Development of a Finite Element Analysis Software for composite materials structures based on a new exponential model

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ABSTRACT

This present work proposed new analytical model and same time the development of an object oriented software for finite element analysis of composite material structures on the basis of this new model to consider the better effect of the transverse shear stresses.

In start, an analytical approach is used to validate the new model by evaluating different type of structures like plate, beam and shell for different boundary conditions and applied load with the comparison of the three dimensional exact solution and check its accuracy with respect to other existing models, like of Ressiner-Mindlin, Reddy, and Touratier.

Secondly, the development of an object oriented software «SuperKAM» for composite material structures on the basis of this new model using C++ with the finite element library DiffPack. This finite element software use a nine nodes bi-quadratic quadrilateral finite element to incorporate C^1 continuity in the numerical analysis.

1 INTRODUCTION

Increased utilization of composite materials in the design of large variety of structures has led to increase research activity in the mechanical characterization, structural modelling, failure, and damage assessment of composite materials. Because the solution of the three-dimensional linear problem with general boundary conditions involves considerable mathematical difficulties, in recent years some bi-dimensional linear theories including transverse shear stresses for multi-layered plates and shell are developed. The approach generally uses a variational principle more often conjunction with an assumed displacement field. Integration with respect to the thickness co-ordinate supplies the governing differential equations consistent boundary conditions in terms of unknown generalized displacements, which are independent of thickness co-ordinate.

Consider a doubly curved shell Ω of constant thickness h is considered. Let (ξ_1, ξ_2, ζ) denotes orthogonal principal curvilinear co-ordinates such that the ξ_1 and ξ_2 curves are lines of curvature on the mid-surface $\zeta = 0$, ζ curves are straight lines perpendicular to the surface $\zeta = 0$ as shown in Fig. 1.

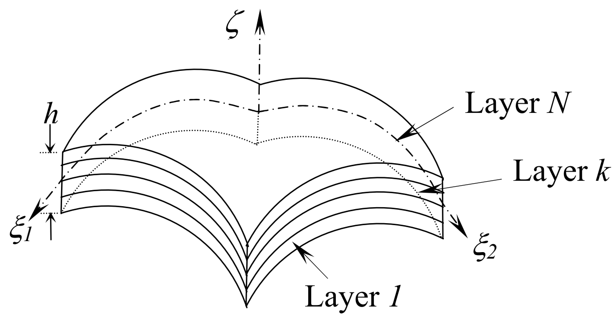

Fig. 1. Geometry of laminated shell.

A large number of shell kinematics are developed by Taylor series development of displacement (Eq. 1), with five variationals unknowns, two membrane displacement u_α, and one transverse displacement *w,* and two are shear stress rotations γ_α. The kinematics are based on the different definitions of shear stress function $f(\zeta)$.

$$\begin{cases} U_\alpha(\xi_1,\xi_2,\zeta)=\left(1+\dfrac{\zeta}{R_\alpha}\right)u_\alpha(\xi_1,\xi_2)-\dfrac{\zeta}{A_\alpha}\dfrac{\partial w}{\partial \xi_\alpha}(\xi_1,\xi_2)+f(\zeta)\gamma_\alpha(\xi_1,\xi_2) \\ U_3(\xi_1,\xi_2,\zeta)=w(\xi_1,\xi_2) \end{cases} \tag{1}$$

where, A_α = metric, R_α = radius of curvature, and $\mu_\alpha=(1+\zeta/R_\alpha)$

$\gamma_\alpha=(1/A_\alpha)w_{,\alpha}+\phi_\alpha$ = transverse shear stress rotation, and, the new transverse shear stress function 'KAM' [1]:

$$f(\zeta)=\zeta e^{-2(\zeta/h)^2} \tag{2}$$

2 ANALYTICAL DEVELOPMENT

For assuring the continuity conditions at interface of layers of laminated or sandwich structures, two different approach are used; first is based on transverse shear stress hypothesis proposed by Whitney [2], and second based on displacement hypothesis using Heaviside step function proposed by Ossadzow [3].
By the virtual power principle, the governing equations are developed as follow;

$$\begin{aligned} &N_{11,1}+N_{12,2}=\Gamma^{(u)} \\ &N_{22,2}+N_{12,1}=\Gamma^{(v)} \\ &M_{11,11}+M_{22,22}+M_{12,12}+M_{12,21}+q(\xi_1,\xi_2)=\Gamma^{(w)} \\ &P_{11,1}+P_{12,2}-R_1=\Gamma^{(\gamma_1)} \\ &P_{22,2}+P_{12,1}-R_2=\Gamma^{(\gamma_2)} \end{aligned} \tag{3}$$

where,

$$\begin{aligned} &\Gamma^{(u)}=I_o\ddot{u}-I_1\ddot{w}_{o,1}+J_1\ddot{\gamma}_1 \\ &\Gamma^{(v)}=I_o\ddot{v}-I_1\ddot{w}_{o,2}+J_1\ddot{\gamma}_2 \\ &\Gamma^{(w)}=I_o\ddot{w}_o+I_1(\ddot{u}_{,1}+\ddot{v}_{,2})-I_2(\ddot{w}_{o,11}+\ddot{w}_{o,22})+K(\ddot{\gamma}_{1,1}+\ddot{\gamma}_{2,2}) \\ &\Gamma^{(\gamma_1)}=J_1\ddot{u}-K\ddot{w}_{o,1}+J_2\ddot{\gamma}_1 \\ &\Gamma^{(\gamma_2)}=J_1\ddot{v}-K\ddot{w}_{o,2}+J_2\ddot{\gamma}_2 \\ &\overline{\Gamma}^{(w)}=-I_1(\ddot{u}+\ddot{v})+I_2(\ddot{w}_{o,1}+\ddot{w}_{o,2})-K(\ddot{\gamma}_1+\ddot{\gamma}_2) \end{aligned} \tag{4}$$

and stress resultant and mass moment of inertia are define as;

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix}=\int_{-h/2}^{h/2}\sigma_{\alpha\beta}\begin{Bmatrix} 1 \\ z \\ f(\zeta) \end{Bmatrix}d\zeta \;,\; \{R_\alpha\}=\int_{-h/2}^{h/2}\sigma_{\alpha\zeta}\{f'(\zeta)\}d\zeta \;,\; \begin{Bmatrix} I_\alpha \\ J_\alpha \\ K \end{Bmatrix}=\int_{-h/2}^{h/2}\rho_o\begin{Bmatrix} \zeta^\alpha \\ f^\alpha(\zeta) \\ \zeta f(\zeta) \end{Bmatrix}d\zeta \tag{5}$$

3 FINITE ELEMENT FORMULATION

The finite element method is a flexible numerical approach for solving partial differential equations. One of the attractive features of the method is the easy straightforward handling of geometrically complicated domains. Using the displacement kinematics (Eq. 1), develops the strain tensor as follow $(\varepsilon_{33}=0)$;

$$\begin{aligned}\varepsilon_{\alpha\alpha} = & -\frac{\zeta R_{\alpha,\alpha}}{\mu_\alpha A_\alpha R_\alpha^2}u_\alpha + \frac{1}{A_\alpha}u_{\alpha,\alpha} + \frac{\mu_\beta A_{\alpha,\beta}}{\mu_\alpha A_\alpha A_\beta}u_\beta + \frac{1}{\mu_\alpha R_\alpha}w + \frac{(\zeta - f(\zeta))A_{\alpha,\alpha}}{\mu_\alpha A_\alpha^3}w_{,\alpha} \\ & + \frac{(f(\zeta)-\zeta)}{\mu_\alpha A_\alpha^2}w_{,\alpha\alpha} + \frac{(f(\zeta)-\zeta)A_{\alpha,\beta}}{\mu_\alpha A_\alpha A_\beta^2}w_{,\beta} + \frac{f(\zeta)}{\mu_\alpha A_\alpha}\phi_{\alpha,\alpha} + \frac{f(\zeta)A_{\alpha,\beta}}{\mu_\alpha A_\alpha A_\beta}\phi_\beta\end{aligned} \tag{6}$$

$$\begin{aligned}\gamma_{12} = & \left(-\frac{\zeta R_{1,2}}{\mu_2 A_2 R_1^2} - \frac{A_{1,2}}{A_1 A_2}\right)u_1 + \frac{\mu_1}{\mu_2 A_2}u_{1,2} + \left(-\frac{\zeta R_{2,1}}{\mu_1 A_1 R_2^2} - \frac{A_{2,1}}{A_1 A_2}\right)u_2 + \frac{\mu_2}{\mu_1 A_1}u_{2,1} \\ & + \frac{A_{1,2}}{A_2 A_1^2}\left[\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)(\zeta - f(\zeta))\right]w_{,1} + \frac{A_{2,1}}{A_1 A_2^2}\left[\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)(\zeta - f(\zeta))\right]w_{,2} \\ & + (f(\zeta)-\zeta)\frac{1}{\mu_2 A_1 A_2}w_{,12} + (f(\zeta)-\zeta)\frac{1}{\mu_1 A_1 A_2}w_{,21} \\ & - \frac{A_{1,2}f(\zeta)}{\mu_1 A_1 A_2}\phi_1 + \frac{f(\zeta)}{\mu_2 A_2}\phi_{1,2} - \frac{A_{2,1}f(\zeta)}{\mu_2 A_1 A_2}\phi_2 + \frac{f(\zeta)}{\mu_1 A_1}\phi_{2,1}\end{aligned} \tag{7}$$

$$\gamma_{\alpha 3} = \left(\frac{(1-\mu_\alpha)}{\mu_\alpha A_\alpha} + \frac{(\zeta - f(\zeta))}{\mu_\alpha A_\alpha R_\alpha} + \frac{f'(\zeta)}{A_\alpha}\right)w_{,\alpha} + \left(f'(\zeta) - \frac{f(\zeta)}{\mu_\alpha R_\alpha}\right)\phi_\alpha \tag{8}$$

A quadrilateral bi-quadratic with nine node is selected for finite element development, which have five engineering degree of freedom on each node (Fig. 2), three displacement U_1, U_2, U_3 and two rotations θ_1, θ_2, means total 45 degree of freedom per element. By the definition of iso-paramatric finite element, deduce the functions of interpolations for displacements and geometry are as follows;

$$\begin{aligned}& N_1(\xi,\eta) = \frac{1}{4}\{\eta(\eta-1).\xi(\xi-1)\};\ N_2(\xi,\eta) = -\frac{1}{2}\{\eta(\eta-1).(\xi^2-1)\}, \\ & N_3(\xi,\eta) = \frac{1}{4}\{\eta(\eta-1).\xi(\xi+1)\};\ N_4(\xi,\eta) = -\frac{1}{2}\{(\eta^2-1).\xi(\xi-1)\}, \\ & N_5(\xi,\eta) = \{(\eta^2-1).(\xi^2-1)\},\ N_6(\xi,\eta) = -\frac{1}{2}\{(\eta^2-1).\xi(\xi+1)\}, \\ & N_7(\xi,\eta) = \frac{1}{4}\{\eta(\eta+1).\xi(\xi-1)\};\ N_8(\xi,\eta) = -\frac{1}{2}\{\eta(\eta+1).(\xi^2-1)\} \\ & N_9(\xi,\eta) = \frac{1}{4}\{\eta(\eta+1).\xi(\xi+1)\}\end{aligned} \tag{9}$$

So displacements can be interpolated as:

$$[U] = [N(\xi,\eta)][q] = [N][q] \tag{10}$$

with [q] is the vector of degree of freedom [45×1] defined as :

$$[q]^T = [(U_1)_1 \quad (U_2)_1 \quad (U_3)_1 \quad (\theta_1)_1 \quad (\theta_2)_1 \quad .... \quad (U_1)_9 \quad (U_2)_9 \quad (U_3)_9 \quad (\theta_1)_9 \quad (\theta_2)_9] \tag{11}$$

$(U_i)_j$ is the displacement of node j in the direction i. Now generalized strain vector can be write using the displacement and their derivatives:

$$[E] = \mathrm{D}[U] = \mathrm{D}[N(\xi,\eta)][q] = [B][q] \tag{12}$$

[B] is the derivative of function of interpolations.

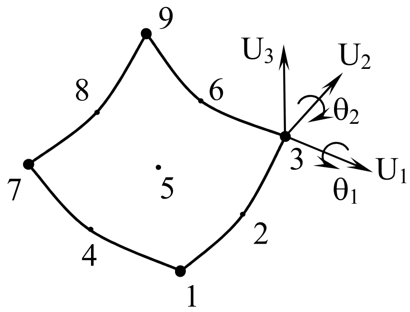

Fig. 2. Bi-quadratic finite element with nine node and five degree of freedom.

The boundary value problem is established by using the principle of virtual power in static:

$$\int_V D_{ij}^* \sigma^{ij} dV = \int_V U_i^* f^i dV + \int_{\partial V} U_i^* F^i d\partial V \tag{13}$$

Using the strain tensor (Eqs. 6,7,8) gives the governing equations of finite element formulations in the form of different matrices as follow [4]:

$$\begin{aligned} &[q^*]^T \left\{ \int_\Omega [B]^T [G]^T [C_G][G][B] A_1 A_2 |J| d\xi d\eta \right\} [q] \\ &= [q^*]^T \left\{ \int_\Omega [N]^T \left(\int_{-h/2}^{h/2} [f] \mu_1 \mu_2 d\zeta \right) A_1 A_2 |J| d\xi d\eta + \int_\Gamma [N]^T \left(\int_{-h/2}^{h/2} [F] d\zeta \right) d\Gamma \right\} \end{aligned} \tag{14}$$

with

$$[C_G] = \int_{-h/2}^{h/2} [G(\zeta)]^T [C][G(\zeta)] \mu_1 \mu_2 d\zeta \tag{15}$$

where [G] is the geometry parameter matrix, [G(ζ)] matrix contains all those terms which depends on the thickness coordinates, and [C] is the matrix of rigidity of material composite. Now we can deduce the weak formulation for an element '*e*':

$$[K^e][q] = [F^e] \tag{16}$$

where $[K^e]$ is the stiffness matrix and $[F^e]$ is load vector.

4 OBJECT ORIENTED APPLICATION "*SuperKAM*"

Diffpack [5] is used to implement the finite element formulation for the development of object oriented application "*SuperKAM*". *Diffpack* is a sophisticated tool for developing numerical software, with main emphasis on numerical solution of partial differential equations, acts as a numerical library consisting of C++ classes. The application "*SuperKAM*" is base on following classes of *DiffPack* as shown in Fig. 3.

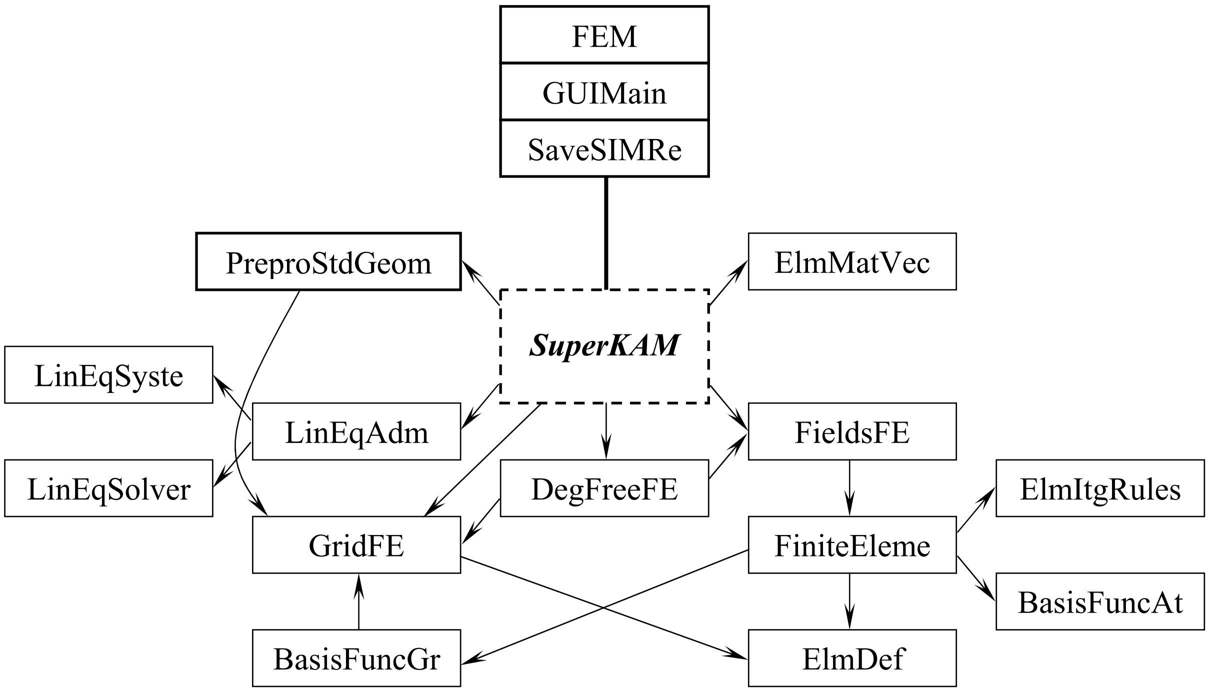

Fig. 3. Schismatic diagram for the utilization of "*DiffPack*" library by the application "*SuperKAM*"

The finite element programming in *DiffPack* is normally based on FEM class. The class *"SuperKAM"* is a derived class of FEM. The method *integrands* of class FEM define the coefficients of rigidity matrix and load vector.

$$[K^e] = \sum_{i=1}^{n} [B]^T [G]^T [C_G][G][B] A_1 A_2 |J| w_i \tag{17}$$

$$[F^e] = \sum_{i=1}^{n} [N]^T [P] A_1 A_2 |J| w_i \tag{18}$$

where w_i is the associated weight at point of integrations, [J] is the Jacobian matrix and [P] is the applied load.

The class SuperKAM derived also the method *GUIMain* to define graphic user interface for input and output. Method *SaveSimRes* stock the output result of finite element analysis. The class *PreproStdGeom* generate mesh of rectangular or triangular type. *ElmMatVec* use to define matrices and vectors, the class *GridFE* is responsible to define the numbering of nodes and elements and coordinates etc. *DegFreeFE* used to define degree of freedom, and the class is a linear equation solver. The method *resultReport* is used to write the finite element analysis results for simulation.

The integration over thickness of different terms of the of the general behaviour matrix consider the aspect of multi-layered structures of composite material.

The graphic user interface (Fig. 4) allows user to define following inputs and outputs:

- Geometry of the structure (plate or shell), number of layers and their material description.
- Applied boundary conditions (like simply supported, cantilever, free or forced displacement).
- Applied load (like uniform, sinusoidal, hydrostatic pressure or concentrated load).
- Type and number of element: triangle (ElmT3n2D/ElmT6n2D) or quadrilateral (ElmB4n2D/Elm9n2D).
- The definition of shear stress function *f(ζ)*, (Touratier [6], Reddy [7], Reissner-Mindlin [8] and "KAM").

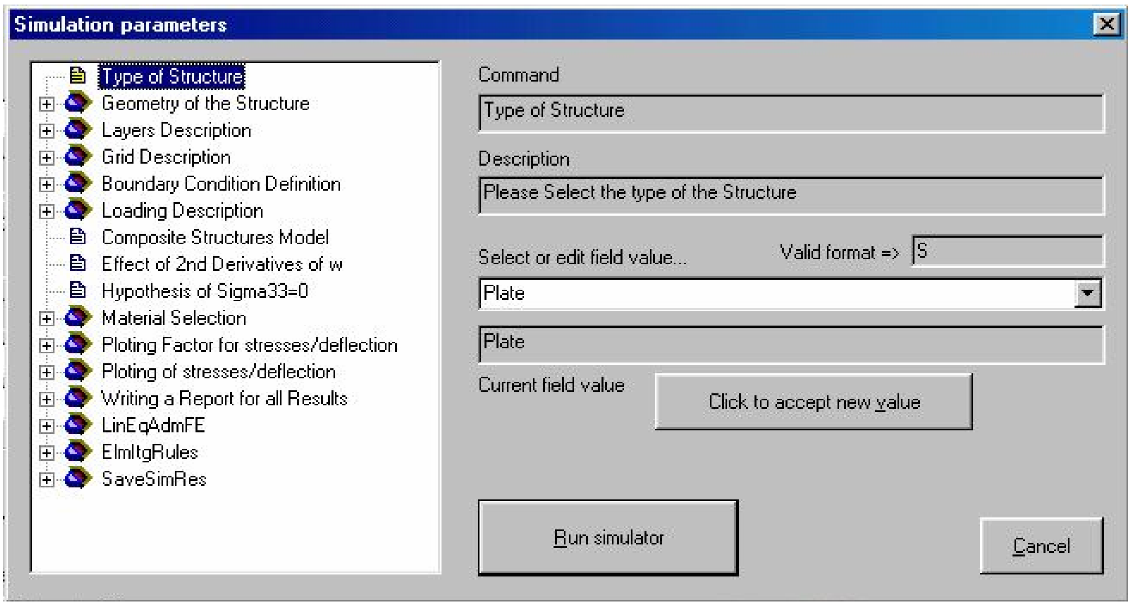

Fig. 4. GUI to select input and output parameters for finite element analysis

5 ANALYTICAL AND NUMERICAL RESULTS

Different boundary value problem are considered to evaluate the precision of new exponential model with other existing model, on different laminated and sandwich structures like beam [9], plates and shell, with the reference of exist 3D exact solutions. Here is some analytical results on plates.

In the light of existing exact 3D solution [10], two different layers orientation are considered with length over height ratio (a/h = 4), for a following material:

$$E_L/E_T = 25,\ G_{LT}/E_T = 0.5,\ G_{TT}/E_T = 0.2,\ \nu_{LT} = \nu_{TT} = 0.25$$

where *L* and *T* respectively are the directions parallel and normal to the fibres. The plate is subjected to a uniformly distributed doubly sinusoidal transverse load $\left(q(\xi_1,\xi_2) = q_o \sin\alpha\xi_1 \sin\beta\xi_2\right)$ with $q_o = 1\ MPa$.

Table 1. Analytical solution of a four layer square plate (0°/90°/90°/0°) under a uniform sinusoidal pressure

Model	w	Error %	σ_{11}	Error %	σ_{22}	Error %	σ_{12}	Error %	σ_{13}	Error %	σ_{23}	Error %
Ex.-3D[10]	1.9540	Ref.	0.7200	Ref.	0.6630	Ref.	0.0467	Ref.	0.2190	Ref.	0.2920	Ref.
KAM	1.9193	1.78	0.7004	2.72	0.6367	3.97	0.0459	1.65	0.2264	-3.39	0.2532	13.29
Tourt.[6]	1.9088	2.31	0.6830	5.14	0.6349	4.24	0.0450	3.59	0.2162	1.26	0.2462	15.70
Reddy[7]	1.8937	3.09	0.6651	7.62	0.6322	4.64	0.0440	5.69	0.2064	5.77	0.2389	18.18

Table 1 and Table 2 give the non-dimensionalized deflection strains and stresses. In the first case (Table 1) results are calculated by the approach of non continuity of transverse shear stresses on layers interfaces, new refined model "*KAM*" gives good approximation as compared to other refine models in the reference of exact 3D solution. In the second case (Table 2), two approaches are used non continuity (NC) and continuity (C), model "*KAM*" still gives very good precision especially in transverse shear stresses with continuity.

Table 2. Flexion of a four layer rectangular (b=3a) plate (0°/90°/0°) under a uniform sinusoidal pressure

Model	w	Error %	σ_{11}	Error %	σ_{22}	Error %	σ_{12}	Error %	σ_{13}	Error %	σ_{23}	Error %
Ex.-3D[11]	2,8200	Ref	1,1000	Ref	0,1190	Ref	-0,0281	Ref	0,3870	Ref	0,0330	Ref
KAM (NC)	2,6838	4,8	1,0974	0,2	0,1038	12,8	-0,0272	3,2	0,2982	22,9	0,0360	-9,2
KAM (C)	2,6994	4,3	1,2215	-11,0	0,1043	12,4	-0,0276	1,8	0,3796	1,9	0,0314	4,8
Tourt.[6]	2,6657	5,5	1,0672	3,0	0,1034	13,1	-0,0268	4,7	0,2851	26,3	0,0355	-7,5
Reddy[7]	2,6411	6,3	1,0359	5,8	0,1028	13,6	-0,0263	6,4	0,2724	29,6	0,0348	-5,5

For checking the performance of this new application "*SuperKAM*" and the precision of new selected element "*ElmB9n2D*" in *DiffPack*, some standard finite element analysis test are done, like convergence and shear locking test, and then precision of new exponential with other models in comparison of experimental results[4] is checked.

Consider a sandwich square plate (*400 mm*) clamped on its four sides, under uniform pressure 1 bar. Material properties of skin are as aluminium ($E = 72\ GP,\ v = 0.33$) and the core has properties of aluminium honey-comb structure in [GPa] is as,

$$E_1, E_2 = 1,\ E_3 = 1000,\ G_{13} = 450,\ G_{23} = 250,\ G_{12} = 1,\ v_{13} = v_{23} = 0.3,\ v_{12} = 0.5$$

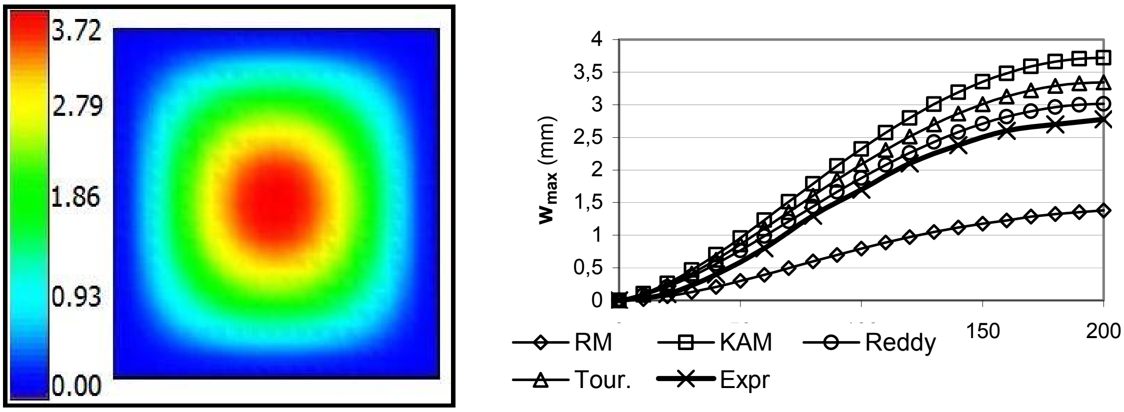

Fig. 5(a),(b). Transverse displacement in plate sandwich (mm)

After having the convergence on displacement at 400 elements, a comparative study of the transverse displacement, strain and stresses is done. Fig. 5(a) shows the graphical simulation of transverse displacement using new exponential model '*KAM*'. A comparison of transverse displacement evolution along the symmetrical axis of the plate sandwich from centre of the plate till clamped end is shown in (Fig. 5(b)). We found that all refined model are closed to each other unlike by RM which is considerable far from the experimental results. It is also found that RM under estimate the transverse displacement unlike refined models.

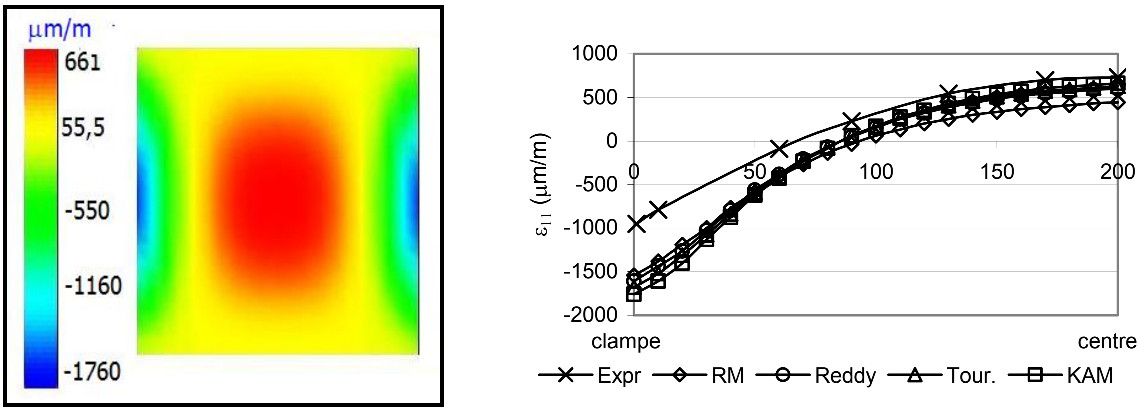

Fig. 6(a),(b). Principle strain ε_{11} in plate sandwich (µm/m)

Fig. 6(a) shows the graphical evolution of principle strain ε_{11} by application "*SuperKAM*" using model "*KAM*". Fig. 6(b) and Fig. 7 shows principle strains ε_{11}, ε_{22} respectively along the axis of symmetry of plate sandwich, from clamped end to the centre, using the different refine models and Ressiner-Mindlin theory. It is found that the model "*KAM*" gives very good results specially for the extreme values (like at the centre and clamped end) as compared to experimental results [4] which are measured by strain gauges. Table 3 shows the principle strain at the centre of plate by different models, with a good approximation (under 10 %) by model KAM.

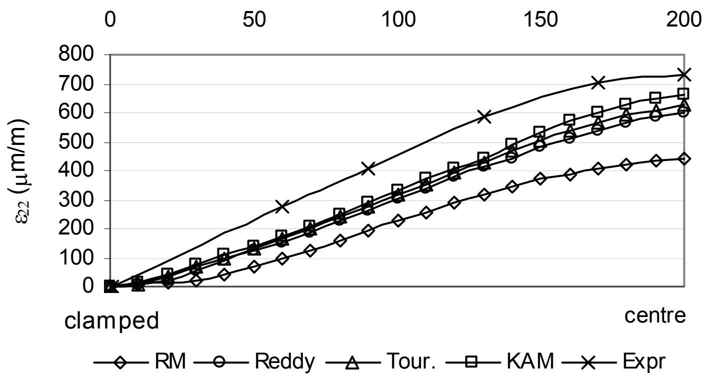

Fig. 57. Principle stain (ε_{22}) in the sandwich plate

Table 3. Principle strain at the center of plate sandwich

	Expr.[4]	RM	Reddy	Tour.	KAM
Principle strain (µm/m)	731	444	597	627	661
Error (%)	Ref.	39,3%	18,3%	14,2%	9,6%

Fig. 8 shows the evolution of principle strains difference using different models with the references of experimental measurements [4] using photo-elastic method. It is very much clear that all models are very much close to each other and model "*KAM*" gives very good precision with experimental especially at the clamped end (Table 4).

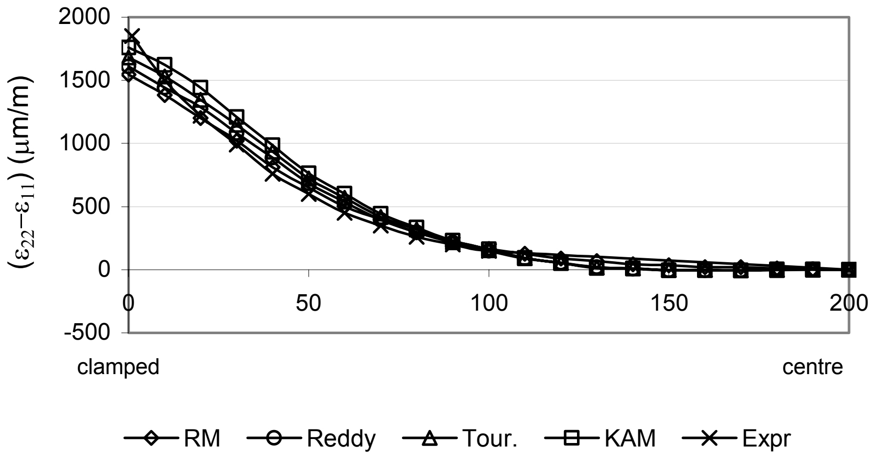

Fig. 8. Difference of principle of strain in sandwich plate

Table 4. Principle strains difference at the clamped end

	Expr.[4]	RM	Reddy	Tour.	KAM
(ε_{11}-ε_{22}) (µm/m)	1850	1545	1610	1680	1760
Error (%)	Ref.	16%	13%	9%	5%

As a second case study for numerical analysis, consider a square (400 mm) laminated tissue plate (glass/epoxy) with twelve layers under different uniform pressure with following mechanical properties as:

$$E_1, E_2 = 19.5, E_3 = 9.75, \ G_{13}, G_{23}=3.75, G_{12} = 4.75, \nu_{13} = \nu_{23} = 0.4, \nu_{12} = 0.25$$

After having convergence on 400 elements by the application "*SuperKAM*" with new model '*KAM*', a comparative study is done on maximum strain at the centre of plate under different uniform pressure as compared to experimental results [13]. Table 5 shows the new refined model '*KAM*' gives very good precision as compared to Reissner-Mindlin theory.

Table 5. Maximum strain under different uniform pressure at the centre of plate

Pressure	RM		KAM		Expr.[13]
	ε_1 (µm/m)	Error (%)	ε_1 (µm/m)	Error (%)	ε_{11} (µm/m)
0,5	305,2	12,8%	326	6,9%	350
1	611,3	12,7%	652,4	6,8%	700
1,5	917	8,3%	978	2,2%	1000
2	1222	12,7%	1304	6,9%	1400
2,5	1528	10,1%	1631	4,1%	1700
3	1833	12,7%	1957,1	6,8%	2100

Conclusion

An object oriented application for finite element analysis "*SuperKAM*" is developed with new refined model "*KAM*" and other existing models to add shear stress rotation, using a nine node element "*ElmB9n2D*" with five degree of freedom for continuity C^1.
Analytical results show that new model "*KAM*" gives very good approximation with other refined models as compared to exact 3D solution especially in the case of transverse shear stresses.
Different numerical analysis on sandwich and laminated plate shows that new model "*KAM*" gives very good precision in the case of extreme values under various boundary conditions as compared to other existing model in the reference of experimental results.

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