

# Monte-Carlo Simulation to Fracture of Cross-ply Laminates at Room and Low Temperature

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## Abstract

A three-dimensional meso-mechanical model was developed by finite element method combined with Monte-Carlo simulation technique. Through this model, the damage process, including four types of micro-rapture events, i.e. the matrix cracking, fiber-matrix interface debonding, delamination and fiber rupture, was simulated in glass-fiber/epoxy cross-ply laminates with a stacking sequence  $[0^\circ/90^\circ/0^\circ]$ . Size effect of the laminates was studied with increasing the number of fibers. The mechanical behaviors including the damage process, stress/strain curve and tensile strength were simulated for a glass-fiber/epoxy cross-ply laminate. Parts of simulated results were compared with the experiment data. Furthermore, the influence of temperature was discussed. The results showed that the increasing of the scale parameter is the essential cause of the mechanical properties trending upward at low temperature.

**Key words:** cross-ply laminates, damage process, finite element, Monte-Carlo simulation, three-dimensional, meso-mechanics.

## 1.Introduction

Fiber-reinforced composites have been widely adopted in many fields. So it is important to predict the overall mechanical behaviors of the laminates from their constituent properties. In recent studies, two meso-mechanical methods have been mainly adopted: representative volume element (RVE) and shear-lag model. The former [1,2,3] is based on the known constituents properties of composites and the assumption that the composites process a periodic structure. It represents the whole laminate with RVE to study the overall properties of the laminate. The model is simple to construct and the simulation results are relatively accurate. However, the statistical characters of fiber strength are neglected in these models and it is difficult to consider clearly. The later [4-7] adopts the shear-lag model combined with

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Monte-Carlo simulation technique to simulate the mechanical behaviors of laminates, including the damage process. In these models, the statistical characters of fiber strength are considered. But these models are usually used to simulate the mechanical behaviors of unidirectional composites. Besides the two kinds of model, there are some other models. For example, some researcher [8] attempted to develop a multi-scale micro-meso-local-global methodology to simulate all damage modes.

In this study, a three-dimensional meso-mechanical model was developed adopting the finite element method combined with Monte-Carlo simulation technique. The numeral calculations were conducted for a glass-fiber/epoxy laminate with a stacking sequence  $[0^\circ/90^\circ/0^\circ]$  at room and low temperature. The results showed that the developed model is effective for predicting the mechanical behaviors of cross-ply laminates.

## 2. finite-element model and simulation procedure

For the sake of simplicity, the cross-ply laminate consisting of three layers sequenced by  $[0^\circ/90^\circ/0^\circ]$  was considered in this study. The finite element mesh was set for fiber and matrix, respectively, as shown in Fig.1.

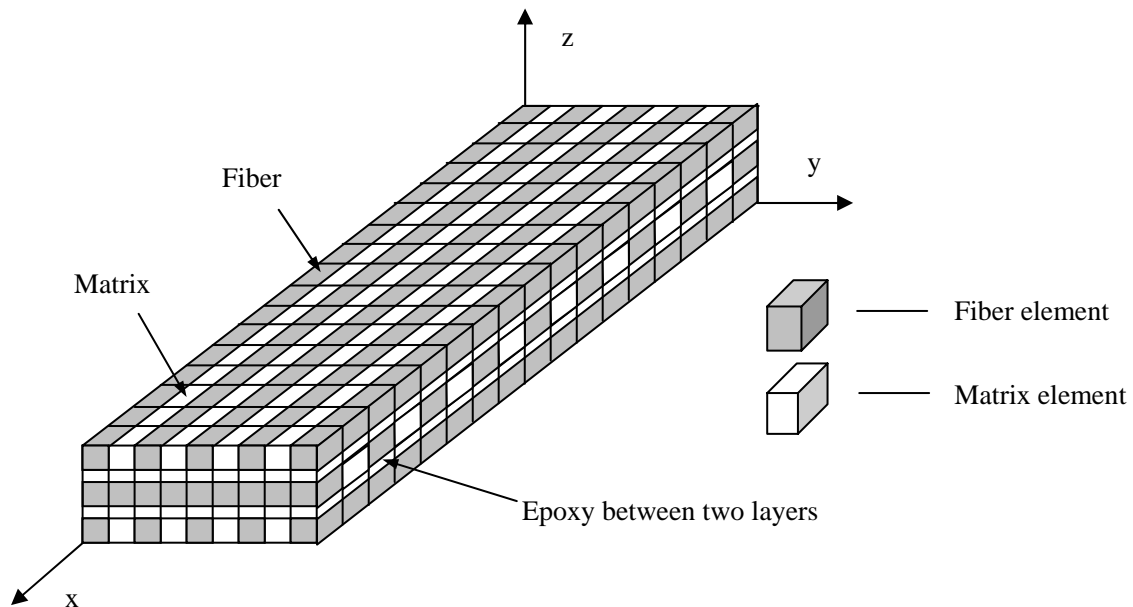


Fig.1. Finite element model used in the present simulation

The fiber was treated as square prism and Eight-node hexahedral elements were adopted for the fiber and matrix, respectively. The FE mesh length in the fiber direction was equal to fiber diameter (e.g.  $10 \mu m$ ), which is sufficiently smaller than the fiber critical length [9]. Through adjusting the thickness of the matrix layer and

the width between the adjacent fibers in a layer, the fiber volume fraction could be controlled easily. It was also assumed that the fiber and matrix are homogeneous isotropic linear elastic.

Displacement conditions were applied to the boundary surface  $x=0, y=0, z=0$ .

$$\begin{aligned} u(0, y, z) &= 0 \\ v(x, 0, z) &= 0 \\ w(x, y, 0) &= 0 \end{aligned} \quad (1)$$

$$u(L, y, z) = u(L, 0, 0)$$

It should be noted that, in this study, the external load was applied by prescribing the displacement  $u(L, 0, 0)$ .

The simulation was carried out following the flow chart shown in Fig.2.

In the simulation, it should be noted that:

1. The strength of the matrix and the interface is considered as definite, while fiber strength is described by two-parameter Weibull distribution  $F(\sigma)$ , as following:

$$F(\sigma) = 1 - \exp\left\{-\frac{L}{L_0} \left(\frac{\sigma}{\sigma_0}\right)^\beta\right\} \quad (2)$$

where  $F(\sigma)$  is the probability that the fiber strength  $\sigma$  is no more than  $X$ , while  $\sigma_0$  and  $\beta$  are Weibull scale and shape parameters, respectively.  $L_0$  is the original gage length at which the single filament tension test and estimation of Weibull parameters are conducted, while  $L$  is the extrapolated fiber length of interest. In the simulation program, the following reverse function based on Eq. (2) is used as the operator assigning the random Weibull number.

$$\sigma = \sigma_0 \left(-\frac{L_0}{L} \ln(1-x)\right)^{\frac{1}{\beta}} \quad (3)$$

where  $x$  is a random number taken from the uniform distribution, and  $\sigma$  is the strengths of the fiber elements.

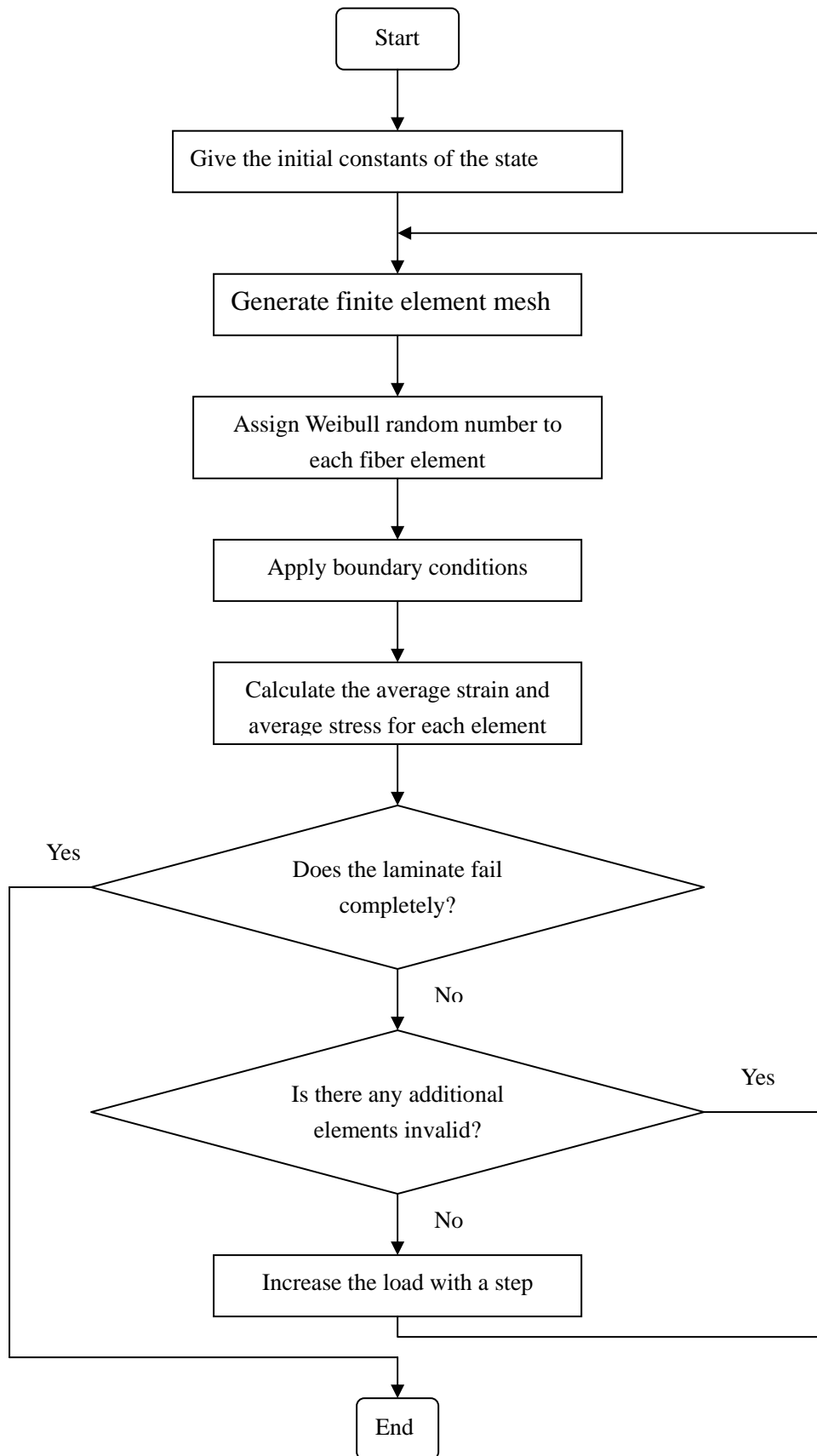


Fig.2. The flow chart of simulation

2. In the simulation, the load step is sufficiently small, so the procedure can be treated as a static procedure. The maximal stress criterion is adopted as failure criterion to judge the breakage of the elements of fiber and matrix.
3. The frictional force at interface after debonding was neglected in the simulation. When the shear stress of interface reaches the interfacial shear strength, the interface debonding will occur. Usually, matrix acts as a medium for stress transfer and the normal stress in matrix is much smaller than that in fiber. So when the interface debonding occurs, the neighboring matrix element will be treated as invalid. Consequently, it wouldn't contribute to the whole structure any more.

### 3.Application to glass-fiber/epoxy cross-ply laminate

The mechanical behaviors of a glass-fiber/epoxy cross-ply laminate with a stacking sequence  $[0^\circ / 90^\circ / 0^\circ]$  were simulated by using the developed model and simulation procedure.

#### 3.1 Material constants

Some material constants from the experiment [10] were used in the simulation. They are listed in Table.1.

Table.1. Material constants at room temperature

	E-glass	Epoxy
Elastic modulus (Gpa)	70	2.75
Shape parameter $\beta$	9.35	-
Scale parameter $\sigma_0^a$ (Mpa)	840	-
The volume fraction of fiber $V_f(\%)$	30~50	-
Coefficient of thermal efficiency $\alpha_L (10^{-6}/k)$	4.8	48
Material strength (MPa)	-	51.9
Interface strength (MPa)	59.5	59.5
Diameter of fiber (mm)	0.01	-

#### 3.2 The size effect in composites

The fiber number arrayed along x and y direction, as shown in fig.1, can influence the overall properties of the laminates significantly. If fiber number is too large, element number will too many to simulate. While if fiber number is too small, the results will not be accurate. So it is very important to consider the size effect of the composites. Calculations showed that for a certain laminate width, the simulated tensile strength of the composites decreases sharply when the laminate length

increases, as shown in Fig.3(a). Here,  $m$  is the fiber number in a  $0^\circ$  layer and  $n$  is the fiber number in a  $90^\circ$  layer. The width and the length of the laminate can be presented by  $m$  and  $n$ . It should be noted that when  $n$  is greater than two times of  $m$ , the tensile strength approaches to a constant. Then a series of laminates with different width ( $m$ ) were calculated and the same results were found. And the constant of different series of laminates with a certain  $m$  decreases with the increasing of  $m$ , as shown in Fig.3(b). When  $m$  is greater than 12, the tensile strength will vary in a small range. So when  $m$  is greater than 12 and  $n$  is greater than  $2m$ , it will be applicable to compare the simulation results with experiment data.

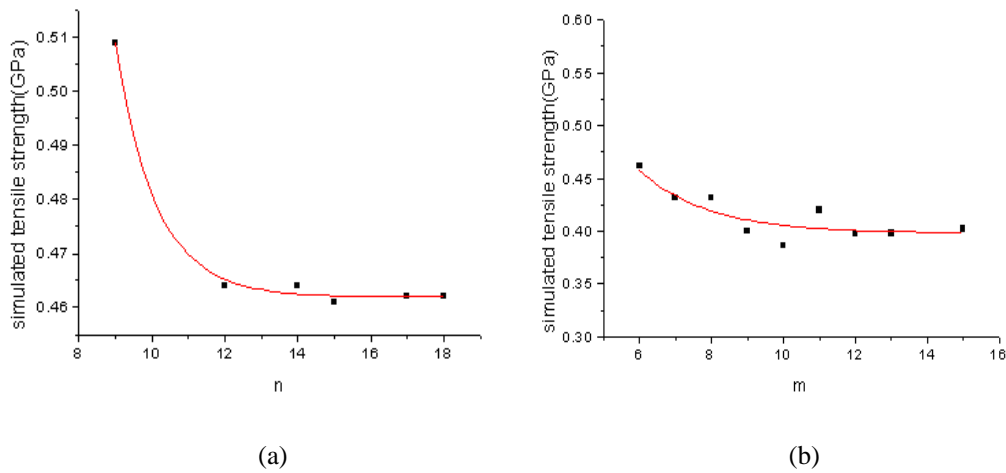


Fig.3. The size effect, (a) The result of calculation with different length  $n$  ( $m=8$ ); (b) The result of calculation with different width  $m$  ( $n \geq 2m$ )

### 3.3 the simulation of the damage process

Generally, there are four types of failure modes in laminates, i.e. matrix cracking, fiber-matrix interface debonding, delamination and fiber rupture. Fig.4 shows the details of damage process with increasing of the applied average strain  $\varepsilon^*$  which corresponded to the average stress  $\sigma^*$  to the end of laminate. It can be seen that, the matrix cracking takes place firstly. The debonding and delamination occur with the increasing of the load. In the meanwhile, the events of matrix cracking develop further. At last, Fiber rupture occurs and then the laminate becomes invalid completely.

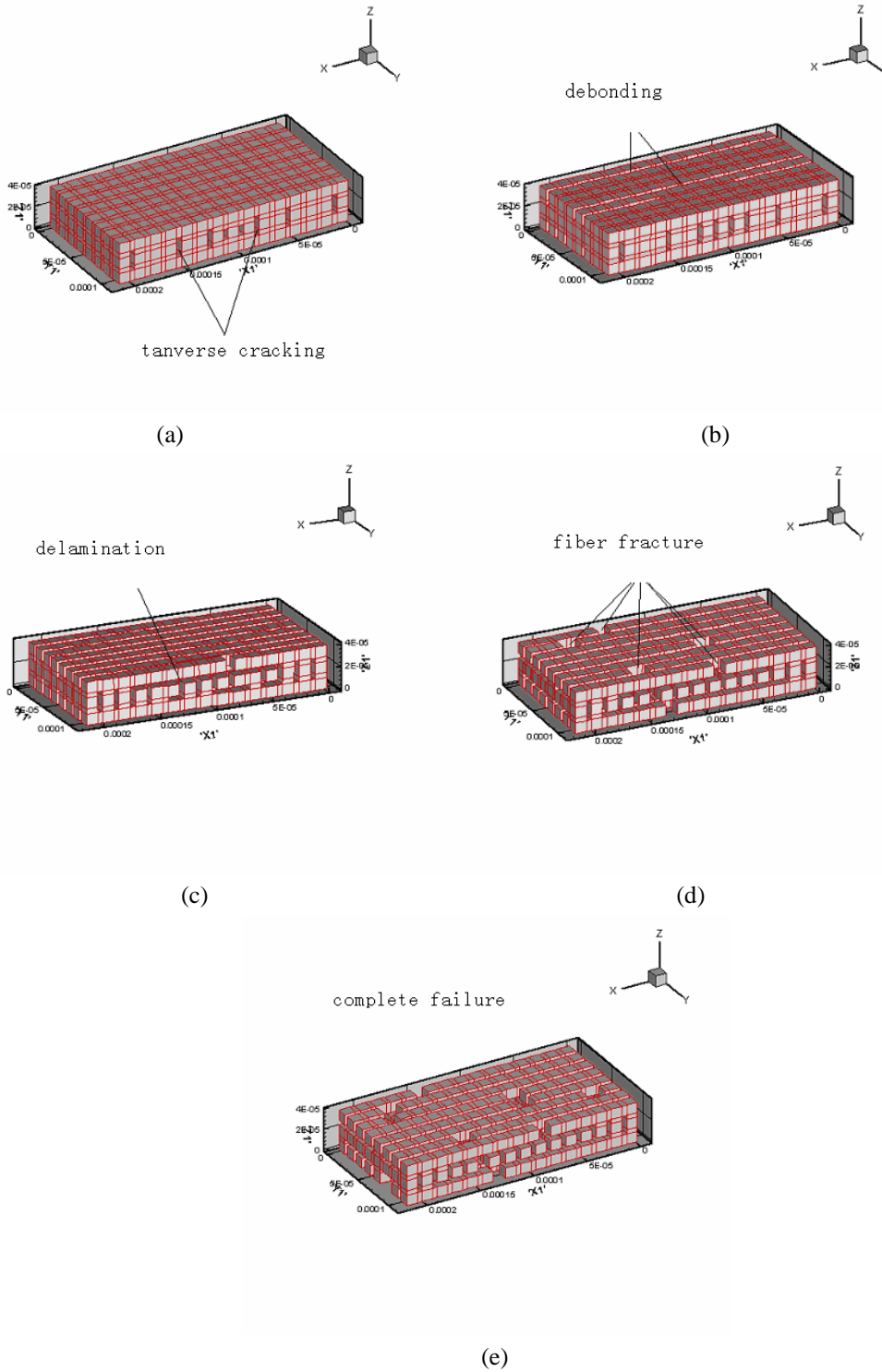


Fig.4. The damage process of the laminate, (a)  $\varepsilon^* = 0.24\%$  ; (b)  $\varepsilon^* = 0.95\%$  ; (c)  $\varepsilon^* = 1.67\%$  ;

(d)  $\varepsilon^* = 2.8\%$  ; (e)  $\varepsilon^* = 3.2\%$

### 3.4 Simulating of stress-strain relations

Fig.5 shows the predicted stress/strain curve at room temperature (296K) in a  $[0^\circ, 90^\circ, 0^\circ]$  laminate (m=12, n=24) and the recorded stress/strain curve from [10].

According to the size effect illuminated in section 3.2, the simulation result is comparable with experimental data. Comparing the two curves, we find that the tensile strength of the simulated laminate (0.40GPa) is bigger than the tensile strength of the experimental specimen (0.36GPa). We attribute this phenomenon to the existence of defects in the tested laminates. Although there are some differences between the predicted and experimental curve, the tendency demonstrates the feasibility of the developed model in this work.

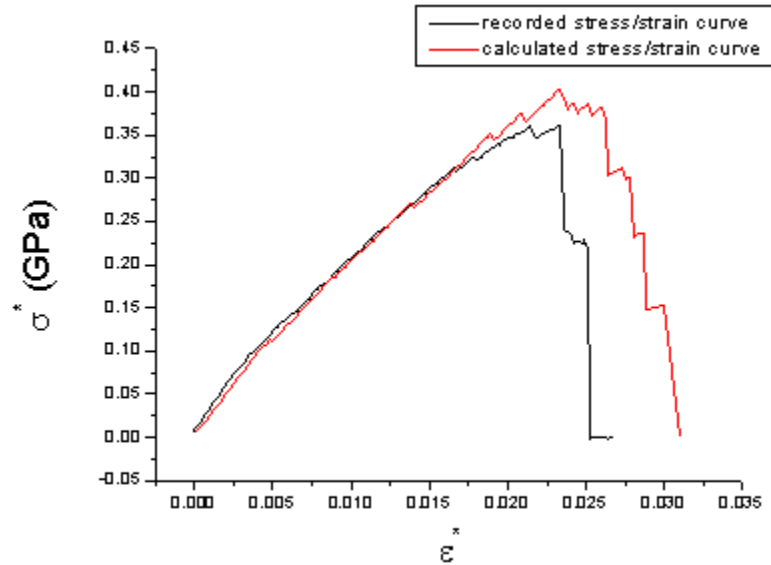


Fig.5. Recorded stress/strain curve and calculated stress/strain curve

### 3.5 The comparison of the mechanical properties at room temperature and low temperature

Table.2. Material constants at low temperature

	E_glass	Epoxy
Elastic modulus (GPa)	70	2.75
Shape parameter $\beta$	9.35	-
Scale parameter $\sigma_0^a$ (MPa)	1120	-
Strength (MPa)	-	155.7
Interface strength (MPa)	238	238
$\Delta T$ (K)	-219	-219

Experiment results showed that some constituent material constants, such as scale parameter, interface strength, will increase when temperature is reduced. The changes



of those constants are listed in table.2 [10]. Results of the simulation are shown in table.3. The results show that the tensile strength is 0.40 GPa at 296 K, and 0.42 GPa at 77 k in the case of only reducing temperature. However, in the case of considering the constituent property changes that result from reducing temperature, the tensile strength is 0.53 GPa at 77K.It is shown that the essential cause of the mechanical properties trending upward at low temperature is not because of reducing temperature, but because of the improving of some constituent properties at low temperature.

Table.3.The results of simulation ( $V_f=0.5$ ,  $m=12$ ,  $n=24$ )

T (K)	Tensile Strength (GPa)
296	0.40(0.36 <sup>*</sup> )
77 <sup>**</sup>	0.42
77 <sup>***</sup>	0.53

\* The value in parentheses is from the experimental data.

\*\* Only reducing temperature, so the material constants in Table.1 were used in simulation

\*\*\* Considering constituent property changes at low temperature, so the material constants in Table.2 were used in simulation

#### 4.Conclusions

A three-dimensional meso-mechanical model and a corresponding procedure to simulate the mechanical behaviors of cross-ply laminates were developed by finite element method combined Monte-Carlo simulation technique. It should be emphasized that an advantage of the developed model is that it is convenient to take into account the statistical characters of fiber strength in simulating mechanical behavior of cross-ply laminates.

By using the developed simulation procedure, size effect of laminate strength was estimated with increasing number of fibers. It was shown that for a certain width (m) of the laminate, the tensile strength decreases sharply when the length (n) of the laminate increases. When n is greater than two times of m, the tensile strength approached to a constant. When m is greater than 12, the tensile strength would vary in a small range. So when m is greater than 12 and n is greater than 2m, it will be applicable to compare the simulation results with experiment data.

The mechanical behaviors including the damage process, stress/strain curve and tensile strength were simulated for a glass-fiber/epoxy cross-ply laminate. Parts of simulated results were compared with the experiment data. The results showed that the developed model and simulation procedure are effective for predicting the mechanical behaviors of cross-ply laminates at various temperatures.

The influence of temperature was discussed. The results showed that the increasing of the scale parameter is the essential cause of the mechanical properties trending upward at low temperature.

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