WOVEN PLY LAMINATE STRUCTURES WITH AN OPEN HOLE : SIMULATIONS AND EXPERIMENTS

Bordreuil Cyril and Christian Hochard

LMA/CNRS, 31 chemin Joseph Aiguier, 13402 Marseille, France

Abstract

In this paper, a model for static loadings of woven ply laminated structure is presented. The analysis is done by introducing tensile shearing couplings. Two main drawbacks are that fibre breaking as to be taken account with a non local criteria, and that damage is coupled with tensile loading in fibre direction. An efficient numerical scheme is presented based on the consistent tangent and some correlations with experiments are done. The results correlate the simulations.

1. Introduction

The woven ply with carbon fibres are present in aeronautical structures (helicopter blade skin for example) due an easier lay-up for complex surfaces. The purpose of this work is to predict the rupture of such material in structures.

Two types of loadings have to be taken into account in order to predict the behaviour of woven ply. The first one is due to fatigue and the other one is due to static loadings. A unified model was proposed for this two types of loadings (Payan, 2004). In this work, only static loadings will be studied. This material shows simple mechanisms of degradation (no transverse rupture and great resistance in respect to delamination). This allows to develop a model at the meso level, i.e the ply, and to make simulations to predict the rupture of structures with woven ply laminates. Two types of phenomena induce the rupture of the ply in traction. One is due to the brittle rupture in the fiber directions and the other one is due to the progressive loss of rigidity in the shear direction (Hochard, 2001).

The macroscopic model to describe the brittle behaviour is a criterion. The criterion developped is non-local (Lahellec, 2004) due to the high heterogeinity of the material at a microscopic level. For some structures, plates with elliptic open holes, this non-local criterion works well. For the loss of rigidity in shear, a continuum based damage model (Ladevèze, 1992) is adapted to the woven ply (Hochard, 2001). It should be noted that tension/shear coupling is introduced in the evolution of damage shear variable with the help of the conjugate thermodynamic forces.

The numerical implementation of these models was developed in details in previous work (Bordreuil, 2004). Some details of the implementation are recalled and the low time computation due to the implementation of the consistent tangent is shown.

The paper will finish with the comparison of prediction from simulations to experimental results on a plate with a hole for different stacking sequencies, subjected to tensile tests as done before by Chang and Chang (1987) for UD ply. A precedent study was done with stacking orientations ranging from $[\pm 22.5]_s$ to $[\pm 40]_s$ degrees (Bordreuil, 2004). Here, the sequencies are chosen for a wider range of orientations: from $[\pm 0]_s$ to $[\pm 45]_s$. The low time computation of the procedure together with good prediction of the rupture allows optimization of structures containing such materials.

2.Behavior of the elementary ply

The model developed is based on the work of Ladevèze for unidirectional ply. The model introduced different damage variables (d_1 ,d_2 ,d_{12}) at the meso scale. In this direction, Hochard adapted this model to the woven ply with balanced and non-balanced warp and fill yarns. The three internal damage variables model the brittle behaviour in fibre direction and the shear damage variable model the progressive degradation of shear modulus due to small microcracks. Under the assumption of plane stresses, and small perturbations, the strain energy is written:

$$E_D^{ps} = \frac{1}{2}\left[\frac{\langle\sigma_1\rangle_+^2}{E_1^0(1-d_1)} + \frac{\langle\sigma_1\rangle_-^2}{E_1^0} - 2\frac{\nu_{12}^0}{E_1^0}\sigma_1\sigma_2 + \frac{\langle\sigma_2\rangle_+^2}{E_2^0(1-d_2)} + \frac{\langle\sigma_2\rangle_-^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0(1-d_{12})}\right] \qquad (1)$$

where < . >$_+$ is the positive part and < . >$_-$ is the negative part. The tension energy and compression energy are split in order to describe the unilateral feature due to the opening and closing of the micro-defects. From this potential, thermodynamic forces associated with the tension and shear internal variables d_i (i=1 and 2) and d_{12} are defined:

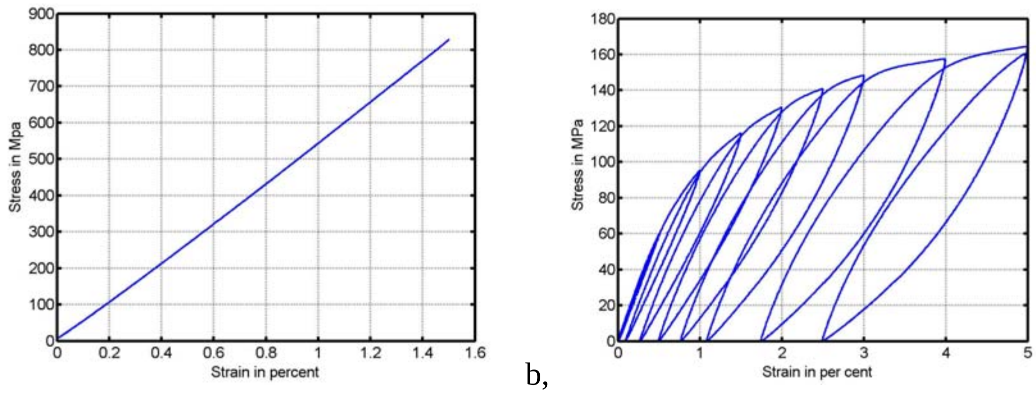
$$Y_{d_i} = \frac{\partial E_D^{ps}}{\partial d_i} = \frac{\langle\sigma_i\rangle_+^2}{2E_i^0(1-d_i)^2}; \qquad Y_{d_{12}} = \frac{\partial E_D^{ps}}{\partial d_{12}} = \frac{\sigma_{12}^2}{2G_{12}^0(1-d_{12})^2} \qquad (2)$$

The development of the internal variables depends on these thermodynamic forces and more precisely on their maximum values during the history of the loading. In traction, the development of d_1 and d_2 is brutal in order to represent the brittle behaviours according to the warp and fill directions

The model is able to take into account some tensile/shear couplings for damage. In this work, experimental results on homogeneous tests are done and then couplings are identified for damage. In the word plasticity, it is the inelastic phenomena such as matrix flow and fiber/matrix sliding that are assumed. To model this phenomena, a classical non-linear hardening is chosen. In case of increasing load, it is not important but for fatigue, kinematic hardening better modelled the behavior in tensile/compressive loading (Bordreuil, 2003). The woven ply is orthotropic but with two different directions with the same behavior. Characterisations are then performed on a [0] and [± 45] woven laminate for fibre and for shear directions. The shear behavior is highly inelastic. These tests are completed by [±30], woven laminate to evaluate the coupling between tensile and shear behavior.

2.1 Fiber directions

The behavior in the fiber direction is shown in **figure** 1(a) :



a, b,

Figure 1 : Behaviour in the direction of the fibre (a) and in shear direction (b) for tensile loading.

It is shown that the behavior is elastic and then brittle rupture occurs. When rupture in one point of a ply occurs then the rupture immediatly propagate in the whole laminate. This brittle behavior is often modelled by a local criteria in stress:

$$\sigma_i = \sigma_{rupt} \quad (3)$$

For homogeneous tests, the stress at rupture is simply related to the laminate stress. The damage variables d_1 and d_2 evolve in the brittle manner. Elasticity is then simply modelled by :

$$\sigma_i = c_{ij}\varepsilon_j \quad (4)$$

When using such a criteria to predict the rupture of a laminate (Bordreuil 2004) it is shown that the rupture of structure are underestimated. When using a local criteria in the fibre direction, we have a "mesh dependence of the solution". It is in the sense that the elastic solution is only discretely catch. In fact, if we have a coarse mesh, we will have a gross estimation of the elastic solution. To circumvent this fact, an internal length must be introduced in the same manner as Withney and Nuismer (1976). The model is introduced and identified in Lahellec (2004) and is more general from the point of view that the criteria is applicable at each quadrature point of a ply of the structure. A non-local criteria was used to predict the rupture. The criteria has the form of the equation (5):

$$\frac{1}{S}\int \sigma_{ii}\, dS = \sigma_{rupt} \quad (5)$$

This criteria is a mean stress criteria reviewed by Isupov and Mikhailov (1998) but that is slightly modified for implementation. This criteria is applicated at every quadrature points. The results for quasi isotropic laminate for woven ply are excellent for different structures and can catch some size effects. In the present work, the criteria, first developped for the laminate will be adapted and implemented at the meso scale,ie the ply.

2.2 Shear direction

The behavior in the shear direction is shown in figure 1 (b). The behavior is complex. There is a competition between elasticity, damage and plasticity during straining. A simple model is developed, for computation purpose, but on physical basis.

2.2.1 **Elasticity and damage**

When a homogenous test is done on [±45], some loadings-unloadings are performed. This allows to see some lost of rigidity of the structure when not considering the hysteresis, mainly due to viscosity and friction between matrix and fibre. First, the elastic-damage behaviour is modelled based on the general strain energy presented at the top of this section and the shear stress is expressed :

$$\sigma_{12} = G_{12}^{0}(1 - d_{12})\gamma_{12}^{e} \quad (6)$$

where σ_{12} is the cauchy stress and γ_{12}^{e} is the engineering strain $(= 2\varepsilon_{12}^{e})$. From the strain energy, associate force to the variable damage in shear d_{12} are defined. The evolution law will be present in the next section due to strong couplings that exist between fibre direction loading and damage and plasticity evolution in shear. The damage evolution is coupled to the equivalent thermodynamic forces that take into account tensile/shear coupling :

$$Y = \alpha_1 Y_{d_1} + \alpha_2 Y_{d_2} + Y_{d_{12}} \qquad and \qquad \underline{Y}(t) = \sup_{\tau \le t}(Y(\tau)) \quad (7)$$

where α_1 and α_2 are the tensile /shear coupling parameters. It must be noticed that the equivalent thermodynamic force does not take into account compressive loading in the direction of the fiber. The coefficient is based on $[\pm 30]$ homogeneous test, figure 2. The evolution of shear damage is then governed by the simple law based on experimental measurement:

$$d_{12} = \left\langle \frac{\sqrt{\underline{Y}} - \sqrt{Y_O}}{\sqrt{Y_c} - \sqrt{Y_O}} \right\rangle_+ \quad (8)$$

Y_0 and Y_c are respectively the threshold for evolution of damage and the critical value of damage.

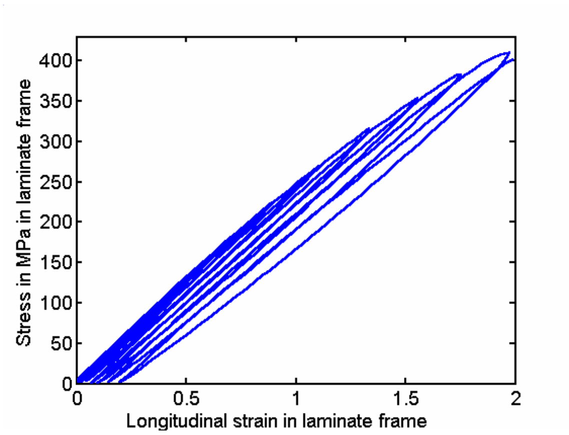


Figure 2 : Homogeneous test to identify tensile/shearing coupling on [+30,-30]_s.

2.2.2 Plasticity

Only inelastic strains are allowed in the direction of shear due to the fact that carbon fiber are much more rigid than resin. As said before, non linear hardening is chosen. The first law developed in Hochard (2001) was a yield criteria expressed in term of effective quantities such that the elastic field is defined by:

$$f = |\tilde{\sigma}_{12}| - r_0 - Kp^{\gamma} \qquad (9)$$

where $\tilde{\sigma}_{12}$ is the effective stress, and r_0 is the elastic limit and p is an internal variable to describe the state of the non linear hardening. This variable is defined as :

$$\dot{p} = (1-d)|\dot{\gamma}_{12}^{p}| \qquad (10)$$

where the superscripts p stands for the plastic part of the strain rate tensor.

2.3 Parameter identification

The identification follows the developpement of Ladevèze and Ledantec 1992, for such model with an estimation of the coupling coefficients based on homogeneous test $[\pm 30]$.

"**Table 1.** Properties of G939/963."

Property	Value
Young Modulus (MPa)	55000
Tensile strength (MPa)	850
Internal length (mm)	2.4
Hardening expoent (MPa)	600
Yield Limit (MPa)	30
Y_0	0
Y_c	4.01
Coupling parameter	.7
Plastic exponent	.39

3 Numerical implementation

In the case of an implicit resolution of the virtual work principle, the two main points are *(i)* the computation of the stresses in the local algorithm and *(ii)* the computation of the consistent tangent operator in order to maintain the second order rate of convergence.

The model introduced different non linearities which were induced by the damage and anelastic strain. The fact that woven plies can not support anelastic strains in the direction of the fibres simplifies the algorithm.

In the first section, the previous woven plies model is described in an incremental framework for the local algorithm. Following this, the way to obtain the consistent matrix is then deduced.

3.1 Incremental model

The notations are such that $_{n+1}$ represents the values of the variables at the end of the time step. The upper-script ^{e} is for elastic strain and ^{p} is plastic (or anelastic) strain. The incremental model is developed in a Euler Backward scheme. The Euler Backward is chosen for its high stability. The unknowns variables of the model are σ_{n+1}, d_{12n+1}, p_{n+1} is the value of the cauchy stress the damage shear variable and the cumulated plastic strain at the end of the increment.

All these variables need to be updated for a time step. Variables d_i are not treated because the material does not damage in the direction of the fibre until brittle fracture occurs, in a monotonic loading, although a criterion is developed for the rupture in the direction of the fibre.

The stresses are ordered in the Voigt notation. In the direction of the fibre the stresses are given by equation (4). In this study, non-linear elasticity in compression due to micro buckling or alignment of the fibre are not yet taken into account. The shear stresses are determined with the help of the elastic relationship:

$$\sigma_{12n+1} = G^0_{12}(1-d_{12n+1})\gamma^e_{12n+1} \tag{11}$$

The two main strains are obtained by the summation of total incremental strains as long as no damage or no anelastic strain are possible until rupture. The elastic shear strain is obtained by computing the difference of the total strain and the plastic strain.

$$\Delta\gamma^e_{12} = \Delta\gamma^t_{12} - \Delta\gamma^p_{12} \tag{12}$$

The main features in computing stresses are the values of the damage at the end of the increment and of the incremental plastic strain.

Damage is given by the equation:

$$d_{12n+1} = \left\langle \frac{\sqrt{\underline{Y}_{n+1}} - \sqrt{Y_O}}{\sqrt{Y_c} - \sqrt{Y_O}} \right\rangle_+ \tag{13}$$

where :

$$\underline{Y}_{n+1} = sup(\underline{Y}_{12n+1} + \alpha_1 Y_{1n+1} + \alpha_2 Y_{1n+2}, Y_n) \tag{14}$$

If the first term is greater than Y_n, then damage will evolve.

If this set of equations is expressed in a displacement based problem, the solution is function of incremental strains. When plasticity is involved, the shear strain increment is then decomposed as in equation (12) and the only unknown variable is the incremental plastic

strain. When plasticity occurs, the overall set of equations must be linearised with respect to the unknown variable (More details of the linearisation can be found in Bordreuil (2004).
The first search is to establish whether or not plasticity occurs due to a predictor step at fixed internal variables (d_{12_n}, p_n). If plasticity occurs the equation has to be solved for the unknown variable $\Delta\gamma_{12}^p$ and a Newton method is developed. The scheme is iterated until the solution is found : $f_{n+1} = 0$. The scheme is summarised in table 2.

"**Table2.** Local algorithm."

1 Elastic predictor

$\sigma^{el} = \sigma_n + C : \Delta\varepsilon$

2 $test = f(\sigma^{el}, p_n, d_{12_n})$

3 Check if plasticity occurs: test>tolor then

- a Compute $\dfrac{\partial f}{\partial \gamma_{12}^p}$
- b Calculate c_p
- c Update internal variables : p_{n+1} and d_{n+1}
- d Update Stress : $\sigma_{n+1} = \sigma_n + C : (\Delta\varepsilon - \Delta\varepsilon^p)$
- e Calculate $test = f_{n+1}$ and go to 3

4 Update internal variables

5 Compute consistent tangent matrix

6 End of the computation

3.2 Rupture criterion

Two types of rupture are observed within the developed model. The first one is due to the progressive loss of rigidity of the material in shear direction. The second one is due to brittle rupture in the direction of the fibres. From the point of view of implementation, the first one does not need any criteria but the second one has to be expressed in terms of rupture criteria. At this point, a criterion in a maximum conjugate damage variable is chosen and, because no damage is involved before rupture, it is equivalent to a stress criterion.

From the point of view of the Euler Backward, the time step has to be recomputed each time the criterion is overlapped. The criterion is a bit exotic because it is non local. This fact induced that the criterion can not be treated in the classical finite element implementation of non linear behaviour of the material at each gauss point. So an alternative treatment of the criteria proposed in Lahellec (2004) must be chosen. The algorithm is performed in post treatment after an increment. Due to the non-linearity in the shear direction, the behaviour in the fibre direction is modified. The first attempt proposed here is to simply stop the computation when the criteria is overlapped. To catch the onset of fracture, it is assumed that the behaviour is locally linear and the force at rupture for the structure is simply obtained by a linear law. In reality and as outlined before the non linear behaviour in shear influences the state of stress in fibre direction. It means that to catch the onset of rupture a simple linear law is only an approximation.

In this model, it is assumed that when one point of a ply reaches the critical stress then the whole laminate is ruptured.

It is important to note that the overall behaviour of brittle fracture and shear damage is embedded in the model. The algorithm treatment is presented in table 3.

Point 1 of the algorithm is done once in the study because this operation is time consuming. The lecture of the coordinate and of the representative volume of a gauss point in the ply in the database is done once assuming that no large change of geometry occurs.

One more time, it must be outlined that the linear law is not relevant when non linearities as damage and plasticity occurs. Nevertheless, it gives good predictions.

To finish, it has to be noted that the result is independent of the mesh when the representative volume see around 5 stress at neighbourhood of the quadrature points.

"**Table3.** Numerical treatment of the non local criteria ."

1	For first increment, Lectures of datas and recognizing neighboor quadrature gauss points
2	Mean of stress for every quadrature point.
3	Test if criteria is overlapped
4	If so computed the force on structure with a linear law
5	End of the computation

3.3 Consistent tangent

Simo et Taylor 1985 have shown that to keep good computational efficiency, namely good rate of convergence with the principle of virtual work, the incremental consistent matrix must be computed. In most cases (anisotropic behavior, large number of internal variables,....), this computation is not an easy task. In our specific case, the difficulties arise from the thermodynamic conjugate force of the damage Y_{d_i} . Details for obtaining the consistent tangent matrix is available in Bordreuil (2004). Even with this simple model, two internal variables, the linearisation is quite tough. It can be emphasised that this type of implementation can be used for carbon fibre UD ply with transverse anelastic strain.

3.4 Implementation in Abaqus

The following model was implemented in Abaqus Standard via a user subroutine umat. For orthotropic behaviour, Abaqus rotates at the time step of concern, all the variables, namely stresses and strains, in the direction of orthotropy. It means that the computation has to be performed in the direction of the ply. Evolution of the orthotropy frame is not considered and perturbation analysis is performed.

One important point concerning the performance of the resolution is the time step. Abaqus auto adapts the time step. If the convergence is less than three iterations in the time step, then the next time step will be increased automatically. If convergence is poor, between three and ten iterations (values by default), then the time step is kept constant. If convergence is really poor, more than ten iterations, then Abaqus reduces the time step in question. It is possible for the user to modify the time step if:

- poor convergence is encountered in the local algorithm.

In order to appreciate the performances of the scheme, two tests were carried out on the structure. The first one corresponded to a small
time step and will be seen as the reference solution. The second one was for authorised large time steps. By comparing the two solutions, the performance of the scheme can be appreciated on a $[\pm 45]_s$ structure strained up to 3mm.

Because the consistent tangent operator was computed, the second order of convergence was assured. The total number of iterations by time step were quite small - 2 iterations for the largest increment.

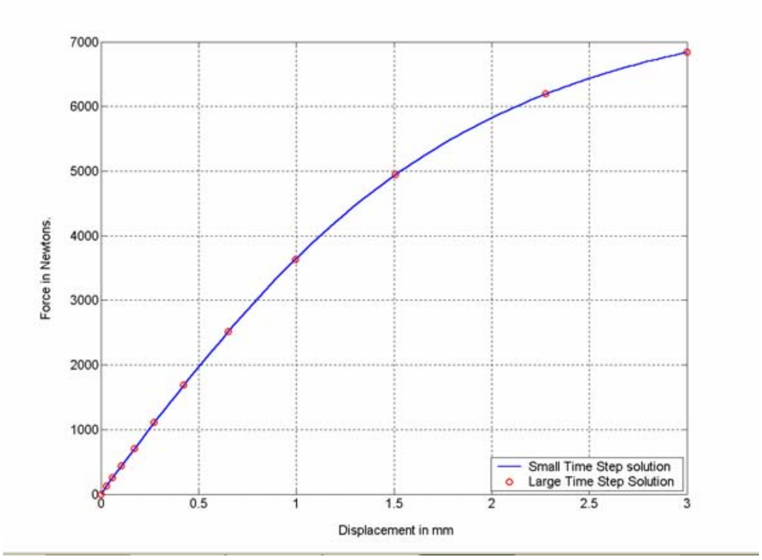


Figure 3 : « Comparison of the performance of the scheme »

4. A structure: Plate with an open hole

The model is tested on a structure. Several assumptions have to be made relating to on numerical and experimental stand points.
The final purpose is to see if the developed model can predict the rupture of the structure.

4.1 The experimental test

A 1.4mm thick plate (160x50 mm) of woven ply G963/939 with a hole of 6.5mm radius is stretched between two dies. Several orientations are tested: $[\pm 45]_s$, $[\pm 40]_s$, $[\pm 30]_s$, $[\pm 20]_s$ and $[0]$.

A tensile monotonic test is performed. Several measurements are made: the force and the relative displacement are measured every second.

This tensile test is performed on the plate with hole under MTS tensile machine. The main difficulties come from the estimation of boundary conditions. The both dies are clamped in the machine.

4.2 Numerical model

In this part, the geometry needed for our model is presented. The whole plate is modelled by a linear quadratic plane stress element. The material data are given in table 1.

The numerical discretised model for the geometry is shown in figure 5. The boundary conditions are clamped on one side and simply strained on the other. Rigid body motions are avoided by clamping certain degrees of freedom.

To model the laminate, different elements are connected to the same nodes. As a consequence, each element with different material properties is submitted to the same displacement and strains. The results for the different configuration are shown on figure 6.

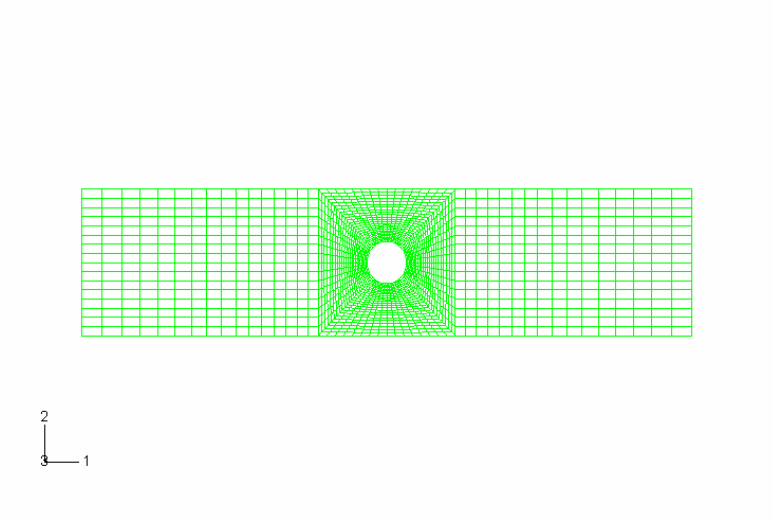


Figure 5 : « Plate with an open hole: FE model »

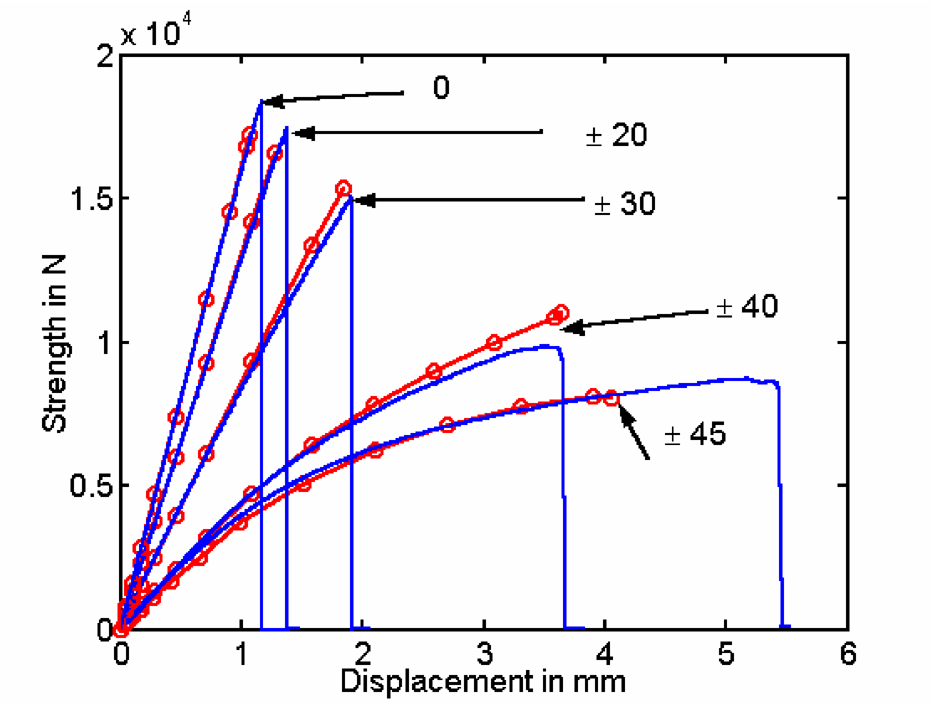


Figure 6 : « Comparison of the numerical simulation and experimental results for an open hole under tensile loading »

In the figure 6, the result are adjusted in order to have the same initial stiffness for the structure. This experimental discrepancies comes from the fact that the length between the dies is around 160mm.

The simulations end for all specimens except $[\pm 45]_s$ due to fiber breaking. The non local model developed originally by Withney and identified in a general manner by Lahellec gives good results when localised at the ply level. It is in the same way intrinsic to the ply. Nevertheless, when analysis the curve displacement-Force for a [+30,-30]_s laminate it seems that the specimen is more non linear that the simulation. When looking on full field strain measurement, it is observed that for such sequence the anelastic deformation and damage are more important that predicted by the model. Nevertheless the force at rupture for such structure are in good agreement.

5 Conclusion

An efficient local algorithm coupled with a non local criteria gives good correlation with experiments on laminated structures. When looking more accurately at the results on figure 5, it seems that simulations are stiffer than experiments, it seems that it is probably due to a lack in our model in the sense that non linear behaviour is taken into account. In particular, attention is focus on coupling between tensile in fiber direction and plasticity and damage.

In a second time, study in compression will be addressed and integrated in our code to complete the feasability of the rupture of woven ply in structure.

References

1. **Hochard C, Aubourg PA, Charles JP** . "Modelling of the mechanical behaviour of woven-fabric CFRP laminates up to failure.", *Composites Science and Technology* 2001;61:221-230
2. **Bordreuil C., Hochard C**." Finite element computation of woven ply laminated composite structures up to rupture". *Applied Composite Materials*, issue 3,vol14,2004.
3. **Ladevèze P, Le Dantec E**. "Damage modelling of the elementary ply for laminates composites". *Composites science and technology*, 1992;43(3):257-268.
4. **Ladevèze P**. "A damage computational method for composite structures". *Computers & Structures*, 1992;44:79-87.
5. **Lahellec N, Bordreuil C., Hochard C**." Un modèle de rupture non local mise en évidence d'une longueur interne sur un stratifié quasi isotrope ". *Comptes Rendus de l'acamédie des sciences*, soumis, 2004.
6. **Payan.J,** "Contribution à l'étude de l'endommagement dans les composites stratifiés sous chargements statique et cycliques", *thèse de l'université de la méditérranée*,2003.
7. **Withney, Nuismer,"**Strain gradient in composite laminate structure " **,** *Journal of composite materials*, Vol 35, p 733-735,1976
8. **Chang, Chang**, "A progressive damage model model for laminated composites containing stress concentrations"**,** Journal of composite materials **,**vol 21, p 834-855**,** 1987
9. **Simo, Taylor**, "Consistent tangent operators for rate independent plasticity", *Computer methods in applied mechanics and engineering*, vol 48, p101-118, 1985