FAILURE CRITERIA FOR NON-METALLIC MATERIALS
PART I: FIBRE REINFORCED PLASTICS

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ABSTRACT

Within the frame of the study “Failure Criteria for Non-Metallic Materials”-Part I funded by ESA, detailed guidelines for the proper use of failure criteria during the different stages of composites spacecraft structures design have been elaborated for fibre reinforced plastics (FRP). Therefore, one important part of this work was the evaluation of the results which the World Wide Failure Exercise (WWFE) has delivered. The test results and the predictions of the different authors given in the WWFE did allow a benchmarking of failure criteria that was necessary to evaluate the enormous amount of existing failure criteria. One conclusion of this study was that more reliable test results must be gathered in order to be able to better benchmark the existing criteria. An overview of the results and a comparison with data from literature is given. A unique high level of biaxial inplane compressive stress was reached compared to test results of other authors but far below of the Tsai-Wu-criterion predictions. However, the expected strength level was not reached as the failure of the specimens was initiated by undesired failures so far. On the theoretical side Puck’s action plane fracture criteria are presented in some detail which seem to be the best tool for describing the brittle fracture in FRP with polymer matrix.

1. INTRODUCTION

Within the frame of the ESA study “Failure Criteria for Non-Metallic Materials” detailed guidelines for the proper use of failure criteria during the different stages of composites spacecraft structures design have been elaborated for non-metallic materials. For the engineers it is of interest what kind of failure will happen and at which load level initial failure or ultimate failure occurs leading to corresponding margin of safety. Part I is focussing on Fibre Reinforced Plastics (FRP) and Part II on Ceramic Matrix Composites [1].

With respect to failure criteria of fibre reinforced plastics (FRP) a significant amount of work has already been performed by universities and industry. Many models exist and a considerable amount of test data is available. The World Wide Failure Exercise (WWFE) has helped to have now a kind of evaluated overview on failure criteria. This Part I shall give a comprehensive overview on failure criteria from the engineering point of view with focus on the most used one of Tsai-Wu and the best one according to the results from the WWFE.

When analysing composite structures it is very important to define in advance the kind of failure which has to be assessed. The definition of failure is in fact an underestimated issue during the failure analysis. Also within the World Wide Failure Exercise it was noticed, that especially initial failure has to be defined in the right way. For instance leakage of a pressurised composite tube often has nothing to do with the initial failure of single lamina within the tube’s wall.

A look on the classification of failure criteria regarding their original aim has to be made before application. Failure criteria can be classified according to: analysis range (initial or ultimate failure), physical structure model (micro, macro or component level) and mathematical approach (limit, polynomial tensor or physically based). It has to be reminded, that experimental data influenced by the material and manufacturing process on the one side and the test specimen and test itself on the other side and also by the physical structure level selected. To ensure a proper design of the composite structure experimental data consistent with the selected failure criteria has to be used.

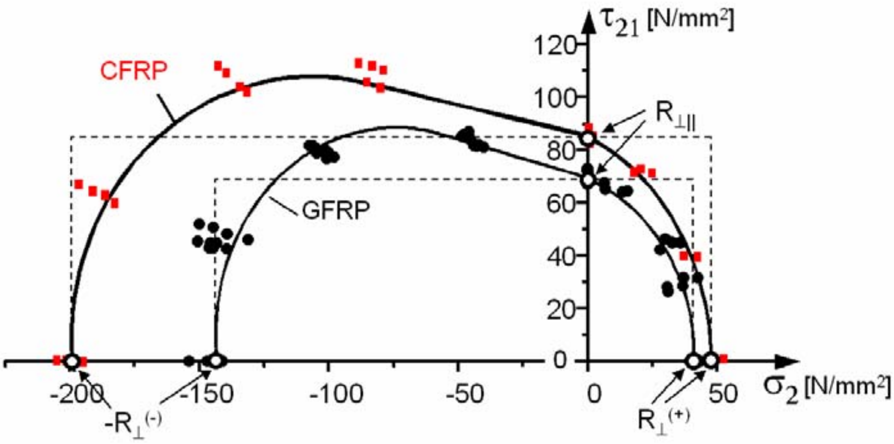
2. FAILURE CRITERIA STATE OF THE ART

In the early days of FRP-science some work was done to describe strengths of laminates with different lay-ups. It was realised that a reasonable way to analyse laminates could only be accomplished on the level of the single lamina of the laminate [2]. The stress analysis for this layer-by-layer failure analysis was performed by the Classical Laminate Theory (CLT). Only using this concept on the level of single unidirectional lamina allows applying failure criteria generally.

In the past three decades many kinds of differing theories have been developed leading to such a variety of criteria that it is not manageable anymore even by the experts. The design engineer has no interest in – and can not accomplish the task of – checking all existing criteria to detect the one which he feels is best suited for his problem. For this reason the different failure criteria need to be verified and benchmarked by independent persons in order to assure that the resulting ranking and recommendations are credible and accepted by the design engineers as well as by the scientific community.

The organisers of the WWFE tackled this incredible task in 1992, probably not realizing that they were heading for a decade of work [3]. Now, twelve years later, the results have been published [4], but unfortunately a ranking was used in which some very different criteria attain quite good marks. The major reason for this ranking is undoubtedly the fact that the organisers of the WWFE could not find enough reliable test results that would have allowed better evaluation and judging the different criteria. Thus, once again, the design engineers left with a choice of criteria. Also, as long as there is no sufficient experimental database a lot of work will possibly be wasted and progress remains slow because everyone will continue in pursuing his own preferences.

One aim of the ESA-study was to create some indubitable reference data against which one can benchmark existing criteria. After a thorough literature study it was decided to design tests measuring the fibre parallel compressive strength of a unidirectional (UD) carbon fibre lamina when high transverse compressive stresses σ_2 are acting simultaneously with the longitudinal compressive stress σ_1. For this case there is almost no experimental data available and it is promising very important results because it belongs to a stress space in which the most widely known criterion (Tsai-Wu) predicts strengths up to three times higher than any other theory which took part in the WWFE.



"**Fig. 1.** Strengths of FRP measured in the (σ_2, τ_{21})-stress space."

Failure criteria are used to predict the strength of a material for any arbitrary state of multiaxial stress with the preliminary knowledge of only some easy to measure strengths. Thus, to benchmark different criteria multiaxial experimental results should be used. In Fig. 1

test results obtained in a large German research project [5] in the (σ_2, τ_{21})-stress space and predictions using Puck's action plane criteria are shown for carbon (squares) and glass fibre reinforced plastics (circles).

Apart from Tsai and Puck most of the other authors who participated in the WWFE use a so called maximum stress failure condition for the unidirectional layer. That means $\sigma_2^{(+)} = R_\perp^{(+)}$, $\sigma_2^{(-)} = -R_\perp^{(-)}$, $\tau_{21} = R_{\perp\|}$. Hence their failure envelope for (σ_2, τ_{21})-combinations is given by the dashed lines in Fig.1. This is in clear contradiction to the well known interaction of σ_2 and τ_{21} as it can be seen from Fig. 1. Further Fig.1 shows that the true failure envelope is not symmetric to any line parallel to the τ_{21}-axis.

The Tsai-Wu Criterion describes an interaction of τ_{21} and σ_2 but uses a symmetric curve which is an ellipse that is shifted on the σ_2 axis due to the different uniaxial transverse strengths $R_\perp^{(+)}$ and $R_\perp^{(-)}$ which must lie on the failure curve. The only failure envelope in the WWFE that gives an unsymmetrical curve for the prediction for the strengths in the (σ_2, τ_{21})-stress area is Puck's Criterion as can be seen in Fig. 1.

The Tsai-Wu Criterion is an interpolation-polynomial without any physical background. It allows no clear differentiation of failure types in a lamina but it is easy to use. Sometimes it delivers good results, but for the load cases selected for the experimental work it seems to be extremely unconservative! The problem here is that no user or design engineer really knows where it is unconservative and where not. The most important advantages and details necessary to correctly use the Puck Action Plane Criteria are outlined in chapter 4.

One part of the presented study was dedicated to the implementation of Puck's physically based Action Plane Criterion in order to be able to use Puck's very realistic criteria with the design software ESACOMP [6] and to make it available for composite designers.

3. THE TSAI-WU CRITERION

The quadratic equation that defines the fracture envelope of the Tsai-Wu Criterion under plane stress conditions is given in Eq. (1) [7].

$$F_{11} \cdot \sigma_1^2 + F_{22} \cdot \sigma_2^2 + F_{66} \cdot \tau_{21}^2 + 2 \cdot F_{12} \cdot \sigma_1 \cdot \sigma_2 + F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 = 1 \qquad (1)$$

σ_1 and σ_2 are the stresses of the UD-lamina parallel and transverse to the fibre direction and τ_{21} is the in-plane shear stress. The coefficients F_{ij} and F_i are defined by points on the σ_1-, σ_2- and τ_{21}-axes through which the failure envelope must pass.

$$F_{11} = \frac{1}{R_\|^{(+)} \cdot R_\|^{(-)}};\ F_{22} = \frac{1}{R_\perp^{(+)} \cdot R_\perp^{(-)}};\ F_1 = \frac{1}{R_\|^{(+)}} - \frac{1}{R_\|^{(-)}};\ F_2 = \frac{1}{R_\perp^{(+)}} - \frac{1}{R_\perp^{(-)}};\ F_{66} = \frac{1}{R_{\perp\|}^2} \qquad (2)-(6)$$

The interaction term F_{12} has a very strong effect on the (σ_1, σ_2)-fracture curve in the compressive σ_1/compressive σ_2 quadrant. In the WWFE Tsai used $F_{12}^* = -0{,}5$ in the formula

$$F_{12} = F_{12}^* \sqrt{F_{11} \cdot F_{22}}\ . \qquad (7)$$

A significant risk in using this criterion is the prediction of extremely high strengths for biaxial inplane compression as it exists in cases of cylindrical vessels with external pressure as for instance in deep submerge applications.

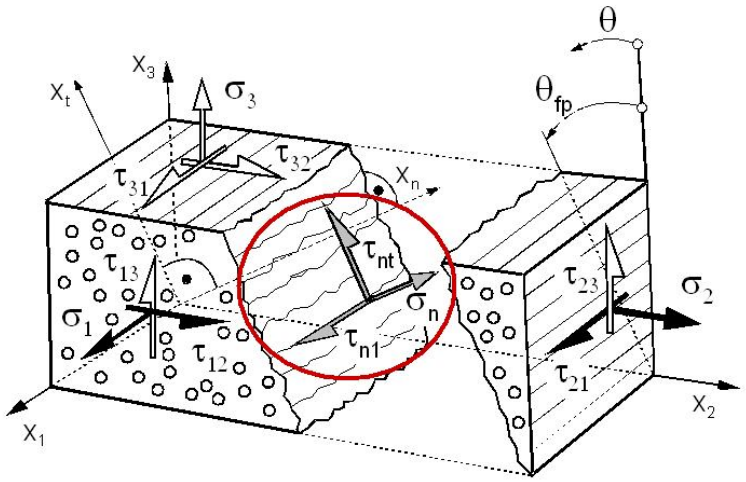
4. PUCK´S ACTION PLANE CRITERIA

Puck's failure theory is based on a physical background. The model can be used to determine the exertion f_E (in earlier papers called 'stress exposure factor') of a lamina, separately for fibre failure (FF) and inter fibre failure (IFF) and thus the corresponding reserve factor or the margin of safety. Moreover, it supplies a lot of information on the mode of failure, which is a very important prerequisite for a realistic post failure analysis of a laminate as e.g. the degradation of the transverse modulus $E_{\perp}$ and the inplane shear modulus $G_{\perp\|}$ in a layer after IFF of Mode A, that allows calculating the stresses and the deformations under post failure conditions [2, 8]. This is described briefly in [9] and in full detail by Knops [10].

Puck's Action Plane Criteria for IFF

In his well known 1980-paper [11] Hashin developed separate invariant-based interpolation criteria for fibre failure FF and inter fibre failure IFF (the so called "Hashin-Criteria"). In that paper he also mentioned, that more realistic IFF-results could be expected, if the IFF-criteria would be based on O. Mohr's [12] fracture hypothesis for brittle fracture. This states that only the normal and shear stress on the fracture plane are responsible for fracture. The problem is, that the fracture plane, on which IFF occurs under a certain stress combination (σ_n, τ_{nt}, τ_{n1} – see Fig. 2) is not known in advance. It must be detected by solving an extremum problem. Hashin in 1980 was afraid that the amount of numerical calculations necessary for detecting the fracture plane which is the plane with maximum IFF-exertion could not be tolerated in engineering practice.

At the beginning of the 1990ies the computing capacity available to design engineers had increased enormously. Therefore, in 1992 Puck started his development work on IFF-criteria [13] based on O. Mohr's hypothesis and a modification by B. Paul (1961) [14]. Hence, Puck's Criteria for IFF are based on the assumption that FRP actually do behave like intrinsically brittle material. The necessary verification for the brittle failure was given by Huybrechts [15] and Kopp [16, 17], who performed tests on prismatic UD specimens under uniaxial transverse compression to determine whether the fracture would occur under an inclined fracture plane, as predicted by Mohr's hypothesis or not. In the experiments the fracture angle θ_{fp} (see Fig. 2) was found to be $|\theta_{fp}|$=53°±3° (just the same as for grey cast iron). Based on these results one can state that the basis of Puck's model has been completely verified.



"**Fig. 2.** Stresses acting on planes perpendicular to the natural axes x_1, x_2, x_3 of the UD-element and on an inclined fibre parallel plane"

As can be found in literature, for instance [8, 9, 18, 19], Puck's formulation allows to predict IFF even for most general 3D states of stress. In this paper the 2D state of stress is focussed which is mostly used when dimensioning FRP components. For (σ_1, σ_2, τ_{21})-combinations

Puck has found an analytical solution for the extremum problem of detecting the inclination angle θ_{fp} of the IFF-fracture plane:

$$\cos\theta_{fp} = \sqrt{\frac{1}{2(1+p_{\perp\perp}^{(-)})}\cdot\left[\left(\frac{R_{\perp\perp}^{A}}{R_{\perp\|}}\right)^2\cdot\left(\frac{\tau_{21}}{\sigma_2}\right)^2+1\right]} \tag{8}$$

$$\text{with}\quad R_{\perp\perp}^{A} = \frac{R_{\perp\|}}{2p_{\perp\|}^{(-)}}\left[\sqrt{1+2p_{\perp\|}^{(-)}\cdot\frac{R_{\perp}^{(-)}}{R_{\perp\|}}}-1\right]\quad\text{and}\quad p_{\perp\perp}^{(-)} = p_{\perp\|}^{(-)}\frac{R_{\perp\perp}^{A}}{R_{\perp\|}} \tag{9 - 10}$$

The stress parallel to the fibres σ_1 has no influence on θ_{fp} [8, 9, 18-20]. Using a modified equation for θ_{fp} Puck could formulate the IFF-failure conditions for (σ_2, τ_{21})-combinations as polynomials in σ_2 and τ_{21} [8, 9, 19]:

Mode A ($\sigma_2 \geq 0$):

$$f_E\big|_{\theta_{fp}=0^\circ} = \frac{1}{R_{\perp\|}}\left[\sqrt{\left(\frac{R_{\perp\|}}{R_{\perp}^{(+)}}-p_{\perp\|}^{(+)}\right)^2\sigma_2^2+\tau_{21}^2}+p_{\perp\|}^{(+)}\sigma_2\right]=1 \tag{11}$$

Mode B ($\sigma_2<0$); $\left|\frac{\sigma_2}{\tau_{21}}\right| \leq \frac{R_{\perp\perp}^{A}}{|\tau_{21c}|}$

$$f_E\big|_{\theta_{fp}=0^\circ} = \frac{1}{R_{\perp\|}}\left[\sqrt{\tau_{21}^2+(p_{\perp\|}^{(-)}\sigma_2)^2}+p_{\perp\|}^{(-)}\sigma_2\right]=1 \tag{12}$$

Mode C ($\sigma_2<0$) ; $\left|\frac{\tau_{21}}{\sigma_2}\right| \leq \frac{|\tau_{21c}|}{R_{\perp\perp}^{A}}$

$$f_E\big|_{\theta_{fp}\ from\ Eq.8} = \frac{\tau_{21}^2}{4\cdot(R_{\perp\|}+p_{\perp\|}^{(-)}R_{\perp\perp}^{A})^2}\cdot\frac{R_{\perp}^{(-)}}{(-\sigma_2)}+\frac{(-\sigma_2)}{R_{\perp}^{(-)}}=1 \tag{13}$$

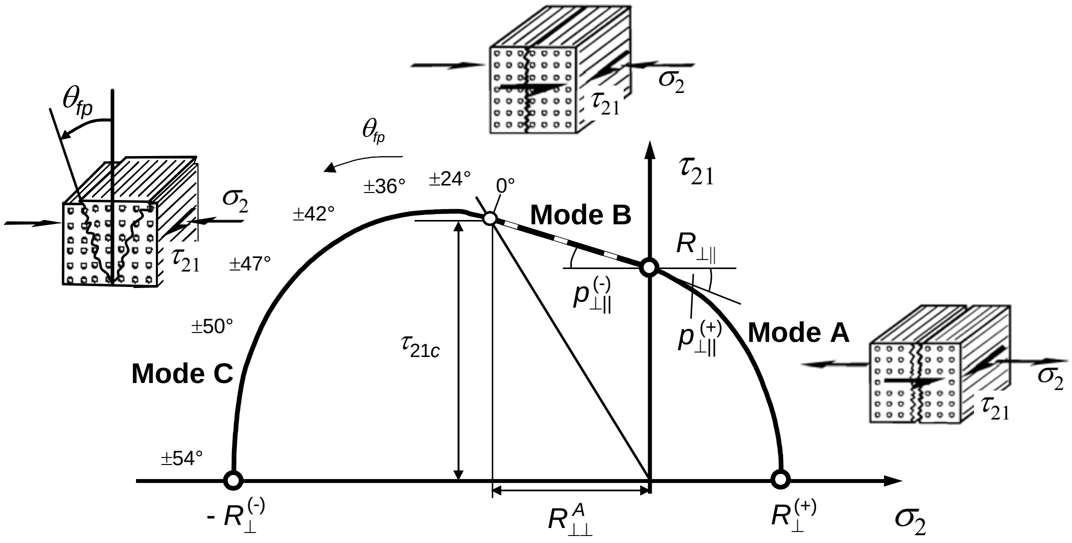
$$\text{with}\quad \tau_{21c} = R_{\perp\|}\sqrt{1+2p_{\perp\perp}^{(-)}}\quad\text{(see Fig. 3)} \tag{14}$$

If the numerical value of the IFF-exertion f_E is set to '1' as it has been done in Eqns. (11) to (13) one gets the fracture conditions from which the fracture curve in Fig. 3 is calculated. If for a certain state of stress the numerical value of the polynomial given in Eqns. (11) to (13) is not equal to '1', this numerical value is the IFF-exertion f_E for the given state of stress.

With Eqns. (11) to (13) Puck accounts for three different modes of IFF: A, B and C (see Fig. 3). Mode A signifies failure in the positive σ_2 range, and Mode B a longitudinal shear failure in the presence of high τ_{21} and moderate σ_2 compressive stresses. For both Mode A and Mode B the fracture plane is the common action plane of σ_2 and τ_{21} (that means $\theta_{fp} = 0°$, see Fig. 2). Mode C prevails, if – under sufficiently high compressive σ_2 stress and τ_{21} combinations – failure occurs on an inclined fracture plane. This fracture is brought about by the transverse shear stress τ_{nt} in cooperation with the longitudinal shear stress τ_{n1} (see Fig. 2).

While IFF developing under Mode A and Mode B may be tolerable, Mode C failures can not be tolerated because of the wedge-effect of the inclined fracture planes, that can initiate delamination in adjacent layers, furthers the risk of fibre local-buckling of adjacent layers causing fibre fracture and can lead up to a catastrophic failure of the whole laminate by massive delamination [8, 18, 19].

The three different sections of the fracture curve in Fig. 3 resulting from Eqns. (11-13) are excellently in line with the test results in Fig. 1. In addition to the basic strengths $R_{\perp}^{(+)}$, $R_{\perp}^{(-)}$ and $R_{\perp\|}$ Puck uses four inclination parameters $p_{\perp\|}^{(+)}$, $p_{\perp\|}^{(-)}$, $p_{\perp\perp}^{(-)}$, $p_{\perp\perp}^{(+)}$ (the latter one just needed for 3D-states of stress). It could be shown in the past [21] that these coefficients can be taken as constants for one type of fibre-matrix combination and vary only very little for different materials. Table 1 lists recommend values. Thus, for the complete Puck theory the 5 basic material strengths ($R_{\parallel}^{(+)}$, $R_{\parallel}^{(-)}$, $R_{\perp}^{(+)}$, $R_{\perp}^{(-)}$, $R_{\perp\parallel}$) as for the Tsai-Wu Criterion and most other failure criteria are sufficient.



"Fig. 3. Predicted fracture curve from Puck's IFF criterion, set up with reasonable material constants"

"**Table 1.** Recommended values for the inclination parameters [21]."

	$p_{\perp\parallel}^{(-)}$	$p_{\perp\parallel}^{(+)}$	$p_{\perp\perp}^{(-)} = p_{\perp\perp}^{(+)}$
GFRP	0,25	0,30	0,20 – 0,25
CFRP	0,30	0,35	0,25 – 0,30

Important: Using Eqns. (11) to (14) normally $p_{\perp\parallel}^{(-)}$ is chosen from Table 1. By doing this $p_{\perp\perp}^{(-)}$ is always fixed by the parameter coupling given in Eq. (10) which is mandatory in this case.

Puck's Criteria for FF

For the well understood fibre failure Puck recommends for (σ_1, σ_2, τ_{21})-combinations to use the maximum stress formulation Eq. (15) he proposed already in 1969 [2, 22]. It expresses the physical idea that fibre failure under multiaxial stresses in a UD-lamina occurs when the fibre parallel stress σ_1 is equal to or exceeds the stress necessary for a failure under uniaxial stress σ_1. From this hypothesis follows the simple FF-condition

$$f_{E(FF)} = \frac{|\sigma_1|}{R_{\parallel}^{(\pm)}} = 1 \qquad \text{with} \begin{cases} R_{\parallel}^{(+)} & for\ \sigma_1 > 0 \\ R_{\parallel}^{(-)} & for\ \sigma_1 < 0 \end{cases} \tag{15}$$

Puck [8, 18, 19] and Fischer [23] worked on a further improvement of the fibre failure criterion. This work is based on a more sophisticated failure hypothesis: FF of a UD-lamina under combined stresses will occur when in the fibres the same stress σ_{1f} is reached which is acting in the fibres at an FF of the lamina caused by uniaxial tensile stress σ_1 or a uniaxial compressive stress σ_1 respectively. Based on this failure hypothesis of maximum normal stress of the fibres the FF-condition for the UD-lamina is:

$$f_{E(FF)} = \left| \frac{1}{R_{\parallel}^{(\pm)}} \left[\sigma_1 - \left(\nu_{\perp\parallel} - \nu_{\perp\parallel f}\, m_{\sigma f} \frac{E_{\parallel}}{E_{\parallel f}} \right) (\sigma_2 + \sigma_3) \right] \right| = 1 \qquad \text{with} \begin{cases} R_{\parallel}^{(+)} & for\,[...] \geq 0 \\ R_{\parallel}^{(-)} & for\,[...] < 0 \end{cases} \tag{16}$$

$f_{E(FF)}$ is the FF-exertion, $\nu_{\perp\parallel}$ is the major Poisson's ratios of the UD-lamina, and $\nu_{\perp\parallel f}$ the major Poisson's ratio of the fibre. $E_{\parallel}$ is the longitudinal modulus of the lamina parallel to the fibres and $E_{\parallel f}$ the longitudinal modulus of the fibres. $m_{\sigma f}$ is a magnification factor for the transverse stress in the fibres (GFRP $m_{\sigma f} \approx 1{,}3$ and for CFRP $m_{\sigma f} \approx 1{,}1$). Puck has found that in case of plane (σ_1, σ_2, τ_{21})-stress, the results of Eq. (15) and Eq. (16) differ only by a few percent. (However, the influence of transverse stresses on FF can become important in the region of combined $\sigma_2<0$ and $\sigma_3<0$ of similar magnitude, where $|\sigma_2|$ and $|\sigma_3|$ can exceed the transverse compressive strength $R_{\perp}^{(-)}$ by a factor of up to 3 or 4 [17]).

Influence of Stresses σ_1 parallel to the fibres on IFF-Strength

Following strictly O. Mohr's hypothesis, stating that only the stresses acting on a common fracture plane are producing the fracture, in case of an IFF only the stresses σ_n, τ_{nt}, τ_{n1} (see Fig. 2) would be responsible for IFF and not the fibre parallel stress, which acts on a plane transverse to the fibres. But as already mentioned in his early work of 1969 [2, 22] Puck expects an influence of high stresses σ_1 on IFF-strength. There is no interaction of σ_1 and the stresses σ_n, τ_{nt}, τ_{n1} in the usual sense of an "cooperation" in bringing about the failure like the interaction shown in Eq. (16). The stress σ_1 does certainly not cooperate directly with σ_n, τ_{nt}, τ_{n1} in producing the brittle fracture because σ_1 is not acting on the fracture plane. Instead σ_1 reduces the IFF-strengths $R_{\parallel}^{(+)}$, $R_{\perp\parallel}$, $R_{\perp}^{(-)}$ ($R_{\perp}^{(-)}$ is proportional to the action plane fracture resistance $R_{\perp\perp}^{A}$) by micro-damage caused by failures of single fibres or small filaments, which occur progressively, when σ_1 comes close to $R_{\parallel}^{(+)}$ or $R_{\parallel}^{(-)}$ respectively.

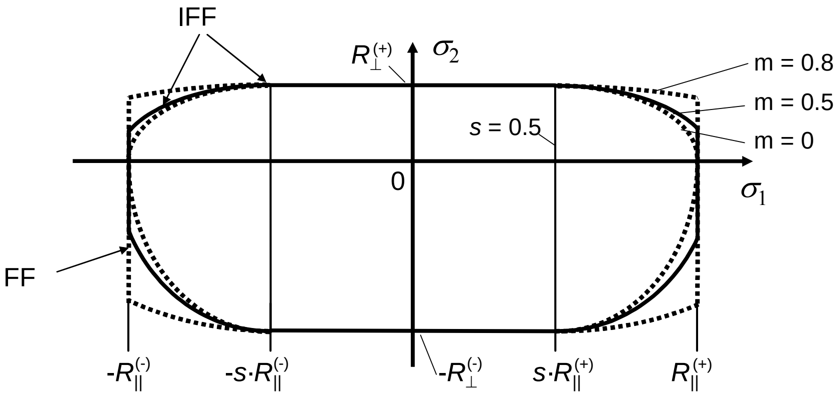
Based on this consideration Puck formulated a degradation factor η_w (w = weakening). All three action plane fracture resistances $R_{\perp}^{(+)}$, $R_{\perp\perp}^{A}$, $R_{\perp\parallel}$ governing IFF are multiplied with the same weakening factor $\eta_w<1$. Decreased IFF-strength means increased IFF-exertion. Therefore, the IFF-exertion f_{E1} respecting a weakening due to σ_1 follows from f_E according to Eqns. (11) to (13) by setting

$$f_{E1} = \frac{f_E}{\eta_w} = f_E \cdot \left[\frac{c \cdot \left(a\sqrt{c^2(a^2 - s^2) + 1} + s \right)}{(ca)^2 + 1} \right]^{-1} \tag{17}$$

with $c = (f_E/f_{E(FF)})$, valid for $1/s \geq c \geq m$. The parameter s marks the starting point of the weakening effect and a is the axis of an assumed elliptical fracture curve given by

$$a = \frac{1 - s}{\sqrt{1 - m^2}} \tag{18}$$

where m is the assumed minimum value of η_w. The effects of s and m can be seen in Fig. 4. Eq. (17) is valid for $1/s \geq c \geq m$. $c > 1/s$ means no weakening and $c < m$ means no IFF before FF. For the time being no test results are available from which figures for s and m could be taken. It is recommended to use preliminary medium figures $s = 0.5$ and $m = 0.5$ [9].



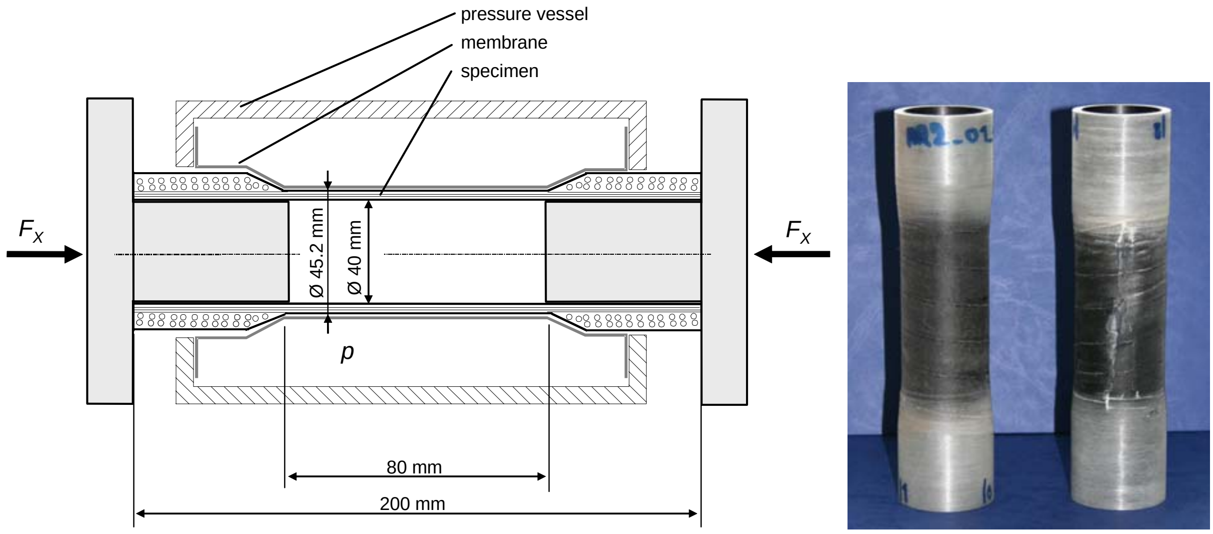
"**Fig. 4.** Possible shapes of the IFF curve depending on the parameters *s* and *m*"

The treatment of the weakening effect in IFF-analysis is described briefly in [9] and in more detail in [20]. The parameters *s* and *m* can be chosen between 0 and 1. This presents an excellent adoptability of the fracture envelope to test results. But, the severe problem at the moment is, that no reliable test results are available for (σ_1, σ_2)-compression as such tests are extremely difficult.

5. TEST-SETUP FOR BIAXIAL INPLANE COMPRESSION

Tubular specimens loaded by axial compression F_X and an external pressure *p* can be used to obtain the desired state of biaxial inplane (σ_1, σ_2)-compression in an inner test layer (see Fig. 5 and 6). The design of adequate tubular specimen is a complex task. An optimization process resulted in a lay-up [89°;(+11°;-11°)$_4$;89°]. The 'circumferentially' wound 89°-layers are made of two plies and have a nominal thickness of 0.3 mm and are made from T300 Carbon fibre (200 tex). The supporting 11°-layers are manufactured as single plies from T700 Carbon fibre (300 tex), their total nominal thickness is 2 mm. The used epoxy resin is Araldite LY556/HY917/D070. Fibre volume content is ≈ 60%.

The 11° supporting layers were used to reach the necessary safety against buckling which is extremely critical for shells under combined external pressure and axial compression which. These layers – in combination with the inside and outside circumferential layers – produce a sort of sandwich effect that results in a rather higher circumferential bending stiffness. Angles larger than 11° would result in the need of too high external pressure, smaller angles would lead to an undesirably high necessary axial compression force to obtain the desired σ_2 stresses in the test layer.



"**Fig. 5.** Test setup principle for biaxial compression loading and photo of test specimen."

"**Fig. 6.** Test setup for biaxial compression loading."

In spite the fact that the "circumferential" 89° test layer was placed on the inside of the tube, due to the external pressure p a certain radial compressive stress σ_3 is acting on the outside surface of the test layer. $|\sigma_3|$ does however not exceed |10| N/mm^2 which appears acceptable.

6. RESULTS & DISCUSSION

So far the specimens were tested under four load cases (TS1-TS4) leading to different (σ_1,σ_2)-compression stresses combinations. The stresses in the test layer were determined for each of the load cases by an FE-analysis. For these FE-analyses a model of the specimen meshed with volume elements (Solid 46 in ANSYS) was used and the averaged external loads at failure of the specimens tested for one load case was applied. The stresses resulting for the different load cases are given together with average load at failure and standard deviation in Table 2.

Inspection of the failed specimen showed that failure is most likely not caused by the desired pure fibre fracture in the test layer. Instead, it is suspected that two different undesired kinds of failure occur. Some evidence of buckling which leads to the destruction of the shell could be observed in strain measurements on a circumference which show characteristically diverging strains in some tests. Characteristics of IFF in Mode C could be seen on some other specimens. This is in contrast to the results with the Puck action plane criterion, which did not predict an IFF-exertion f_E in any layer higher than 50% of the exertion for FF in the test layer. Failure in the outer 89° layer should not occur according to different diameters of the inner 89° layer (40.3 mm) and the outer layer (44.9 mm). The exertion for FF due to external pressure is 11% higher in the inner layer (44.9/40.3=1.11) than in the outer layer. Microscopic investigations showed that the 11° layers have very high void content which could explain inaccurate predictions as the void content strongly degrades critical strength for Mode C IFF.

"**Table 2.** Average external loads at failure and calculated stresses σ_1, σ_2 in the inner test layer."

	Load at failure		Stresses at failure in the inner 89° layer		valid specimens
	F_X [N]	p [MPa]	σ_1 [MPa]	σ_2 [MPa]	
TS1	0	18,5	-811	-16,1	7
STAND [%]	-	13,3			
TS2	79450	31,1	-880	-37,5	5
STAND [%]	15,5	16,9			
TS3	128990	35,6	-989	-51,9	6
STAND [%]	5,9	5,8			
TS4	147400	30,5	-823	-53,3	5
STAND [%]	6,4	7,1			

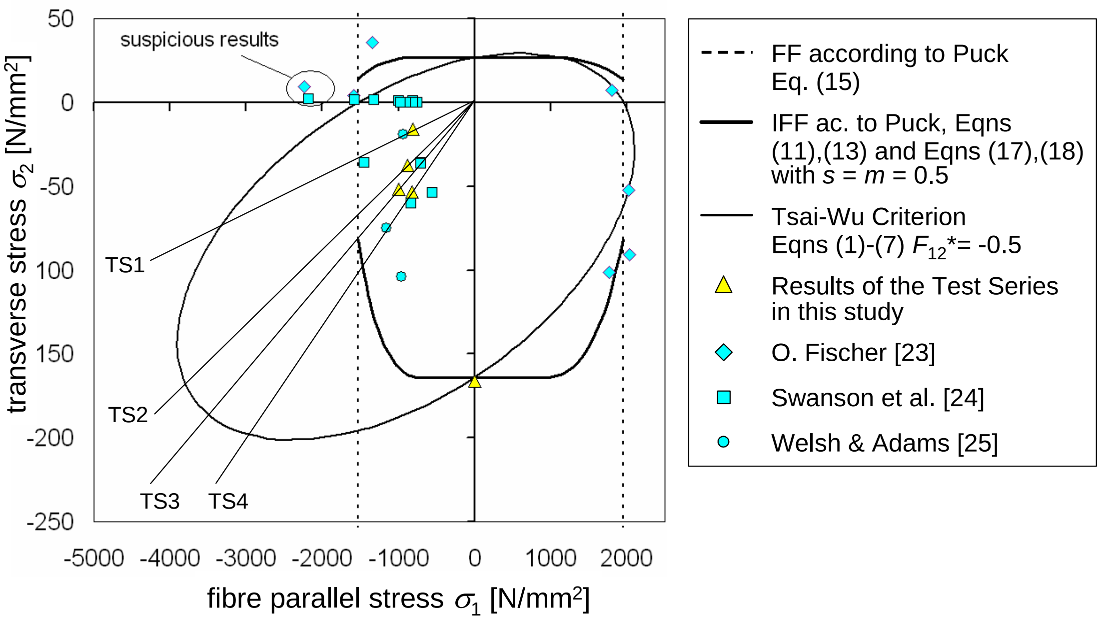
To solve the current problem of specimens failing much earlier than expected several approaches of improvement are under investigation: The lay-up shall be modified in order to obtain a test tube which withstands the external pressure even after the failure of the inner test layer. This would allow proving definitely fibre fracture as the cause of failure in the test layer. The ±11° supporting layers shall be substituted by pure resin. The strain to a Mode C failure in the pure resin is expected to be much higher than that of the unidirectional layer. Voids appeared in the ±11° layers can be avoided by a high quality resin casting process.

7. TEST RESULTS FROM LITERATURE FOR BIAXIAL COMPRESSION

In the literature the highest reported strengths under biaxial (σ_1, σ_2)-compression are from Swanson et al. [24, Fig. 10]. They were measured on tubular specimens with a quasi isotropic 0°/90°/±45°-lay-up. Unfortunately, each of these test points belongs to a single tested specimen and Swanson also suspects a circumferential buckling to cause the failure.

Statistically more secured results are reported from Welsh and Adams [25]. They performed tests on cruciform specimens made from plates with a $[0°/90°]_{10}$ lay-up. It is not sure whether the stress distribution in their test region is homogenous enough as it should be for tests used to actually benchmark failure criteria. To the knowledge of the authors the results which Welsh and Adams report allow to draw the most complete “fracture curve” measured so far. But because of the 0°/90°-fibre orientation of the test specimens it is very questionable if the failure curves are applicable to unidirectional layers. Following the exertion analysis of these results it is estimated that buckling was involved in the tests as well.

Swanson et al. and Welsh and Adams report their fracture stresses in global stresses (σ_x, σ_y) on the tested laminate and thus their results can not directly be compared to the predictions of existing failure criteria. For this reason the results are transformed into stresses for the UD-lamina using information given on the specimen’s geometry and the lamina properties for the fibre matrix combination tested (AS4/3501-6) as given in Part A of the WWFE [26].



“**Fig. 7.** Stresses at failure in one layer, calculated from external loads or stresses.”

From the fracture stresses (σ_1, σ_2) in the layers with different fibre direction the highest σ_1 stresses are taken as strength values and plotted in Fig. 7. In this figure the results of Swanson et al (squares), Welsh and Adams (circles), Fischer (rhombi) [23] and the test results of this study (triangles, calculated by FEA) are compared with the predicted failure envelopes of the Tsai-Wu Criterion and Puck´s Theory.

As one can see from Fig. 7, in none of the quoted works the measured strengths under combined biaxial (σ_1, σ_2)-compression stresses gets close to the uniaxial compressive strength $R_{\parallel}^{(-)}$ for high transverse compression as given by [25]. Compared to these stresses, the stresses achieved in the presented test campaign represent already a remarkable improvement. The stresses come from at least 5 valid tests per load ratio. The ability to further develop the present specimen in order to avoid buckling and other undesired failures before reaching the fibre compressive strength at presence of a high transverse compression stress σ_2 is absolutely given.

Of course, none of the results in Fig. 7 can be used to adapt failure criteria or evaluate the predictions made by them. Another question arises from these results: as up to now it has not been possible to reach the 'real' fibre strength under combined compressive loading, not even with the most sophisticated test set-ups, it is difficult to imagine in which real structure stresses much higher than those achieved in the tests could be reached without a prior failure of the structure – because each specimen is nothing else than an simple FRP structure thoroughly designed to withstand very high (σ_1, σ_2)-compressive stresses. However, of course this consideration does not relieve us from the task of finding the true strengths under combined (σ_1, σ_2)-compressive stresses, but it still shows that all criteria would lead to un-conservative predictions as even the uniaxial compression strength $R_{\parallel}^{(-)}$ predicted as the strength by all maximum stress criteria has not been reached so far in any tests.

8. CONCLUSION

As the results from the literature and the presented new experimental results have shown that the determination of the strength for biaxial (σ_1, σ_2)-compression remains a very difficult task. However, it can be stated that the large ellipse in the Tsai-Wu failure envelope predicting strengths 4 times higher than any test result (Fig. 7) is unrealistic.

In spite of the encountered difficulties, one shall continue to elaborate the test set-up and do other verification tests because only in this way a major problem in the dimensioning of FRP can be solved. There is not a lack of theories but a lack of reliable test results to benchmark the available theories. Thus, it is difficult for the design engineer to know which criteria to trust on the basis of the summary [27] of the WWFE. Also Part C of the WWFE recently published and taken into account the contribution of the so-called post-runner of the exercise [28, 29] has not helped to clarify the situation.

Therefore, further testing is necessary as it is in no ones interest to follow one of the advices [30] given in connection with the WWFE which was simply to use several criteria simultaneously and then trust the highest calculated exertion. This would mean that the calculating effort reaches a maximum, and the design will most of the time be very conservative. A conservative design, without knowing how conservative it is, will always represent a risk for the designing engineers and maybe even more important, the weight and costs of the designed parts will in most cases be too high, because the light weight potential is not fully exploited.

ACKNOWLEDGEMENTS

The presented work was funded by the European Space Agency (ESA) under the contract no. 16162/02/NL/CP.

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