OPTIMIZATION OF COMPOSITE PATCHES USING THE DANG VAN'S FATIGUE CRITERION AS COST FUNCTION

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ABSTRACT

This paper describes the application of genetic algorithms to the optimization of composite patches bonded on structures to reinforce them. Usual patches are rectangular or circular. More complex shapes are presently defined with a genetic algorithm. The objective is to calculate the shape as well as the position and the ply orientations of a patch by minimizing a fatigue criterion in a zone to be relieved. The shape itself is modelled with a spline curve, thus enabling to define complex geometries with few design variables, that is the coordinates of control points. Composite ply orientations are modelled by angles which can be equal to usual discrete values. The Dang Van's fatigue criterion is used as objective function in practice. Constraints are defined by some forbidden zones that cannot be covered by the patch and a maximum surface of the patch. A bidimensional modelling is considered and solved by finite element calculation. Applications are performed for a carbon-epoxy patch perfectly bonded on an aluminium plate with a circular notch at its centre. Patches resulting from optimizations ensure both local and global reinforcement effects: the patch is close to the zone to be relieved and it advantageously modifies the stress field within the whole structure as well.

1. INTRODUCTION

Composite material patches are used in aeronautics and civil engineering for repairing structures suffering damages or impacts [1] [2] [3] [4]. In practice, they can be affixed to their substrate either with bolted [5] or with adhesive [6] [7] [8] joints. High stress concentration takes place however near the bolts. This drawback is avoided using bonded patches because stress concentrations are lower in this case. Externally bonded patches have therefore proved to be an effective method to repair cracks and defects in aircraft structures, as shown by the Australian defence [7].

An alternative of repairing structures is to prevent them from fatigue or damage effects using similar bonded patches, but before defect appearance. Thanks to the load transfer from the structure to the patch, the stress amplitude under or near the patch is reduced and the fatigue life is therefore increased. Usual patches have rectangular or circular shapes, but one can expect that more complex geometries could bring about better reinforcements. So the aim of this work is to examine the optimal design of bonded composite patches which could be used in structure reinforcement. Shape, location and ply orientation are considered as variable in this optimization problem.

Performing repair or reinforcement can be difficult in practice because some zones of the structure must not be modified, for example to preserve screw access in aeronautical structures. The proper design requires therefore the use of specific tools to define optimal characteristics of patches in terms of geometry, number of plies, ply orientations, location on the structure… A patch optimization program based on genetic algorithms (GAs) is developed in this work to reach this goal. The choice of GAs is *a priori* relevant compared to other optimization approaches because of some characteristics of the problem. Indeed, composite ply angles are often restricted to some discrete values such as +45, 0, -45, 90 degree, and GAs are well suited to optimize functions of discrete variables. They were successfully developed in various fields, especially in the case of laminate optimization [9] [10].

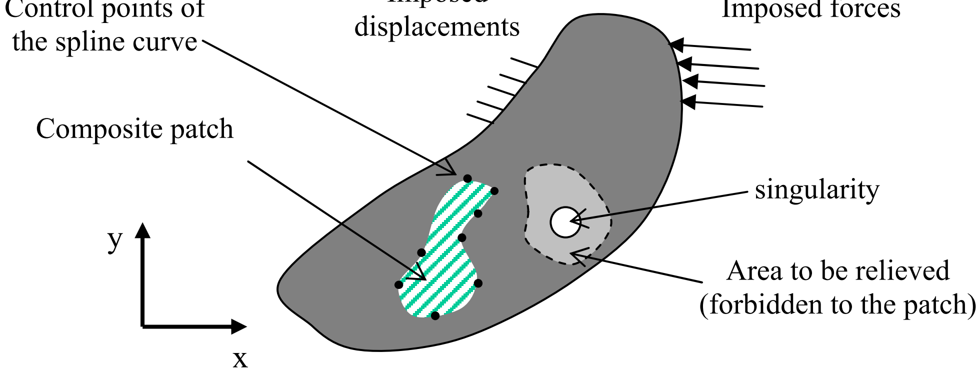
The method is based on the Darwin's principle "survival of the fittest" [11]: an initial population of potential solutions is first randomly created, and the natural evolution of the population is simulated in order to converge towards the best solution. GAs are based on operators which simulate the natural selection, crossover and mutation.

In the present work, the Dang Van's fatigue criterion [12] is used to assess the quality of reinforcement induced by the patch. The algorithm is associated with the ANSYS finite element package to calculate the objective function of any reinforcement configuration.

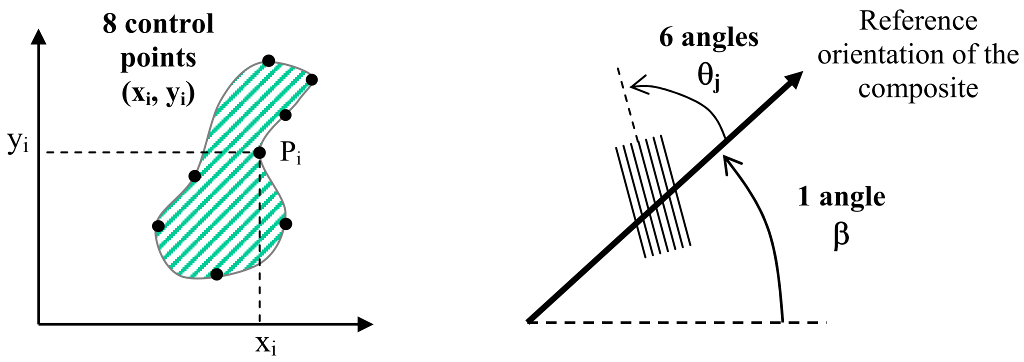
For a bidimensional modelling, the shape and position of any patch can be defined by few variables thanks to a spline curve governed by some control points. The geometry of the patch as well as the ply orientations are simultaneously optimized by the algorithm to reduce the stress level in the area to be relieved.

2. MODELLING OF A REINFORCEMENT

A loaded structure can be reinforced because of the presence of geometrical or material singularities, such as holes, section changes or material weaknesses. The composite patch is bonded onto the surface of the structure to reinforce these critical zones. Some additional technological constraints can be taken into account in the patch design problem, like a limitation of the surface of the patch or some conditions about the location of the patch onto the structure. For the sake of simplicity, the problem here is bidimensional (Fig. 1). The patch is perfectly bonded on the plane structure and no load transfer zone between patch and substrate is considered: stresses are therefore directly transferred from the substrate to the composite. The shape of the patch and its location on the structure are defined by a spline curve with eight control points. This number was chosen to ensure various potential shapes for patches. Note that a specific procedure has been developed to avoid loops during the construction of the spline curve. The number of plies is equal to six in the present case for the sake of simplicity, but this ply number could also be considered as variable. The stacking sequence is therefore modelled by six angles θ_j which are to be chosen among a set of four usual values: 0, 45, -45 and 90 degrees. A last angle β defines the global orientation of the patch with respect to the geometry of the structure (Fig. 2). Thus any patch is defined by 17 continuous variables and 6 discrete ones.



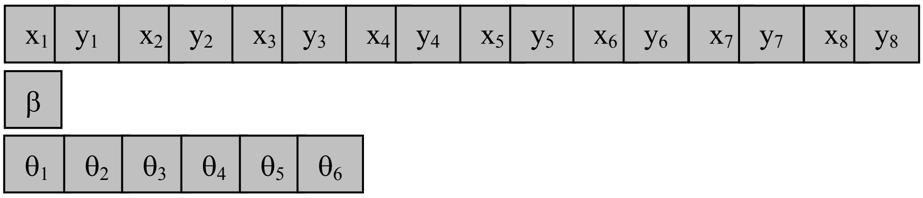
“**Fig. 1.** Bidimensional modelling of a plane reinforcement problem.”



"**Fig. 2.** Design variables for the composite patch."

3. OPTIMIZATION USING GENETIC ALGORITHMS

The first step in to create randomly an initial population of potential solutions. Any individual completely defines reinforcement configuration. It is characterized by the value of its so-called *chromosome*, whose *genes* correspond to the design variables of the problem (Fig. 3).



"**Fig. 3.** Chromosome with 17 genes defining an individual."

Some usual genetic operators are employed to create step by step the successive generations (Fig. 4).

i) The efficiency of all individuals of the initial generation is first estimated through its *objective function* F_i, calculated here with the Dang Van's fatigue criterion.

ii) An 'elitist' approach is used in the algorithm: the best individuals in a population are automatically kept in the next generation. This ensures the convergence of the algorithm.

iii) A 'selection' operator is applied: some individuals are selected in the population as parents of the next generation; non-selected individuals die. The selection process is based on survival probabilities P_i calculated for each individual from its objective function. For a population with N individuals, the survival probability P_i of any individual i is defined by:

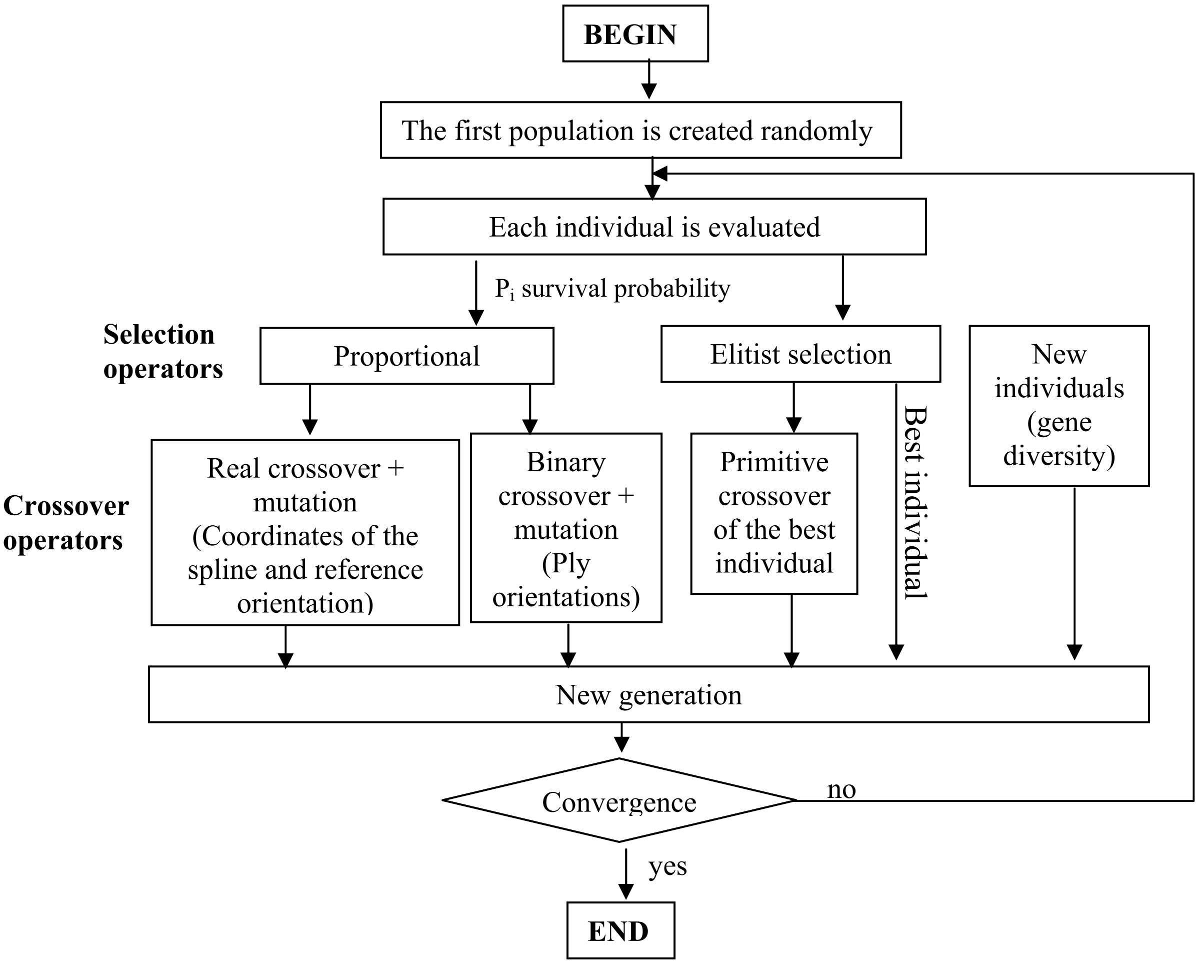
$$P_i = \frac{F_i}{\sum_{j=1}^{N} F_j} \tag{1}$$

A random procedure is then used for each individual to decide on its survival. Note that even individuals with small survival probabilities can be selected as parents. This method gives a chance to any configuration.
An *elitist* approach is also used for the selection: the best individuals are automatically selected as parents in the next generation.

iv) ‘Crossover’ operators are then applied to selected parents: genes of parents are crossed to produce children.
A binary crossover is applied to the genes which take discrete values (ply orientations θ_j). A so-called ‘uniform crossover’ is presently used: the number of ‘crossing points’ is equal to the number of discrete genes minus one (Fig. 5).
A real crossover is applied to continuous variables (angle β and coordinates x_i, y_i). The genes of a child are a weighted average of the parent ones:

$$\text{Gene}_{\text{child}} = w\,\text{Gene}_{\text{parent 1}} + (1-w)\;\text{Gene}_{\text{parent 2}} \qquad (2)$$

where the weighting w randomly lies between 0 and 1 for each crossover (Fig. 6).



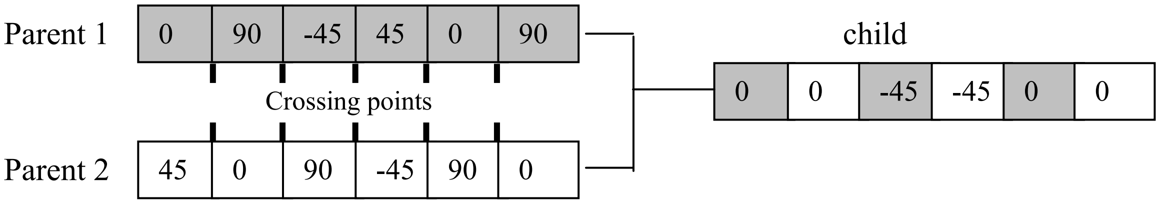
“**Fig. 4.** Principle of the genetic algorithm.”

Mutations can randomly modify gene values. They produce children which have sometimes much stronger efficiency than theirs parents. The mutation operator is governed by a ‘mutation probability’ which defines the chance for a gene to be modified.

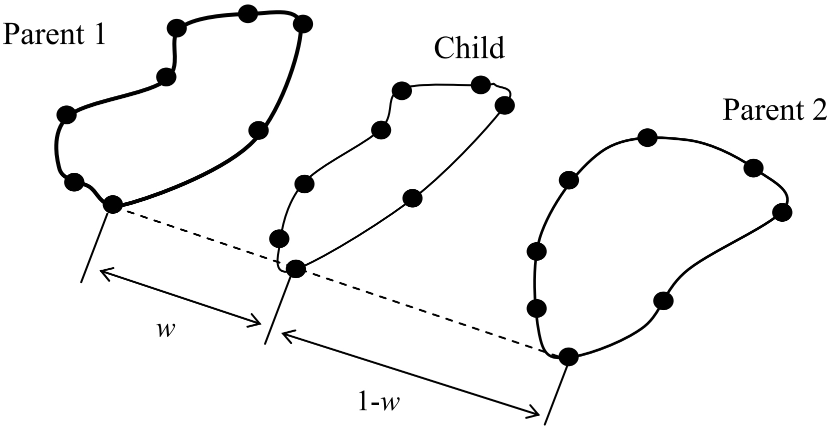
v) The algorithm convergence speed can be strongly improved by a so-called ‘primitive crossover’ of the best individual. It consists in small changes of the gene values of this best individual: if a best configuration is found, it is considered as a child. This operator accelerates the algorithm convergence. Indeed, if the optimum is in the vicinity of the best solution of the generation, this operator finds it immediately.

vi) Finally, new individuals are randomly introduced in each generation to ensure gene diversity during the evolution process.

The algorithm runs until the objective function of the best individual in successive generations converges. Note that a geometric procedure is implemented in the genetic algorithm in order to avoid loop in the spline curve construction. Indeed, the genetic algorithm must not have children which have an incompatible geometry. Moreover, children characterized by a surface greater than 20% of the surface of the structure are killed.



“**Fig. 5.** Uniform crossover for discrete genes.”



“**Fig. 6.** Real crossover applied to spline curves.”

4. OBJECTIVE FUNCTION BASED ON THE DANG VAN'S FATIGUE CRITERION

The goal here is to delay the appearance of cracks due to fatigue in a zone of the structure which must be relieved. A suitable fatigue criterion must be used to reach this goal. Among all multiaxial fatigue criteria, the Dang Van's criterion is one of the most popular. It is presently used to assess the efficiency of reinforcement solutions. This criterion is based upon a 'critical plane' concept: in the sense that it requires to find out the most damaged material plane according a specific damage indicator definition [12].

For a material plane with normal *h*, the damage indicator proposed by Dang Van is a linear combination of the hydrostatic pressure $p(t)$ and the alternate shear stress $\tau_{ha}(t)$ at the same time *t*. This alternate shear stress $\tau_{ha}(t)$ represents the deviation of the shear stress vector with respect to its mean value during the cyclic period. The method needs the review of all possible material planes in order to assess their own damage indicator. The critical plane is obtained through the maximization problem of this indicator.

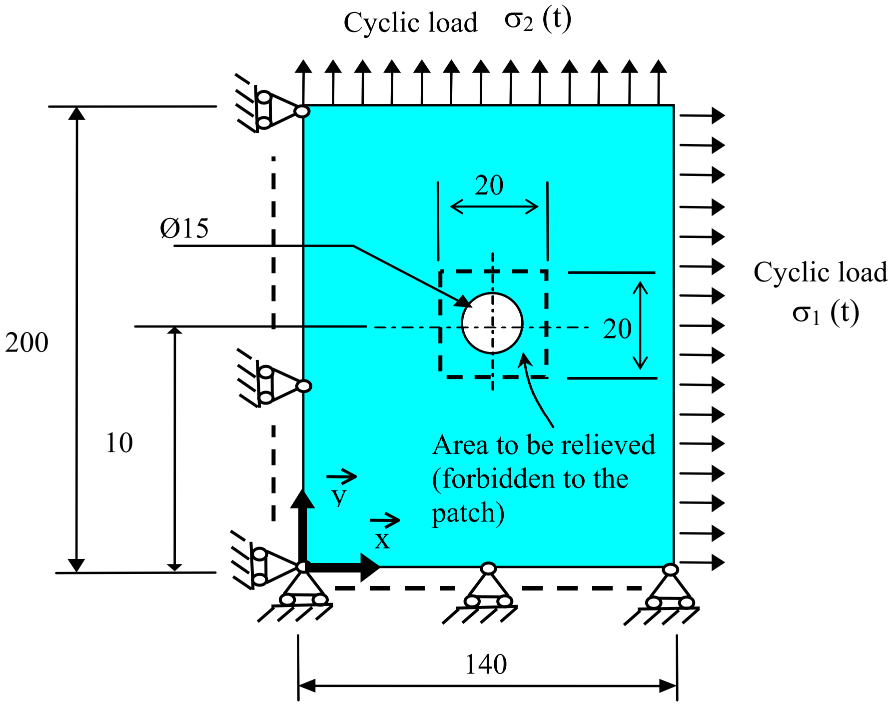
Finally, during the cyclic loading of the structure, the method requires to find out the critical point (x, y) in the zone to be relieved. So the objective function OF_i of any individual *i* is obtained from a triple optimization:

$$OF_i = \underset{x,y}{Max}\ \underset{h}{Max}\ \underset{t}{Max}\left(\left\|\vec{\tau}_{ha}\right\| + \alpha\, p(t) - \beta\right) \tag{3}$$

where α and β are coefficients which are related to basic fatigue characteristics called 'endurance limits', such as σ_D, τ_D, σ_0 under respectively fully reversed tensile-compressive tests, fully reversed torsion test, and fully reversed zero to maximum tensile tests. In practice, α and β can be obtained with two simple fatigue tests.

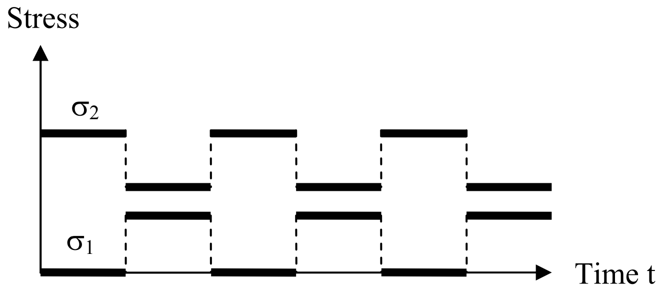
5. NUMERICAL MODELLING OF A REINFORCEMENT PROBLEM

Fig. 7 presents a reinforcement problem. It consists in a non-proportional biaxial tensile test on a 200 mm × 140 mm × 4 mm rectangular plate with a 15 mm hole at its centre. This plate is made of aluminium which is assumed to be linear elastic ($E = 70$ GPa, $\nu = 0.3$). The composite patch is made of 6 unidirectional plies characterized by four constants in the orthotropic axes: $E_x = 181$ GPa, $E_y = 10$ GPa, $\nu_{xy} = 0.28$, $G_{xy} = 7$ GPa. The total thickness of the patch is 0.75 mm. The patch must reinforce a 20 mm × 20 mm square shaped neighbourhood around the hole. This square zone is forbidden to the patch. This square zone corresponds in practice to the screw accessibility area. The patch surface is presently limited to 20% of the aluminium plate surface.



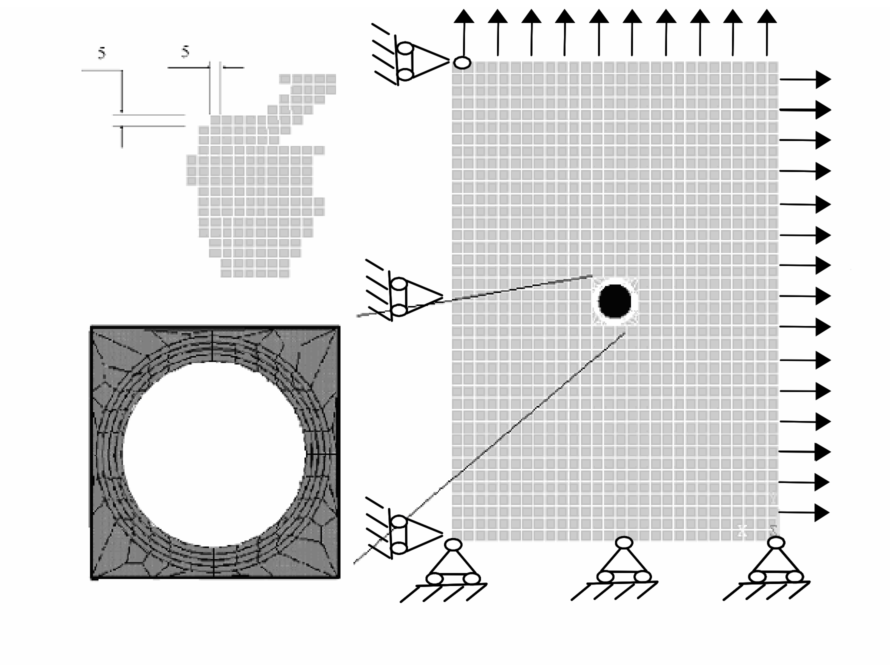
"**Fig. 7.** Reinforcement problem."

The loading under consideration is presented on Fig. 8. The two in-plane loading components $\sigma_1(t)$ and $\sigma_2(t)$ are cyclic. They are assumed to be piecewise constant and divided in two steps. Each of them has the same duration. Such a loading cycle is rather simple. Actual ones used in aeronautics would be more complicated but the same design procedure would apply. σ_1 is equal to 50 (respectively 100 MPa) during the first (respectively second) part of the cycle. σ_2 is equal to 200 MPa (respectively 150 MPa) during the first (respectively second) part of the cycle. It can be noted that this cycle corresponds to a non-proportional loading.



"**Fig. 8.** Fatigue loading applied to the holed plate."

The Ansys 7.0 package is used to compute the objective function. The substrate and the composite patch are meshed by eight-noded shell elements. Perfect bonding between the substrate and the composite is obtained by merging the nodes of both the composite and the substrate. For the sake of simplicity, a regular mesh is chosen over the structure apart from the vicinity of the hole which is meshed with a specific procedure (see Fig. 9).



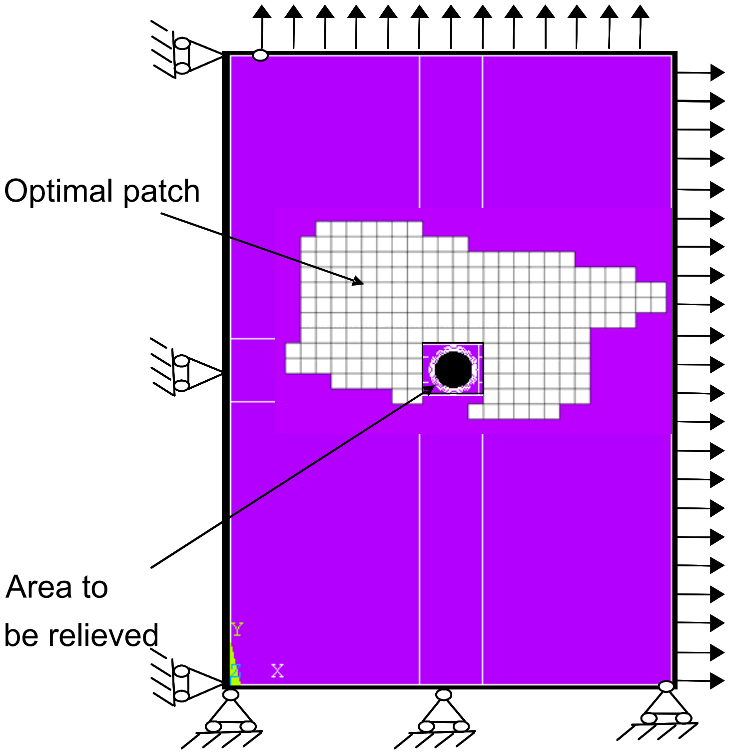
“**Fig. 9.** Mesh of patch and structure.”

The mesh density has been obtained with some preliminary calculations which have been carried out to get a good compromise between the calculation time and the accuracy of the results. The iterative Pre-conditioned Conjugate Gradient solver is chosen. The stress cycle is discretized into two stages in order to evaluate the Dang Van’s criterion. Indeed, the stress cycle must be discretized with relevant discretizations which represent correctly the stress cycle applied on the structure. For the sake of simplicity, the Dang Van’s criterion is averaged over the area to be relieved, whereas Eq. 3 involves a maximization over x and y.

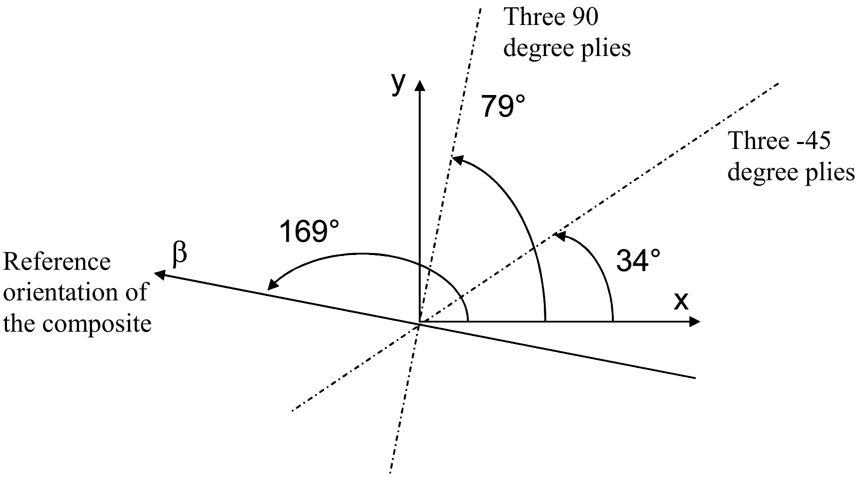
5. EXAMPLE OF OPTIMIZATION

The genetic algorithm is applied to obtain the most efficient composite patch in the above example. It converges after a one day calculation with a 3.06 GHz/1 Go Ram computer. Fig. 10 shows the shape of the patch resulting from the optimization. The global orientation β of this composite is equal to 169 degrees and the laminate sequence is: [90 90 -45 90 -45 90]. Fig. 11 presents the stacking sequence of the optimal patch. The reduction of the Dang Van’s criterion is 7.24%: during the cycle, the damage indicator is maximum for the second stage of the loading, corresponding to $\sigma_1 = 100$ and $\sigma_2 = 150$ MPa. During the optimization process, the stage corresponding to the maximum damage indicator is not always the second one: it depends in fact on the patch characteristics. Note that a circular composite patch with quasi-isotropic properties would reduce the Dang Van’s criterion of 6.19%.

It can be noted in Fig. 10 that the optimal patch does not completely surround the area to be relieved. This means that the patch influence on this area is not only ‘local’: the reinforcement is in fact not only the consequence of the reduction of the stress in the immediate neighbourhood of the patch. The optimal patch is logically located near the area to be relieved, but it covers also some places far away from the singularity (for instance near the right-hand part of the structure). In fact the patch modifies the stress field in the whole structure. Optimal patches combine advantageously both ‘local’ and ‘global’ effects. Note that the ply stacking sequence contributes to both these ‘local’ and ‘global’ effects thanks to its direction dependent properties.



"**Fig. 10.** This optimal composite patch improves the fatigue life of the structure by reducing the Dang Van's fatigue criterion of about 7.24% in the area to be relieved."



"**Fig. 11.** Stacking sequence of the optimized composite patch."

6. CONCLUSION

Aging structure prevention has received an increased interest for about thirty years [1] [2] [3] [4]. A recent solution consists in the reinforcement with bonded composite patches. The goal is to transfer the load from the structure to the patch. Usual patches have rectangular or circular shapes. It has been shown in this work that more complex geometries could be advantageously proposed. An optimization method based on genetic algorithms has been used to optimize shape, location and ply orientation of the patch. The shape itself is modelled with spline curves, thus enabling to define a configuration with just a few design variables (the coordinates of the control points). An application of the method has been performed to a structure submitted to a multiaxial fatigue loading. Results show that a significant reduction of the Dang Van's fatigue criterion in the area to be relieved can be obtained thanks to both local and global reinforcement effects. The experimental mechanical response of such patches is presently under progress.

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