geometry and Loading Conditions

The investigated double-walled pipe configuration is considered representative for a straight section of a cryogenic pipeline with a pressurised fluid being transported at cryogenic temperatures inside the inner pipe and an insulating vacuum being confined within the gap between the inner and the outer pipe. The geometrical parameters of the pipe section are given in Table 2.

Apart from the internal stresses, which arise from the thermal expansion mismatch between the two pipes, and the stresses caused by the pressure differences, the pipe is designed to withstand a bending and a torsion moment acting at the same time. The magnitudes of these moments along with the other load parameters are summarised in Table 3.

Table 2: Geometrical properties of the investigated pipe.

| | |
|---------------------------|---------------------|
| Pipe length | $L = 2,000.0$ mm |
| Mean radius of inner pipe | $R^{(i)} = 16.0$ mm |
| Mean radius of outer pipe | $R^{(o)} = 31.5$ mm |

Table 3: Loads applied to the investigated pipe.

| | |
|--|-----------------|
| Total applied bending moment | $M_B = 200$ Nm |
| Total applied torsional moment | $M_T = 300$ Nm |
| Atmospheric pressure on outer pipe | $p_o = 0.1$ MPa |
| Fluid pressure on inner pipe | $p_i = 0.5$ MPa |
| Environmental temperature (outer pipe) | $T_o = 300$ K |
| Fluid temperature (inner pipe) | $T_i = 20$ K |

3. GENETIC ALGORITHM

3.1 Introduction

Genetic algorithms are based on a directed, but nevertheless random search of the parameter space and rely on the simulation of natural phenomena such as inheritance and mutation for finding parameter sets that lead to an advantageous overall behaviour [4,5]. Hence, these algorithms require the evaluation of a large number of configurations, and it was considered infeasible to perform Finite Element analyses for each trial configuration. The analytical framework outlined in Section 2 allows for a fast evaluation of designs with respect to the 'fitness' of individual pipes, which was expressed in terms of the total component mass. The implementation of the algorithm was based on the C++ library GALib [6].

From the viewpoint of the genetic algorithm the whole pipe arrangement is treated as an individual [7]. Therefore, the 'chromosome', that is, the data packet containing all laminate parameters, has to carry information about the configurations of both pipes. The 'genes', that is, the individual data members of the chromosome, have to store the ply orientations and the thicknesses of both pipes' laminates. To keep the size of the chromosome fixed, it is assumed that the number of plies in each laminate is constant.

3.2 Configuration of the Chromosome

For avoiding unnecessary scatter in the laminate parameters both the ply orientations and the ply thicknesses are varied in fixed increments. The allowable values for the ply orientations are in the range of -90° to $+85^\circ$, with increments of 5° . The ply thicknesses can be chosen from a range of 0.01 mm to 1 mm in steps of 0.01 mm. Since only a limited number of discrete values are allowed for each laminate parameter, it is possible to represent all genes as integers which are within the range of 0 to k_i-1 , where k_i is the number of possible discrete values of the optimization parameter represented by the considered gene (i).

For a symmetric laminate only the properties for the plies of one half are needed for the description of the whole stacking sequence. Furthermore, a balanced stacking is prescribed by means of a $\pm\alpha$ stacking order. This means that the properties of one ply fully describe a pair of two consecutive plies. Due to these facts the size of the chromosome is only one quarter of the size necessary to store the parameters of all plies in the laminate individually.

3.3 Implementation of the Constraints

Most of the optimization constraints are fulfilled by means of the chromosome configuration. These constraints are the ones of symmetric and balanced stacking, the requirement of not having more than four consecutive plies with equal orientations (with the exception of $\pm 90^\circ$ plies) and the limitation of using discrete angle and thickness values.

All remaining constraints are implemented by a simple 'death penalty' method [8]. This means that the fitness value of an individual is set to zero if an optimization constraint is not met.

In Fig. 2 the procedure of fitness determination is depicted. After decoding the chromosome with the help of suitable look-up tables, the laminate thicknesses and the validity of the stacking sequences are checked. Then the laminate stiffness matrices and the interaction of the pipes are evaluated for the subsequent stability and stress analyses. If a given pipe arrangement does not fail due to loss of stability or ply failure the fitness of the individual is evaluated and returned to the genetic algorithm.

4. DISCUSSION

In Section 3, two different lay-ups that may be considered as being close to an optimum configuration were presented. They both had the same total mass owing to an identical thickness of both pipes, but different lay-ups regarding the orientation and the thickness of the individual laminae. While the mass is the same and both solutions provide sufficient safety against first ply failure and buckling the respective safety factors are quite different. The limiting factor is in both cases the safety factor against elastic buckling, which is equal to 1.51 in the case of design (a) and 1.58 in the case of design (b). The higher safety of design (b) is even more pronounced with respect to first ply failure. In both cases the outer pipe is stressed more with the safety factor for first ply failure being 7.0 for design (a) and 13.7 for design (b). Taking into account that both configurations have the same total mass, design (b) may be considered the preferable one.

As far as similarities of the two designs are concerned it has to be noted how the circumferential bending stiffness of the outer pipe is provided by laminae with fibres that are oriented in the circumferential direction. This arrangement is obviously beneficial for resisting the pressure difference between the external atmospheric pressure and the insulating vacuum between the two pipes.

5. CONCLUSIONS

Optimisation by means of Genetic Algorithms proved to be a valuable tool for the design of coupled composite pipes. While it was impossible to empirically foresee the influence of changing individual lay-up parameters due to the interaction of the two pipes, the employed genetic algorithm demonstrated its ability to generate potentially optimal parameter sets for suitable lay-up configurations.

Prior to the implementation of the evolutionary optimisation procedure the presented problem was tackled by means of a heuristic iterative design approach that led to a valid design with an inner pipe wall thickness of 0.6 mm and an outer pipe wall thickness of 1.2 mm [3]. The resulting mass was only 2% larger than the one of the ‘optimised’ configurations. This fact can be interpreted in two ways: firstly, it is entirely possible to come up with a feasible design by application of sound engineering principles and successive refinement, if given enough time. But, secondly, it is also possible to achieve similar and even better results in an automated laminate design approach driven by a genetic algorithm. The possibility of quickly adapting the optimisation algorithm to changing loading and boundary conditions may be considered a great advantage whenever many different configurations have to be designed.

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