FINITE ELEMENT MODELLING AND DYNAMIC ANALYSIS OF LASER-WELDED SANDWICH PANELS

Evgeny Barkanov[1], Eduards Skukis[1] and Hans Kolsters[2]

[1] Institute of Materials and Structures, Riga Technical University, Kalku St. 1, LV-1658, Riga, Latvia
[2] Department of Aeronautics, Royal Institute of Technology, Teknikringen 8, SE-100 44, Stockholm, Sweden

ABSTRACT

Numerical modelling and dynamic analysis of laser-welded sandwich panels are carried out using the broken line sandwich finite elements. To homogenize material properties of complex sandwich core with evenly spaced vertical webs and low-density core material, two approaches have been applied. The first is connected with the rule of mixture and the second – with a calculation of equivalent stiffnesses. In order to describe the rheological behaviour of viscoelastic core materials under dynamic loading, the complex modulus representation is used. Dynamic characteristics of laser-welded sandwich panels are evaluated by the method of complex eigenvalues and from the resonant peaks of the frequency response function. To estimate correctness of the methodology developed, the ANSYS solutions for empty I-core steel sandwich beams and frequency response experimental measurements for different types of laser-welded sandwich beams with longitudinal and transverse webs and foam materials have been performed. Good agreement between different theoretical results and experiment is observed. With the purpose to increase the damping properties of laser-welded sandwich panels with a low-density core material, an influence of the cross-section parameters on their dynamic behaviour has been studied additionally.

1. INTRODUCTION

Laser-welded sandwich panels (Fig. 1) with evenly spaced vertical webs and low-density core material are used recently in advanced ship constructions [1,2] combining high specific stiffness and strength with other advantages such as impact resistance, thermal-acoustic insulation and the possibility to integrate different systems inside. Such sandwich constructions are not only well established in areas where low structural weight is a prerequisite but is also gaining ground where automated production and modular assembly are equally important. Due to considerable increasing of applications of steel-composite lightweight sandwich panels in last time, the present investigations are devoted to the development of finite element models and corresponding methods of dynamic analysis for such structures.

2. FINITE ELEMENT MODELLING

In the present investigation the finite element method has been used to model and analyse sandwich structures with a complex core.

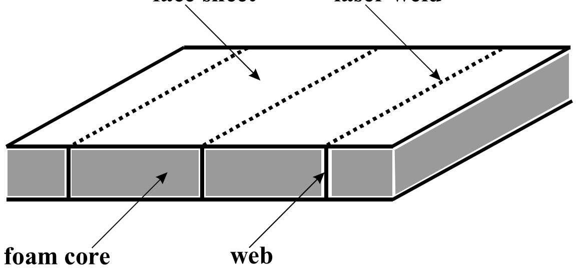

Fig. 1. Laser-welded sandwich panel.

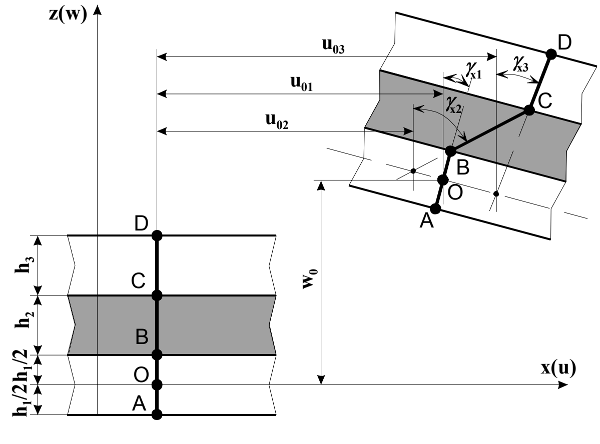

Fig. 2. Broken line model of sandwich.

Finite element model of sandwich panel

In the present approach, the finite element modelling of I-core steel sandwich panels with low-density foam is based on the first-order shear deformation theory including rotation around the normal. In this case the widely known expressions of displacements have the following form:

$$u = u_0 + z\gamma_x \quad , \quad v = v_0 + z\gamma_y \quad , \quad w = w_0$$

where u_0, v_0, w_0 are the displacements in a reference plane, z is the coordinate of the point of interest from a reference plane, γ_x, γ_y are the rotations connected with the transverse shear deformations. Then this hypothesis is applied separately for each layer of sandwich (Fig. 2). This case corresponds to the broken line model [3] and satisfies to the following displacement continuity conditions between the layers:

$$\begin{array}{lcl} u^{(1)} = u^{(2)}\Big|_{z=z_1} & , & u^{(2)} = u^{(3)}\Big|_{z=z_2} \\ v^{(1)} = v^{(2)}\Big|_{z=z_1} & , & v^{(2)} = v^{(3)}\Big|_{z=z_2} \\ w^{(1)} = w^{(2)}\Big|_{z=z_1} & , & w^{(2)} = w^{(3)}\Big|_{z=z_2} \end{array}$$

where in the brackets, the numbers of layers are given.

Equivalent material properties of sandwich core

In the case of application of the present finite element for a modelling of sandwich constructions with a non-regular core, the non-homogeneous material properties of sandwich core should be homogenized. Two approaches [4] have been applied to obtain equivalent material properties (Table 1). The first is connected with the rule of mixture and the second – with a calculation of equivalent stiffness.

Applying the first approach, the volume fraction of steel for I-core steel sandwich panel is introduced

$$V_{st} = \frac{t_w}{2p}$$

where t_w is the thickness of web and $2p$ is the web-pitch (Fig. 3). To carry out the dynamic analysis, an equivalent density of sandwich core should be known. It can be determined as

$$\rho_c = \rho_{st} V_{st} + \rho_{foam}(1 - V_{st})$$

where ρ_{st} and ρ_{foam} are the densities of steel and foam respectively. It is necessary to note that the value of out-of-plane shear modulus in YZ-plane cannot be determined by the rule of mixture since in this case the core acts together with the face's plates and therefore they cannot be separated.

Another approach uses equivalent stiffnesses determined for I-core steel sandwich panel with low-density foam and presented in Table 2. Both approaches give approximately the same values of orthotropic material parameters excluding the case where determination of in-plane shear modulus is examined. In this case the values obtained by the rule of mixture are considerably lower than values obtained by the second approach.

Table 1. Equivalent material properties of sandwich core (Fig. 3).

Notation	Calculation by equivalent stiffness	Calculation by role of mixture
E_{xc}	$\frac{A_{xc}}{t_c}$	$E_w V_{st} + (1 - V_{st}) E_{foam}$
E_{yc}	$\frac{A_{yc}}{t_c}$	$\frac{E_w E_{foam}}{E_w (1 - V_{st}) + E_{foam} V_{st}}$
G_{xyc}	$\frac{A_{xyc}}{d^3}$	$\frac{G_w G_{foam}}{G_w (1 - V_{st}) + G_{foam} V_{st}}$
G_{xzc}	$\frac{S_x t_c}{d^2}$	$G_w V_{st} + (1 - V_{st}) G_{foam}$
G_{yzc}	$\frac{S_y t_c}{d^2}$	-
υ_{xyc}	υ_{xf}	$\upsilon_w V_{st} + (1 - V_{st}) \upsilon_{foam}$

Table 2. Equivalent stiffnesses of sandwich core (Fig. 3).

Notation	Equivalent stiffness
A_{xc}	$\frac{E_w t_c t_w}{2p} + E_{foam} t_c \left(1 - \frac{t_w}{2p}\right)$
A_{yc}	$E_{foam} t_c$
A_{xyc}	$G_{foam} t_c$
S_x	$G_w t_w \frac{\frac{d}{2p}\frac{d}{t_c}\frac{t_f}{t_w} + \frac{1}{6}\left(\frac{t_c}{2p}\right)^2}{\frac{t_f}{t_w} + \frac{1}{3}\frac{t_c}{2p}\frac{t_c}{d}} + G_{foam}\left(1 - \frac{t_w}{2p}\right)\frac{d^2}{t_c}$
S_y	$\frac{1}{\frac{1}{6E_f I_f}\frac{1}{p}\left(p - \frac{t_w}{2}\right)^3 + \frac{1}{12E_w I_w} pd\left[\left(\frac{t_f}{d}\right)^3 - 3\left(\frac{t_f}{d}\right) + 2\right]} + G_{foam}\left(1 - \frac{t_w}{2p}\right)\frac{d^2}{t_c}$ $I_f = \frac{t_f^3}{12}$ - the moment of inertia of face plate $I_w = \frac{t_w^3}{12}$ - the moment of inertia of web

Viscoelastic damping model

To describe the rheological behaviour of viscoelastic foam materials under dynamic loading, the complex modulus representation [5] is used. Using this model, the constitutive relations will be expressed in the frequency domain as follows

$$\sigma_0 = E^*_{foam}(\omega)\varepsilon_0 = E_{foam}(\omega)\left[1 + i\eta_{foam}(\omega)\right]\varepsilon_0 \quad ; \quad \eta_{foam}(\omega) = \frac{E''_{foam}(\omega)}{E_{foam}(\omega)}$$

where σ_0 and ε_0 are the amplitudes of the harmonically time-dependent stress and strain, respectively, E^*_{foam} is the complex modulus of elasticity, E_{foam} is the real and E''_{foam} is the imaginary part of the complex modulus of elasticity, η_{foam} is the material loss factor and ω is the frequency. In this case the storage and loss moduli of viscoelastic foam are defined in the frequency domain by an experimental technique. The curve fitting procedure is used then to approximate experimental data with the purpose to apply these dependencies in the numerical analysis. Taking into account the viscoelastic foam properties, the equivalent orthotropic material parameters (Table 1) become the complex frequency-dependent values: $E^*_{xc}(\omega)$, $E^*_{yc}(\omega)$, $G^*_{xyc}(\omega)$, $G^*_{xzc}(\omega)$, $G^*_{yzc}(\omega)$.

3. DYNAMIC ANALYSIS

The forced vibration equation of a structure with the frequency-dependent viscoelastic damping presented by the complex modulus model appears as follows in a matrix form:

$$\mathbf{M}\ddot{\mathbf{X}}^* + \mathbf{K}^*(\omega)\mathbf{X}^* = \mathbf{F}(t)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}^*(\omega) = \mathbf{K}(\omega) + i\mathbf{K}''(\omega)$ is the complex stiffness matrix, $\mathbf{X}^*$ and $\ddot{\mathbf{X}}^*$ are the complex vectors of displacements and accelerations, and $\mathbf{F}(t)$ is the load vector. The matrix $\mathbf{K}(\omega)$ is determined using the storage modulus $E(\omega)$, while $\mathbf{K}''(\omega)$ is found using the imaginary parts of the complex modulus $E''(\omega) = \eta_E(\omega)E(\omega)$. In the present paper dynamic characteristics, eigenfrequencies and corresponding loss factors, of laser-welded sandwich panels are evaluated by the method of complex eigenvalues and from the resonant peaks of the frequency response function.

Free vibration analysis

Damped eigenfrequencies and corresponding loss factors in this case are determined from the free vibrations of sandwich structure

$$\mathbf{K}^*(\omega)\overline{\mathbf{X}}^* = \lambda^*\mathbf{M}\overline{\mathbf{X}}^*$$

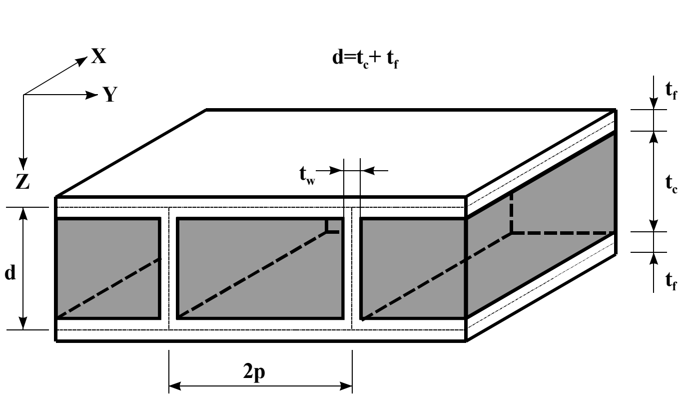

Fig. 3. Geometry of laser-welded sandwich panel.

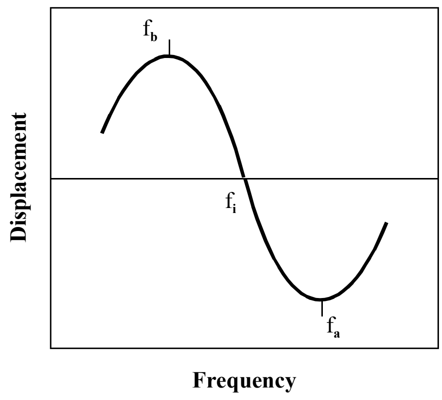

Fig. 4. Frequency response.

where $\lambda^* = (\omega^*)^2$ is the complex eigenvalue, $\overline{\mathbf{X}}^*$ is the complex eigenvector and $\omega^* = \omega + i\omega''$ is the complex eigenfrequency. The real part ω represents the damped eigenfrequency of a structure and the imaginary part ω'' specifies the rate of decay of the dynamic process. Solution of this non-linear generalised eigenvalue problem starts with a constant frequency ($\omega = \text{const}$). Then at each step the linear generalised eigenvalue problem with $\mathbf{K}^*(\omega) = \text{const}$ is solved by the Lanczos method [6], which is programmed in a truncated version, where the generalised eigenvalue problem is transformed into a standard eigenvalue problem with a reduced order symmetric tridiagonal matrix. Orthogonal projection operations are employed with greater economy and elegance using elementary reflection matrices. The iteration process terminates, when the following condition is satisfied

$$\frac{|\omega_{i+1} - \omega_i|}{\omega_i} \le \xi$$

where ξ is the desired precision and ω_{i+1} is the real part of eigenfrequency of a structure calculated from the linear generalised eigenvalue problem with the storage and loss moduli for the frequency ω_i, which was obtained from the same equation in the previous step. The modal loss factors of a structure for each vibration mode are determined by the following relation

$$\eta_n = \frac{\lambda''_n}{\lambda_n}$$

This approach gives the possibility to preserve an exact mathematical formulation for the damping model examined and to calculate structures with high damping.

Frequency response analysis

In the case of harmonic vibrations $\mathbf{F}(t) = \overline{\mathbf{F}}e^{i\omega t}$, solution of the forced vibration equation is found in the form $\mathbf{X}^*(t) = \overline{\mathbf{X}}^* e^{i\omega t}$ and the system of complex linear equations is obtained

$$\left[\mathbf{K}^*(\omega) - \omega^2\,\mathbf{M}\right]\overline{\mathbf{X}}^* = \overline{\mathbf{F}}$$

where ω is the frequency for which the response of a structure is calculated, $\overline{\mathbf{X}}^*$ is the complex amplitude of the displacements and $\overline{\mathbf{F}}$ is the amplitude of applied force. The system of complex linear equations is solved by Gauss algorithm [7] for each frequency. Dynamic characteristics of sandwich structure, eigenfrequencies and corresponding loss factors, can be easily obtained from the frequency response (Fig. 4). The eigenfrequencies $f_n = \omega_n / 2\pi$ of a structure present the points of the real part of the response spectrum, where the amplitude of the displacements is zero, but the corresponding loss factors can be determined by analysing the resonant peaks at frequencies f_a and f_b for a particular mode as follows

$$\eta_n = \frac{1 - (f_b / f_a)^2}{1 + (f_b / f_a)^2}$$

This method takes a considerable computing time, since the dynamic stiffness matrix $[\mathbf{K}^*(\omega) - \omega^2\,\mathbf{M}]$ must be recalculated, decomposed and stored at each of the numerous frequency steps. On this reason for structures modelled by a great number of degrees of freedom and in the case of a great number of desired dynamic characteristics to be calculated, it is more efficient to use the results of the free vibration analysis. The frequency response analysis may be successfully applied in the case, when it is necessary to determine a small number of desired dynamic characteristics, or when the eigenfrequency of the undamped structure is already known and only it recalculation and determination of the corresponding loss factor for the damped structure is necessary. This approach also gives the possibility to preserve an exact mathematical formulation for the damping model examined and to calculate structures with high damping.

4. EXPERIMENTAL SETUP AND ANALYSIS

An experimental setup for determination of the dynamic characteristics, eigenfrequencies and corresponding loss factors, of laser-welded sandwich panels is presented in Fig. 5 and consists of the following items:

- Dytran Model 5800A2 Instrumented Impulse Hammer,
- Dytran Model 3032A Miniature LIVM Quartz Shear Accelerometer,
- Dytran Model 4105C LIVM Current Source Amplifiers,
- Microstar Laboratories IDSC-1816 Data Acquisition and Anti-Aliasing Board,
- PC + MATLAB Software.

To make measurements, each specimen is suspended in two long lightweight strings for the achievement of free-free boundary conditions. An accelerometer is attached to the end of sandwich panel and the impulse hammer is used for excitation at the opposite end.

Upon impact the time histories of the accelerometer and force transducer in the hammer's head were recorded for ten seconds and then Fourier transformed after data reduction and signal conditioning. Using visual inspection of both time histories as well as the frequency response, each specimen was tested until ten good measurements were obtained. To determine the resonance frequencies and corresponding loss factors from the frequency response plot, MATLAB code has been developed using a Nyquist plot [8]. In order to capture resonance frequencies of up to 1000 Hz and to obtain a frequency resolution of about 0.125 Hz, the sampling frequency of the data acquisition board was set to 12800 Hz and a lowpass filter with a cut-off frequency of 3200 Hz was used.

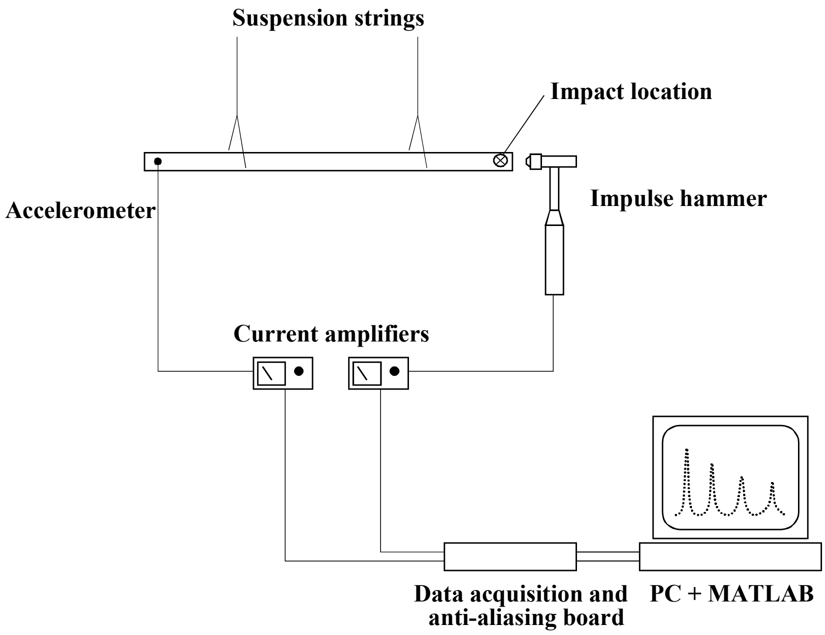

Fig. 5. Experimental setup.

5. RESULTS & DISCUSSION

To estimate correctness of the methodology developed, the ANSYS solutions for empty I-core steel sandwich beams and frequency response experimental measurements for different types of laser-welded sandwich beams with longitudinal and transverse webs and foam materials have been performed. As an example, two types of laser-welded sandwich beams are examined in the paper:

I. sandwich beam with I-core longitudinal webs and rigid polyurethane foam (PU145),

II. sandwich beam with I-core transverse webs and without foam material inside.

These sandwich beams have the following geometrical parameters: the length *L*=2 m, the width *B*=0.18 m, the height *H*=0.044 m, and the following cross-section dimensions (Fig. 3): $t_f = 0.002$ m, $t_c = 0.04$ m, $t_w = 0.004$ m, $2p = 0.12$ m. The material density and mechanical properties are as follows:

- steel: $E_f = E_w = 210$ GPa, $\upsilon_f = \upsilon_w = 0.3$, $\rho_f = \rho_w = 7800$ kg/m^3,
- PU145: $E_{foam} = 48$ MPa, $G_{foam} = 10$ MPa, $\upsilon_{foam} = 0.3$, $\rho_{foam} = 145$ kg/m^3.

The laser-welded sandwich beams have been modeled by 8 broken line sandwich beam finite elements to get convergence for ten first eigenfrequencies. The first eigenfrequencies obtained from experiments, ANSYS and methodology developed are given in Tables 3 and 4 where a good agreement between different theoretical results and experiment is observed. It is necessary to note that the large number of local modes makes difficult considerably the realization of experiment. On this reason, some experimental eigenmodes have been lost but some local modes can be presented in the spectrum. An absence of some theoretical eigenfrequencies in Tables 3 and 4 is connected with an impossibility to obtain the torsion modes using sandwich beam finite element that is clearly seen from Fig. 6. It is necessary to note also that the following equivalent material properties of sandwich core have been used for the sandwich beam type II: $E_{xc} = E_{yc}$ and $G_{xzc} = G_{yzc}$ since the transverse stiffening is applied in this case. Unfortunately, it was not possible to obtain a good correlation between theoretical and experimental modal loss factors since additional mechanisms of energy dissipation have been presented in the construction in time of experiment. But theoretical model includes only the dissipation mechanism connected with the viscoelastic material properties of foam.

With the purpose to increase the damping properties of laser-welded sandwich panels with a low-density core material, an influence of the cross-section parameters (Fig. 3) on their dynamic behaviour has been studied. Numerical simulation is carried out for the simply supported I-core steel ($E = 210\,\text{GPa}$, $\upsilon = 0.3$, $\rho = 7800\,\text{kg/m}^3$) and aluminium ($E = 71\,\text{GPa}$, $\upsilon = 0.32$, $\rho = 2800\,\text{kg/m}^3$) sandwich beams (*L*=2 m, $t_c = 0.04$ m) with

Table 3. Eigenfrequencies (in Hz) of sandwich beam (type I).

Experimental	Theoretical			
	Free vibration analysis		Frequency response analysis	
	By equivalent stiffness	By role of mixture	By equivalent stiffness	By role of mixture
79.9	81.5	81.5	81.5	81.5
213.1	219.2	219.7	219.2	219.7
319.9	-	-	-	-
394.6	414.4	416.2	414.4	416.2
587.9	-	-	-	-
-	656.5	661.1	656.6	661.2
746.4	-	-	-	-
913.0	931.3	940.3	931.6	940.5

Table 4. Eigenfrequencies (in Hz) of sandwich beam (type II).

Experimental	ANSYS	Theoretical	
		Free vibration analysis	Frequency response analysis
		By equivalent stiffness	By equivalent stiffness
31.7	33.8	32.0	31.6
-	45.2	-	-
47.9	51.6	48.7	47.6
67.6	72.5	69.1	66.9
84.4	90.5	87.0	83.2
103.8	110.4	108.0	101.6
121.3	118.6	-	-

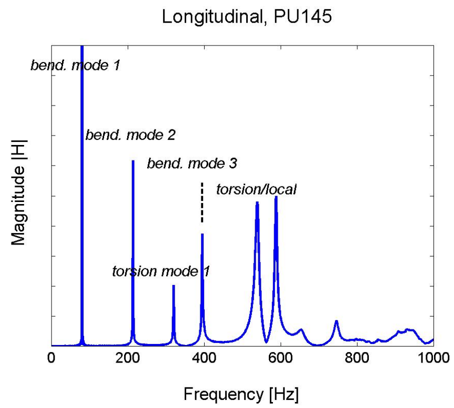

Fig. 6. Experimental frequency response of sandwich beam (type I).

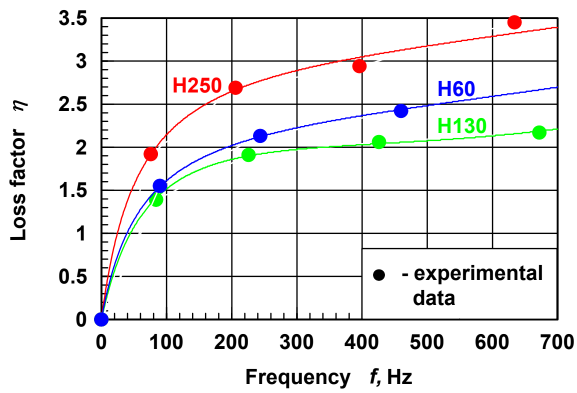

Fig. 7. Dependence of the loss factor on frequency for Divinycell.

Divinycell H250 (Fig. 7: $E_{foam} = 280$ MPa, $G_{foam} = 108$ MPa, $\rho_{\text{foam}} = 250\,\text{kg/m}^3$) used as a core material.

An influence of the web spacing, thickness of face layers and webs on the five first modal loss factors of aluminium and steel sandwich beams is demonstrated in Fig. 8 and 9. It is clearly seen that with an increasing of the web spacing and thickness of face layers, as well as with a decreasing the web thickness, the damping properties of I-core aluminium and steel sandwich beams are improved. This is connected with an increasing of the shear effect in the core layer of sandwich.

5. CONCLUSIONS

The broken line sandwich finite element model has been developed to analyse laser-welded sandwich panels with evenly spaced vertical webs and low-density core material inside. To evaluate the dynamic characteristics of such structures, the algorithms for the method of complex eigenvalues and frequency response analysis have been developed. This technique gives the possibility to preserve the frequency-dependence of viscoelastic core materials and to analyse structures with high damping. Material data in this case are taken from the frequency domain that is why the data from experiments are used directly in the finite element analysis after curve fitting procedure.

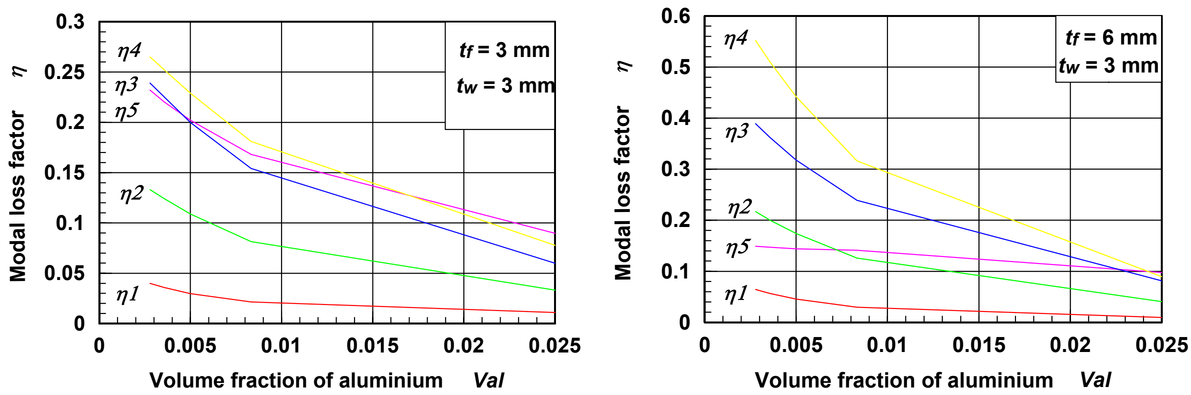

Fig. 8. Modal loss factors of I-core aluminium sandwich beams with different thickness of face layers.

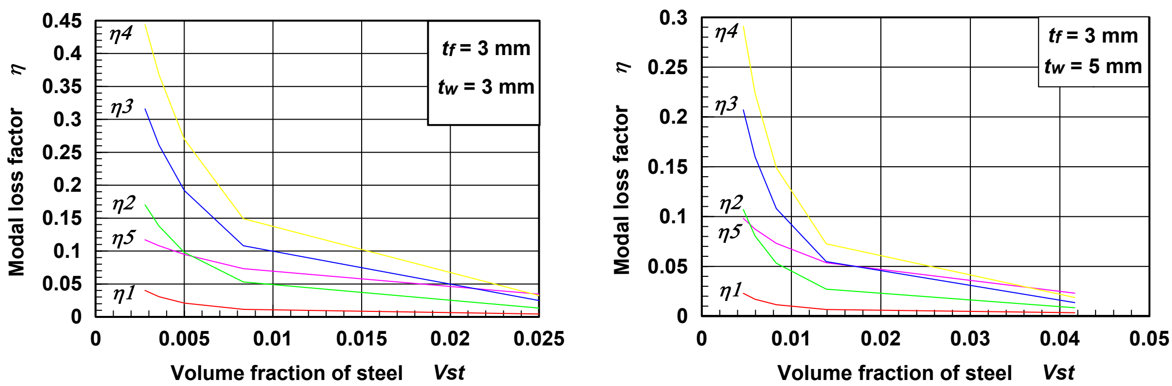

Fig. 9. Modal loss factors of I-core steel sandwich beams with different thickness of webs.

The correctness of the methodology developed have been estimated by the ANSYS solutions for empty I-core steel sandwich beams and frequency response experimental measurements for different types of laser-welded sandwich beams with longitudinal and transverse webs and foam materials. Good agreement between different theoretical results and experiment is observed.

After investigation of the damping properties of I-core steel and aluminium sandwich panels with a low-density core material it was established that only one way exists to increase the damping properties of laser-welded sandwich panels. This is increasing the shear effect in a sandwich core by decreasing the core stiffness or increasing the thickness of face layers. The change of stiffness properties of a sandwich core can be achieved varying the web thickness and their spacing. However in some cases, by increasing the thickness of face layers it is not possible to get an increasing the modal loss factors and we observe the inverse effect. These cases relate to the dynamic analysis of sandwich panels with high stiffness properties of sandwich core (small web spacing and/or large web thickness).

ACKNOWLEDGEMENTS

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