A TWO-DIMENSIONAL APPROACH TO COMPUTE TRANSVERSE SHEAR STRESSES IN BONDED COMPOSITE PATCHES

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ABSTRACT

The aim of this work is to propose a two-dimensional approach to stress calculations in bonded joints. The main interest here is to take into account some two-dimensional phenomena among which the effect of the Poisson's ratio. In this paper, rectangular bonded composite patches are considered. Stress components through the thickness of the three layers of the structure (substrate, joint, composite) are assumed to be piecewise constant. Equations of equilibrium are obtained under this assumption. They lead to a system of differential equations which is solved with the finite difference method. This two-dimensional approach provides an accurate estimation of transverse shear stress components in the adhesive. Boundary effects as well as the influence of some material properties can be investigated. Location of stress concentrations can therefore be detected.

1. INTRODUCTION

Bonded composite patches are used as structural repairs in several fields such as civil engineering for damaged concrete structures [1] or aeronautics for components which exhibit damages, defects or impacts [2] [3] [4]. A similar case of application consists in using patches for the prevention of damage appearance. The fatigue life of such reinforced structures is expected to be increased and expensive repairs are also expected to be avoided. The present work deals with one of the main key-points in reinforcement problems: the shear stress peak near the ends of the patches. This peak may cause the failure of the joint. Indeed, some studies show that 53% of significant defects on bonded structures proceed from adhesive bond failures [5] [6].

In the present study, a rectangular composite patch is bonded near the area to relieve in order to reinforce it. This rectangular shape is presently chosen for the sake of simplicity. Comparing with fastened reinforcement [7], bonded joints avoid high stress concentrations. Considering a perfect bonding with direct load transfer from the substrate to the patch, it has been shown in a recent study that the geometry and the ply orientations of the composite patch can be optimized with a genetic algorithm [8]. The objective is in fact to design suitable patches in such a way that the stress flow is deviated from the sensitive area where cracks are expected to appear. The crack appearance is therefore delayed. It has been checked that these optimal composite patches lead to some significant stress level reductions in the areas to be relieved [8]. The shape of these optimal composite patches is somewhat unexpected and cannot be guessed *a priori*.

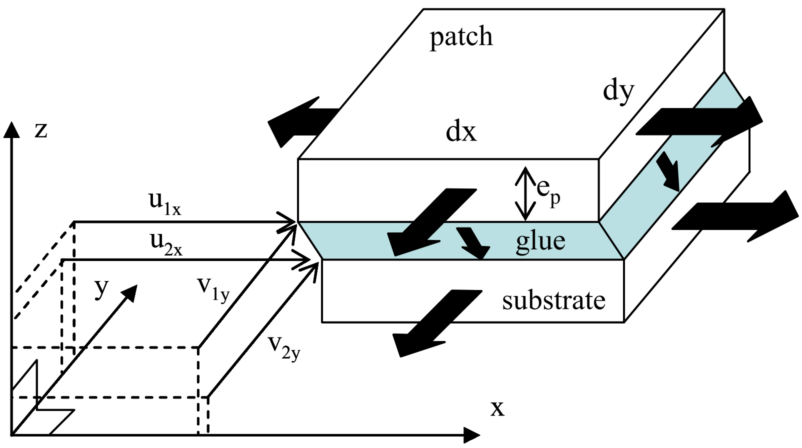
In Ref [8], the shear stress in the bonded joint is not taken into account in the design procedure of the patch. According to usual one-dimensional theories, a shear stress peak is however expected to take place near the free boundary of the bonded joint. The issue here is to study this concentrated shear stresses considering that the bonded joint as well as the composite are plane and not unidimensional.

It must be emphasized that no suitable models have been developed in such bidimensional cases to the knowledge of the authors. It seems that only unidimensional theories have been proposed in the literature [9] [10]. The finite element method (FEM) can be used to obtain either two- or three-dimensional stress distributions through the joint [11], but the FEM does not respect the boundary conditions without a finer mesh near the free edge of the patch. So FEM is not a completely satisfactory solution to estimate the shear stress in the adhesive. The aim of this work is to develop a 2D-dimensional model based on equilibrium equations which can be simply discretized and solved with the finite difference method. Calculations presented herein have been developed with the Matlab 6.5 package.

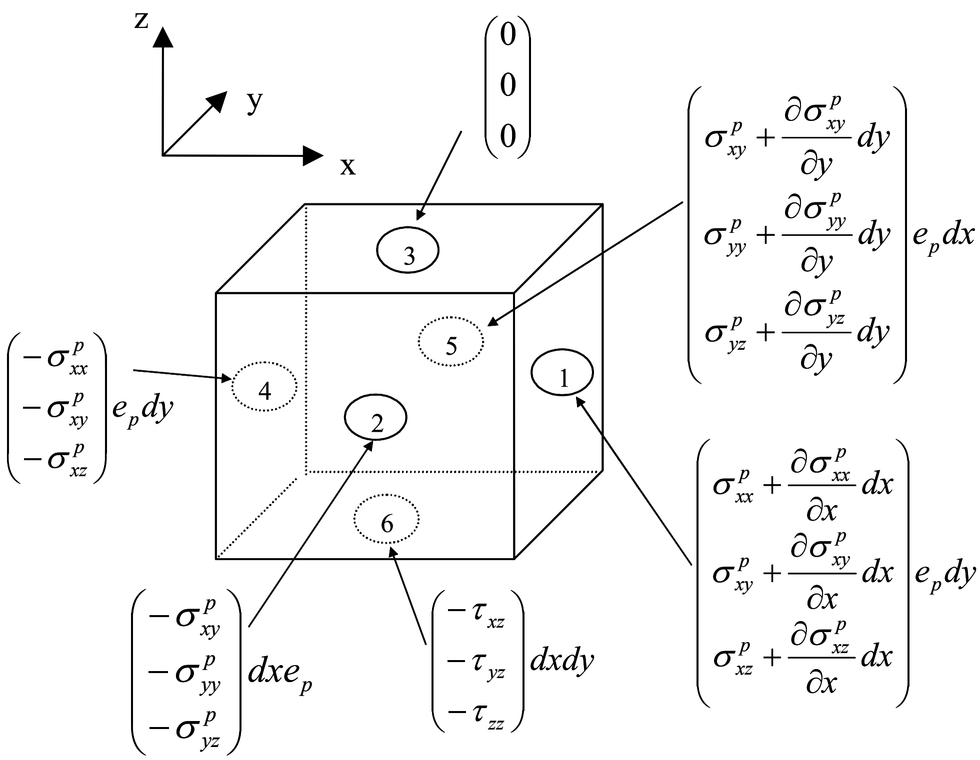
2. THEORY

The studies dealing with the load transfer between substrate and composite are mainly based on a one-dimensional approach in different cases of geometry and loading [9] [10]. In this case, equations of equilibrium are developed in only one direction and are suited to the description of the behaviour of reinforced beams for instance. Because of the plane shape of a patch, it is expected that a two-dimensional approach is more relevant.

In the present study, a rectangular aluminium plate is reinforced with a single bonded composite patch. It has the same width of the aluminium plate. The thicknesses of the adherents are supposed to be constant and small compared to the lateral dimensions of the bonded region. Stresses are considered as piecewise constant through the thicknesses of the reinforced structure. The equations of equilibrium are obtained considering a small part of the reinforced structure shown in Figs. 1 and 2.



"**Fig. 1.** Modelling of a small part of reinforced structure."



“**Fig. 2.** Equilibrium of a small part of composite.”

Assuming that the stress components are functions of x and y only, the equilibrium leads to the following system of nonlinear differential equations.

$$\frac{\partial \sigma_{xx}^{p}}{\partial x} e_p - \tau_{xz}^{a} + \frac{\partial \sigma_{xy}^{p}}{\partial y} e_p = 0$$
$$\frac{\partial \sigma_{yy}^{p}}{\partial y} e_p - \tau_{yz}^{a} + \frac{\partial \sigma_{xy}^{p}}{\partial x} e_p = 0 \qquad (1)$$

where σ_{xx}^{p} (x,y), σ_{yy}^{p} (x,y) and σ_{xy}^{p} (x,y) are in-plane stress components in the composite, τ_{xz}^{a} (x,y), τ_{yz}^{a} (x,y) are shear stresses in the adhesive and e_p is the thickness of the bonded composite plate. Note that the present two-dimensional study leads to both the τ_{xz}^{a} and τ_{yz}^{a} stress in the adhesive whereas usual one-dimensional approaches only give τ_{xz}^{a}.

3. UNIAXIAL TENSILE LOAD

3.1. Equations of equilibrium

The equations of equilibrium are used in the case of a tensile test along the x-axis (see Fig. 3). For the sake of simplicity, shear stresses σ_{xy}^{p} in the composite are neglected. Indeed it has been checked through some preliminary calculations carried out with a finite element programme that these shear stresses are negligible compared to the other stress components. Introducing elastic stress-strain equations in the equations of equilibrium leads to the following system of differential equations:

$$\frac{\partial^2 \sigma_{xx}^p}{\partial x^2} = \frac{G_a}{e_a e_p}\left(\frac{1}{E_{xx}}\sigma_{xx}^p - \frac{\nu_{xy}}{E_{xx}}\sigma_{yy}^p - \frac{1}{E}(\sigma_{xx}^\infty - \frac{e_p}{e_s}\sigma_{xx}^p) + \frac{\nu}{E}(\sigma_{yy}^\infty - \frac{e_p}{e_s}\sigma_{yy}^p)\right)$$
$$\frac{\partial^2 \sigma_{yy}^p}{\partial y^2} = \frac{G_a}{e_a e_p}\left(\frac{1}{E_{yy}}\sigma_{yy}^p - \frac{\nu_{yx}}{E_{yy}}\sigma_{xx}^p - \frac{1}{E}(\sigma_{yy}^\infty - \frac{e_p}{e_s}\sigma_{yy}^p) + \frac{\nu}{E}(\sigma_{xx}^\infty - \frac{e_p}{e_s}\sigma_{xx}^p)\right) \quad (2)$$

with $$\sigma_{xx}^\infty e_s = e_p \sigma_{xx}^p + e_s \sigma_{xx}^s \quad (3)$$

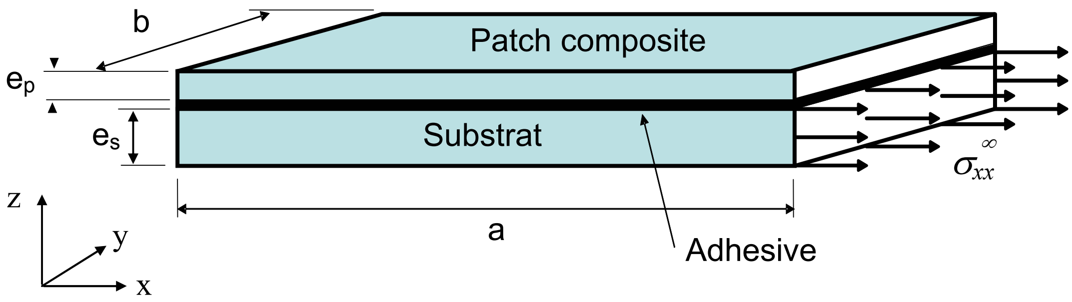
σ_{xx}^∞ is the stress component applied along the boundary of the substrate, $\sigma_{xx}^s(x,y)$ is the longitudinal stress in the substrate, e_a and e_s are the thicknesses of the adhesive and the substrate respectively. The Poisson's ratios of the adhesive and of the composite are directly involved in the above system of differential equations. Indeed, the difference of these coefficients between patch and substrate induce some additional stress components in the transverse direction. No analytical solution of the system is available to the knowledge of the authors. Hence difference method has been used in the present work to solve it.
Eqs. 2 are discretized assuming that:

$$\frac{\partial^2 \sigma_p^{xx}}{\partial x^2} = \lim_{\Delta x \to 0} \frac{\sigma_p^{xx}(x+\Delta x, y) - 2\sigma_p^{xx}(x,y) + \sigma_p^{xx}(x-\Delta x, y)}{(\Delta x)^2}$$
$$\frac{\partial^2 \sigma_p^{yy}}{\partial y^2} = \lim_{\Delta y \to 0} \frac{\sigma_p^{yy}(x, y+\Delta y) - 2\sigma_p^{yy}(x,y) + \sigma_p^{yy}(x, y-\Delta y)}{(\Delta y)^2} \quad (4)$$

Then Eqs. 2 are solved with a suitable programme. The following boundary conditions are prescribed: longitudinal and transversal stress components are equal to 0 along the free edges of the composite patch; shear stress components τ_{xz}^a and τ_{yz}^a are respectively equal to 0 along the longitudinal and transversal free edges of the adhesive. It can be noted that along the two transversal free boundaries of the composite patch, the longitudinal stress distributions correspond to the solution of a unidimensional differential equation.

3.2. Applications

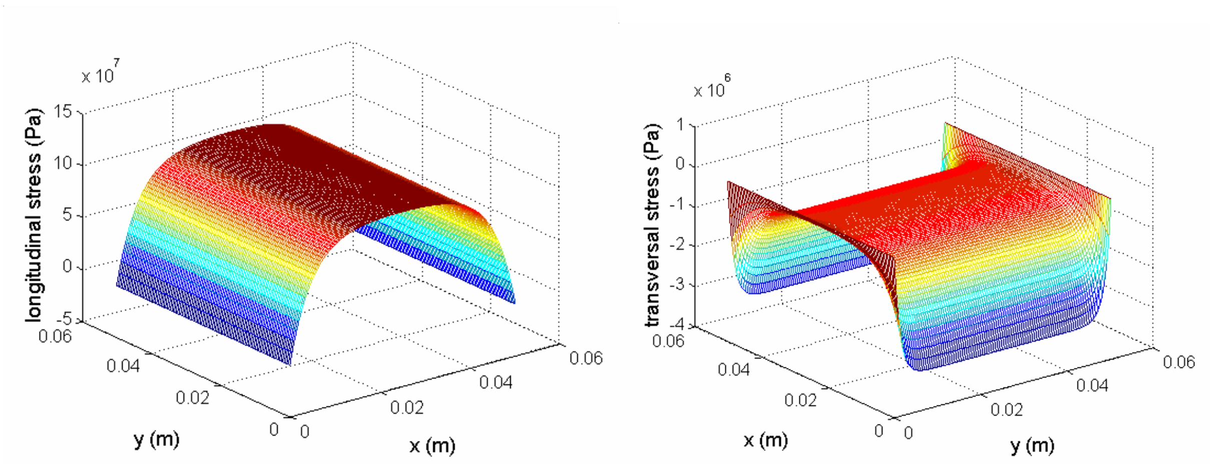
Let us consider the aluminium specimen (E = 70 GPa, ν = 0.3) shown in Fig. 3. Its dimensions are 50 × 50 × 4 mm^3. The composite patch is made in carbon/epoxy (E_x = 181 GPa, E_y = 10 GPa, ν_{xy} = 0.28, G_{xy} = 7 GPa). The patch has 6 unidirectional plies and a total thickness of 0.75 mm. The patch dimensions are therefore 50 × 50 × 0.75 mm^3. The adhesive is supposed to be isotropic and linear elastic (E = 10 GPa, ν = 0.3). Its thickness is equal to 0.3 mm. The bonding between the three layers is supposed to be perfect. The tensile stress σ_{xx}^∞ applied to the aluminium plate is equal to 70 MPa (see Fig. 3). In the present calculation, the pitch $\Delta x = \Delta y$ = 0.5 mm.
Two examples are detailed in the following. They correspond to two distinct fibre orientations with respect to the tensile axis *x*: 0° and 90° unidirectional carbon-epoxy composites are analysed in these two cases respectively. It is therefore expected to emphasize the effect of the Poisson's ratio because it is significantly different in each of these two cases.



"**Fig. 3.** Rectangular composite patch under tensile stresses"

3.2.1. 0° carbon-epoxy composite patch

The 0° carbon-epoxy composite patch is first analysed. The fibre direction and the tensile axis are the same in this case. Fig. 4 represents the longitudinal and the transversal tensile stress distributions in the composite (σ_{xx}^p (x,y) and σ_{yy}^p (x,y) respectively). They are provided by the finite difference programme. Because of the small difference between the Poisson's ratios of the aluminium and of the composite in this case (0.28 and 0.30 respectively), the longitudinal stresses σ_{xx}^p are quasi-equal to the analytical distribution calculated with the one-dimensional case. The transversal stress component is a compressive one because the shrink of the composite patch is lower than the shrink of the aluminium. This phenomenon brings about a load transfer in the transversal direction *y*. After Eq. 1, the differentiation of these stress fields provides the stress components in the adhesive τ_{xz}^a (x,y) and τ_{yz}^a (x,y). Fig. 5 shows these shear stress distributions. As can be seen, shear stress peaks are observed at the four corners of the structure.



"**Fig. 4.** Distribution of σ_{xx}^p (x,y) and σ_{yy}^p (x,y). 0° unidirectional composite patch. Tensile test along the x-axis."

The shear stress τ_{xz}^a in the x-z plane of the adhesive is due to the longitudinal stress in the substrate. The maximum value is reached near the free edge of the adhesive. The shear stress τ_{yz}^a in the y-z plane in the adhesive can be studied in more details. This stress component is due to the load transfer caused by the Poisson's ratio effect. As in the preceding case, the maximum value is obtained near the boundary of the adhesive.

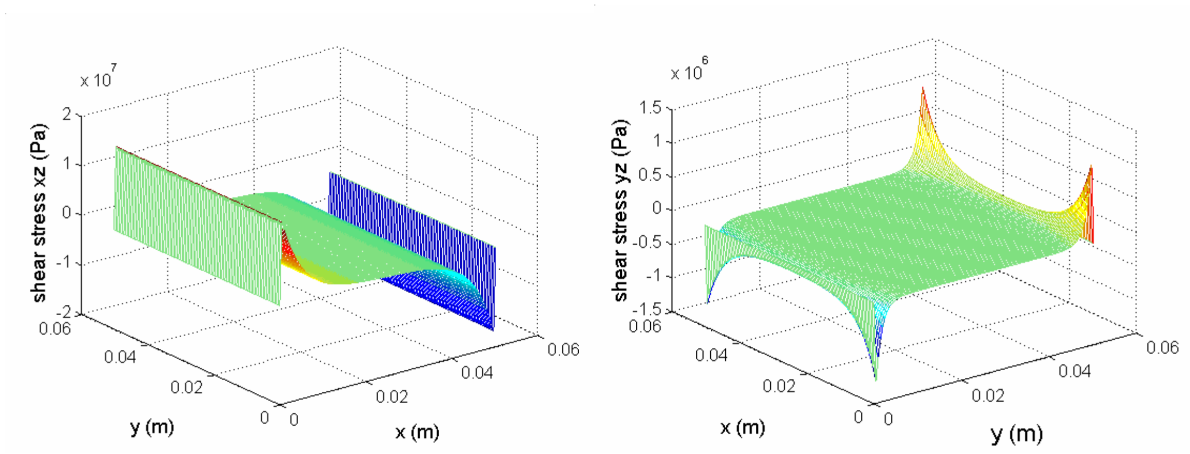
It is proposed to compute an equivalent shear stress in the adhesive as follows

$$\left\|\overrightarrow{\tau_z^a}\right\| = \sqrt{\tau_{xz}^{2\ a} + \tau_{yz}^{2\ a}} \tag{5}$$

The following simple failure criterion based on a maximum shear stress can be proposed

$$\tau_z^{a\ \text{limit}} = \sqrt{\tau_{xz}^{2\ a} + \tau_{yz}^{2\ a}} \tag{6}$$

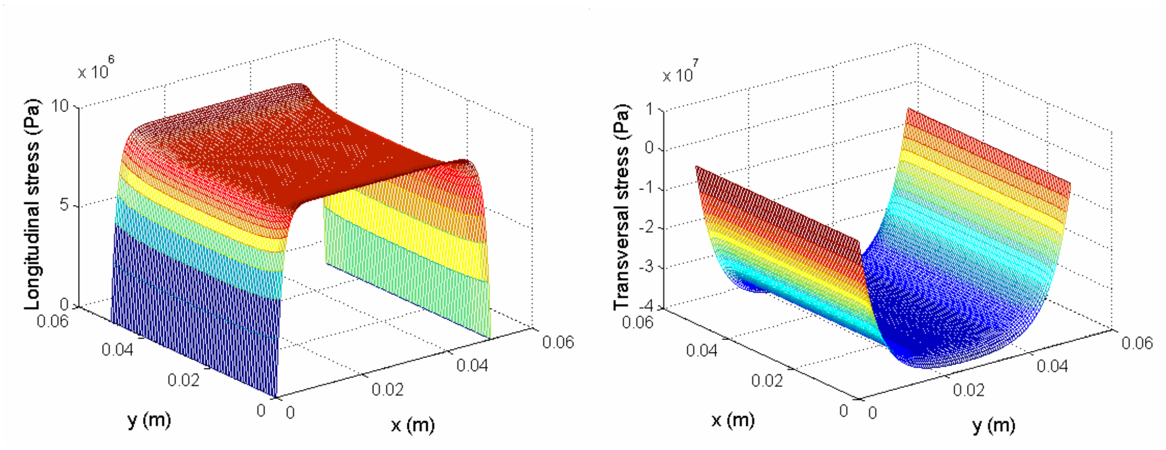
The maximums of this equivalent shear stress are located in the four corners of the adhesive. In this case of tensile test, the failure is therefore *a priori* expected to occur at the corners.



"**Fig. 5.** Evolutions of the longitudinal and transversal shear stresses τ_{xz}^a (x,y), τ_{yz}^a (x,y) in the adhesive for a tensile test along the x-axis and a 0° composite patch."

3.2.2. 90° carbon-epoxy composite patch

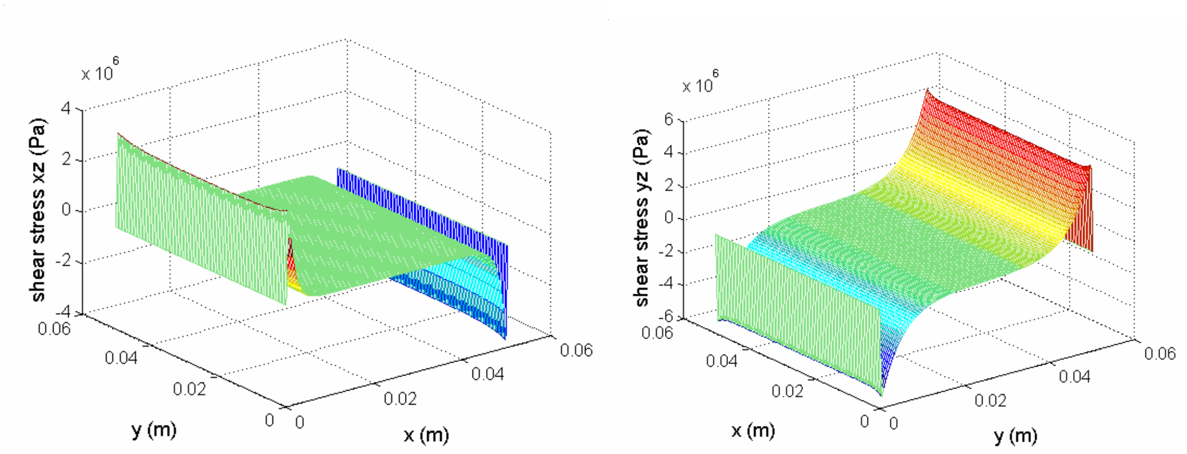
The aluminium plate is here reinforced with a 90° carbon-epoxy composite patch. This corresponds to a fibre direction perpendicular to the x-axis of the tensile test. The material properties as well as the dimensions are the same as in the previous case. It is expected that the Poisson's ratio effect is here more important than in the previous case. Indeed, the Poisson's ratio of the aluminium plate is 20 times greater than the Poisson's ratio of the composite in this case. The composite shrinks scarcely in the transversal direction. This phenomenon brings about an important load transfer length in the transversal direction. Fig. 6 shows the longitudinal and transversal stress distributions in the composite σ_{xx}^p (x,y) and σ_{yy}^p (x,y).



"**Fig. 6.** Distribution of σ_{xx}^{p} (x,y) and σ_{yy}^{p} (x,y). 90° unidirectional composite patch. Tensile test along the x-axis."

The longitudinal stress distribution clearly depends on *y* in this case. This variation is due to the Poisson's ratio effect. Transverse stresses in the patch σ_{yy}^{p} are in fact greater than the longitudinal stress σ_{xx}^{p}. The stress at the centre of the structure can be easily calculated. A ratio of -6 is found between both components.

Shear stress distributions in the adhesive are now analysed (see Fig. 7). In this case, the shear stress peak in the y-z plane is more important than the shear stress peak in the x-z plane. Using Eqs. 2 and 3, it can be calculated that the critical zones of the adhesive are located at the corners of the patch. Obviously, a unidimensional modelling cannot model this phenomenon. The shear stress along the transverse direction cannot be neglected and a two-dimensional approach is relevant in this case. In this case too, a unidimensional approach cannot correctly model the shear stress along the transverse direction.



"**Fig. 7.** Evolutions of the longitudinal and transversal shear stresses τ_{xz}^{a} (x,y), τ_{yz}^{a} (x,y) in the adhesive for a tensile test along the x-axis and a 90° composite patch."

4. SHEAR LOAD

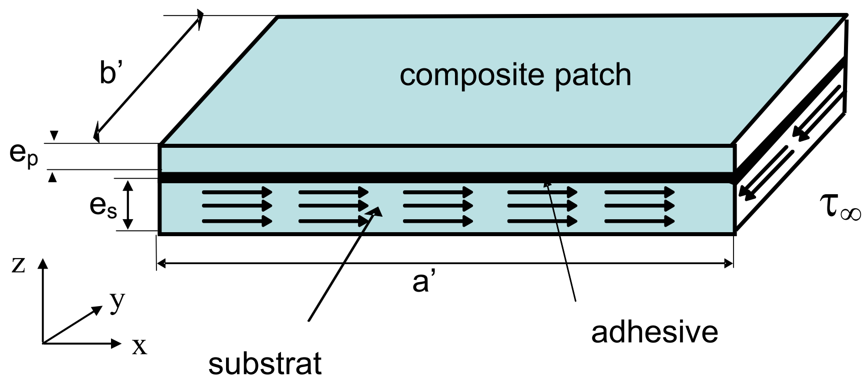
4.1. Equations of equilibrium

Let us now examine a state of shear stresses (see Fig. 8). The substrate is subjected to in-plane shear stresses and a unidimensional analysis is not sufficient to evaluate the stress state in the composite patch and in the adhesive. Equations of equilibrium (Eq. 1) are applied in this case. For the sake of simplicity, the longitudinal and transversal stress components σ_{xx}^p and σ_{yy}^p in the composite are neglected. Introducing elastic stress-strain equations in the equations of equilibrium leads to the differential equation below (Eq. 8) which governs the shear stress distribution τ_{xy}^p in the patch.

$$\frac{\partial^2 \tau_{xy}^p}{\partial x^2} + \frac{\partial^2 \tau_{xy}^p}{\partial y^2} - \gamma^2 \tau_{xy}^p = -\alpha^2 \tau_\infty \qquad (7)$$

with $\tau_{xx}^\infty e_s = e_p \tau_{xx}^p + e_s \tau_{xx}^s$, $\gamma^2 = \frac{G_a}{e_a}\left(\frac{1}{G_p e_p} + \frac{1}{G_s e_s}\right)$ and $\alpha^2 = \frac{G_a}{e_a e_p G_s}$ (8)

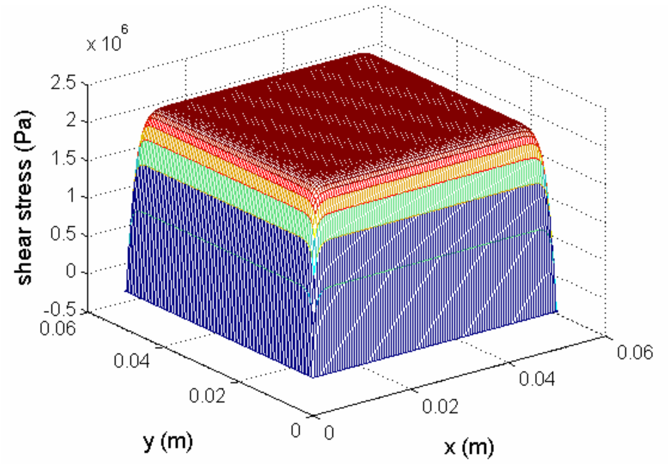
Finite differences are used in this case too to solve the above differential equation.



"**Fig. 8.** Rectangular composite patch under shear stresses"

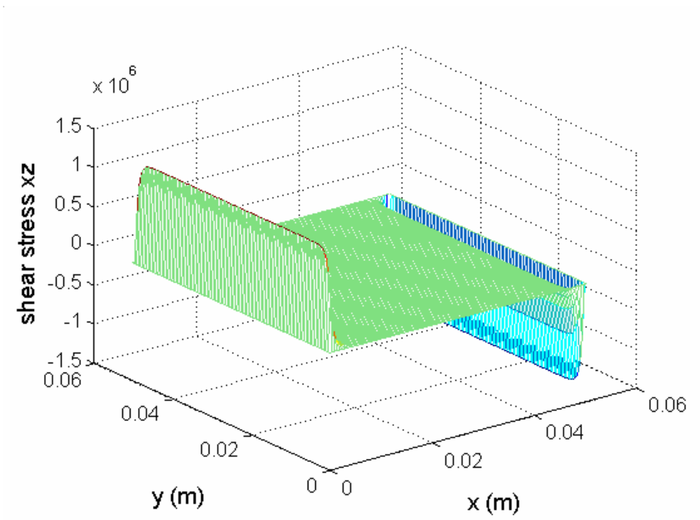
4.2. Application

The composite patch has the same material and geometric characteristics as above (see section 3.2.). Instead of a tensile load, a shear load τ_∞. in the x-y plane is applied to the substrate. This shear stress τ_∞ is equal to 10 MPa. The shear stress distribution in the patch denoted τ_{xy}^p and shown in Fig. 9 cannot be found with usual unidimensional approaches.



"**Fig. 9.** Distribution of the in-plane shear stress τ_{xy}^{p} (x,y) in the patch for a shear test."

The maximum value of the shear stress in the patch is reached at the centre. Shear stresses in the adhesive can then be evaluated. Because of the x-y symmetry, the shear stress distributions in planes x-z and y-z are the same. Fig. 10 shows the distribution of τ_{xz}^{a} in the adhesive. As can be seen, this shear distribution depends on both the x- and y-coordinates. These shear stresses are due to the load transfer between aluminium plate and composite. Such a phenomenon is well-known thanks to simple models developed within one-dimensional approaches [5] [6] and longitudinal loading.



"**Fig. 10.** Evolution of the shear stress τ_{xz}^{a} (x,y) in the adhesive for a shear test. The graph is similar for the component τ_{yz}^{a} (x,y)."

5. CONCLUSION

The main conclusions of the present study are as follows:

- the equations governing the bidimensional equilibrium of a bonded composite patch can be found in some simple loading cases, namely tensile and shear stresses;
- these equations can be integrated with the finite difference method;
- when tensile stresses are considered, results found show that the difference between the Poisson's ratios of both the substrate and the composite patch induce some

transverse stress concentrations near the boundary and at the corner of the bonded joint;
- transverse shear peaks also take place near the free edges of the bonded joint when the substrate is subjected to in-plane shear stresses.

This present approach can be extended to composite patches of any shape, allowing the proper design of composite patches with optimized shapes. The displacement/strain distributions onto composite patches can also be measured with full-field measurement methods and compared to their theoretical counterparts, thus allowing a validation of the present models. Both tasks are presently under progress.

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