EXPERIMENTAL VALIDATION OF COMPOSITE MATERIAL STRUCTURE OPTIMISATION BY GENETIC ALGORITHM

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ABSTRACT

After a short introduction to genetic algorithms and to the main improvements made to their basic form, this paper presents in detail the optimisation strategy as well as experimental results obtained on stacking sequence optimisation of composite plates and blade models subjected to combined loads with respect to ply continuity. The manufacturing did not raise major difficulties and significant weight saving were obtained : about 25% for the plates and more for the blade models. An experimental validation has been carried out, within the limits of the finite element programme assumptions used in this work. Experiments also shown that some numerical and manufacturing problems can occur, and suggest improvements which will have to be take into account to find most feasible and accurate solutions.

1. INTRODUCTION

The limits of the classical optimisation methods led researchers to imagine techniques based on genetic algorithms inspired by the mechanisms involved in biological evolution. Many authors showed thereafter that it was possible to apply such methods to search optimal solutions in various fields of applications. In this context, the goal of this work was to associate a genetic algorithm with a finite element programme in order to be able to optimise complex shape structures taking into account manufacturing technological constraints, and to bring a beginning of experimental validation about the solutions obtained by this procedure. After a short introduction to genetic algorithms, the main improvements made to their basic form in order to increase the convergence speed are presented. Then, one will detail the optimisation strategy as well as the experimental results obtained on some structural elements, thus showing the expected benefits of this approach.

2. GENETIC ALGORITHM IMPROVEMENTS

The basic genetic algorithm (GA) [1][2][3] can be summarised in a succession of operations presented in *figure 2*. It consists in modifying a population of chromosomes during successive generations in order to select the most adapted ones. A chromosome represents here a possible solution of the problem, and is encoded by a set of strings, representing each one an optimisation variable. The research is initialised by building up a population from which the chromosomes are randomly distributed over the whole variable range. At each generation, the most fitted chromosomes are selected and their strings duplicated by crossing in order to build another population which will again be evaluated, until convergence is reached. The mutation consists in introducing a certain degree of variability from one generation to another, to avoid a convergence towards possible local minima. In order to increase the convergence rate of the basic GA, the following improvements were included :

- real string coding to remove the binary encoding and decoding operations;
- ranking selection [3] according to the best chromosome performances in order to generate a new population by string crossover;

- primitive crossover [4] which consists in incrementing each best chromosome progression step to explore the vicinity of the solutions found at each generation;
- multi-elitist selection [3][5] which consists in recopying, in all new generation resulting from the reproduction step (crossover and mutation), the fitest chromosomes enhanced by the preceding generation.

These improvements were tested and validated on the optimisation of laminate stacking sequences subjected to in-plane loads, whose solutions are known for several simple loading cases [6]. In our work, the fitness function evaluation is carried out using a finite elements (FE) programme dedicated to our needs and directly coupled with the GA in order to minimise the computing time. This FE programme computes the solution assuming linear elastic constitutive equations as well as small displacements and strains. It is based on the use of triangular (DKT3: Discrete Kirchoff Triangular) or quadrilateral (DKQ4: Discrete Kirchhoff Quadrilateral) composite plate elements with 5 degrees of freedom per node [7]. The fitness function F is the product of the partial fitness functions f_j related to each subdomains (containing one or more elements) [8], such as :

$$F = \prod_{j=1}^{Nsd} f_j = \prod_{j=1}^{Nsd} \frac{1}{\left|1 - \mathrm{Max}\{c_i\}\right|} \qquad (1)$$

$\{c_i\}$ being the set of values given by the Tsai-Hill failure criterion of the most stressed plies of the element of rank i belonging to the j^{th} subdomain.

3. PLY CONTINUITY

For an isotropic material structure, optimisation consists in determining the material thickness in function, for example, of the stress level at each point. The structure geometry and loading generally induce stress gradients resulting in continuous thickness variation. It remains generally feasible by conventional manufacturing processes. For a composite material structure however, the thickness variation could be managed only by adjusting locally the number of plies for a given stacking sequence. Except the optimisation of stacking sequence for the whole structure, the designer has to divide the structure into a certain number of subdomains, in which the stacking sequences can be optimised independently by the GA. However, in order to find feasible solutions, it is necessary that the ply angles of the adjacent subdomains are identical. In other words, it is necessary to respect ply continuity, which can be carried out in two ways. The first one [11] consists in optimising the stacking sequences first independently. Ply continuity is ensured at a second stage. Another strategy is presently used. Ply continuity is ensured at each generation. It allows indeed a gradual convergence of the population towards feasible solutions, under the continuous action of the major technological constraint lying in ply continuity enforcing.

The sizes and the positions of the subdomains remaining unchanged during the optimisation process, this strategy initially consists in building up, after structure meshing, the subdomain connectivity table. *Figure 1* illustrates the procedure employed, for example, in the case of balanced laminates. After listing the number of plies in each subdomain for each half-stacking sequence, the minimal number of plies $N_{p\,min}$ common to the whole subdomains is searched during the first stage of the procedure (step 1 in *figure 1*). Then, for each level corresponding to the ply rank in the involved stacking sequence, one calculates the average value $<\theta_i>$ of all ply angles, which one then assigns to this level. The average ply angle $<\theta_i>$ is thus made common to all considered subdomains. Other strategies could of course be used [10]. This angle is, by definition, a continuous variable, but it is preferable, for manufacturing reasons, to seek the nearest value among a set of discrete values (for example: 0, 15°, 30°...). After stacking sequence harmonisation, one seeks again, at the second stage (step 2 in *figure 1*), the minimal number of plies common to the remaining subdomains. Then, one identifies (at this stage as at the following ones) the stacking subsequences for which the subdomains have common borders using the subdomain connectivity table. For each subset of common subdomains, the corresponding stacking subsequences are then harmonised in the same way as previously. The procedure (step 3 in *figure 1*) is then repeated until all the stacking sequences are harmonised.

SD 1 (8 plies)	SD 2 (6)	SD 3 (10)
SD 4 (10)	SD 5 (14)	SD 6 (8)
SD 7 (16)	SD 8 (6)	SD 9 (4)

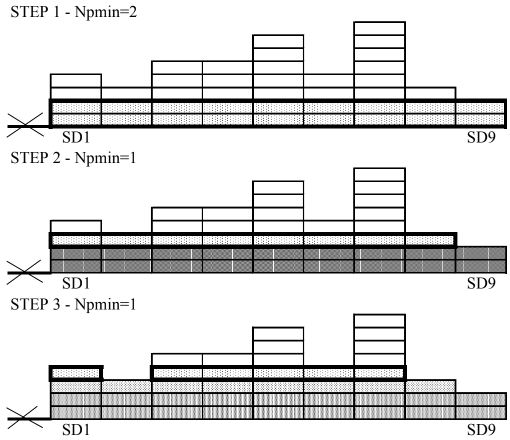

Figure 1 : laminate stacking sequence harmonisation procedure

4. GLOBAL OPTIMISATION PROCEDURE

The optimisation method is described by the flow chart in *figure 2*. It starts with the structure meshing, with respect of the borders defining the different subdomains. Each subdomain can have one or more elements depending on the mesh density. One then builds up the connectivity table to be able to ensure automatically the plies continuity between adjacent subdomains. The mesh remains unchanged during the optimisation procedure.

Before starting, it is necessary to initialise the ply numbers in each subdomain. That can be carried out in a random way but, in this case, the values are, most of the time, very far away from the required solution. In order to be closer to the final solution and to save computing time, the method presently used consists in starting by initialising, in an iterative way, the ply numbers in each subdomain. To do this, one calculates, from the mechanical properties of the material used for the basic ply, the equivalent properties (stiffness and strength parameters) of a quasi-isotropic laminate $[(0/90/45/-45)_m]_s$. The choice of a quasi-isotropic sequence allows to distribute the ply angles more uniformly on their variation range which lies between -90° and 90°. This amounts to inject an equivalent of a "broad band noise" in the genetic algorithm, in order to find more quickly the solution. The determination of sub-sequences number *m* is then performed with an iterative procedure: a finite element calculation determines the stress state from which the fitness function is evaluated to find *m*. Each chromosome (of variable length) is encoded as shown in *figure 3* : it is composed of a string sequence specifying, for each subdomain *j*, $j=1..N_{SD}$, the number of plies n_j, followed by the laminate stacking sequence $[(\theta_{j1}/.../\theta_{jk})]_s$, $k=1,\ n_j$. The procedure itself has two overlapping loops :

- the first one related to ply numbers determination,
- the second one related to the determination of each subdomain laminate stacking sequence by the GA,

followed by the ply continuity enforcing routine.

The ply angles of the stacking sequences are then initialized. The starting angles (0, +45°, -45° and 90°) are reallocated in a random way in a variation range of more or less 30°, centered on the initial values, in order to introduce a sufficient variability from one potential solution to another. This also contributes to distribute the chromosomes more uniformly over the variable definition domain. The first finite elements calculation determines the fitness function of each chromosome, which passes then through the classical steps of selection, crossover and mutation, and then by the more specific procedures described in *chapter 2* below (rank selection fir instance) in order to increase the convergence speed.

The ply continuity is then ensured between the various subdomains before rating each solution with FE calculations. When the convergence is reached (*ie* a part of chromosomes are gathered among the most powerful solutions) one checks that the required number of plies is correct. In the contrary case, the ply number is computed again before seeking again the optimised angles. Finally, the algorithm delivers a solution for which the ply continuity is respected with feasible ply angles. This optimisation scheme was tested successfully on a rectangular plate embedded at one end and subjected to bending loads at the other one [9]. A closed-form solution is indeed available in this case within the beams theory

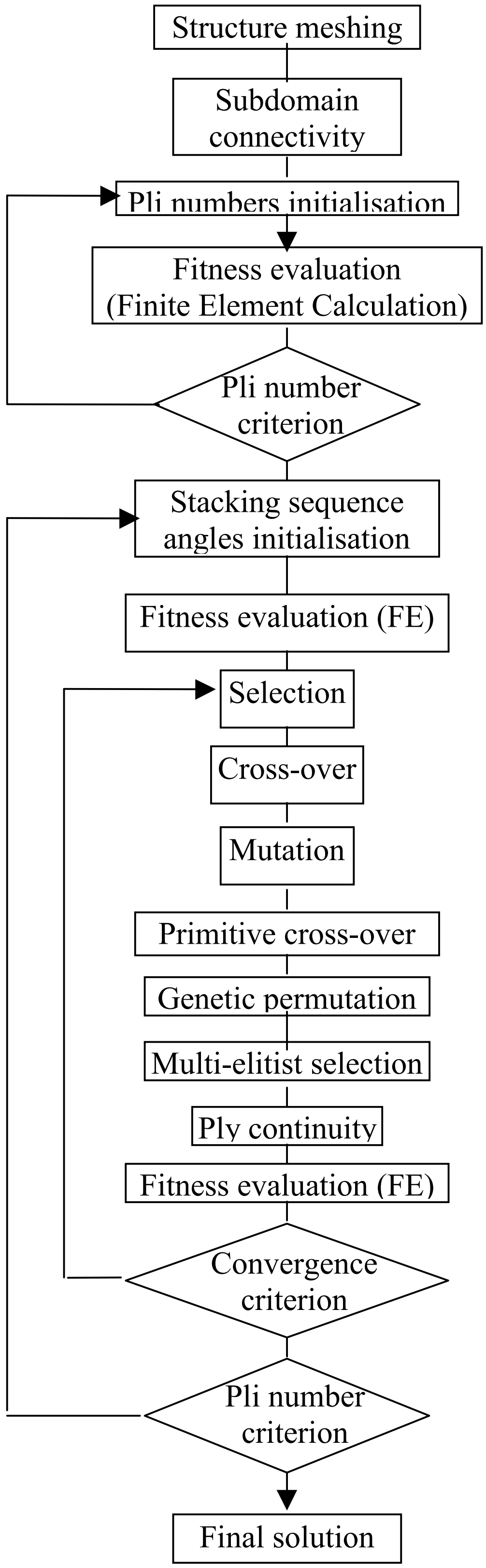

Figure 2 : optimisation procedure

flow chart

n_1	θ_{11}	….	θ_{1n1}	….	n_N	θ_{11}	….	θ_{1nN}

Figure 3 : variable string sequence

5. OPTIMISATION OF PLATES AND BLADE MODELS SUBJECTED TO COMBINED LOADS

The relevance of this optimisation scheme was tested on two types of structures having sufficient degree of complexity and representative of industrial components (see *figure 4*):

- the first one is a series of three plane plates of 400 mm x 400 mm (actually 450 mm, 50 mm being reserved for embedding), embedded on one edge and subjected to a combined torsion and bending loads (F_A=200 N and F_B=-300 N) at the opposite edge. The material employed is a unidirectional carbon T800/ 5245 epoxy resin (see *table 1*);
- the second one is related to two blade models of 1000 mm length and 168 mm in chord, embedded at one end and subjected to a combined torsion and bending loads (F_A=450 N and F_B = -900 N) at the other end. The structure is composed of external skins (extrados and intrados), and of two longitudinal spars located at 20 % and 75 % in chord, wrapped around low density polyurethane foam cores (60 kg/m^3). The material employed is a unidirectional carbon T300/M10 epoxy resin (see *table 1*). Contrary to preceeding material, this one was selected in order to be able to use the polyurethane foam as lay-up gauges as well as ply compacting by foam compression during the curing process.

The objective was to bring a beginning of experimental validation and to highlight possible problems related to the precision of the solutions obtained.

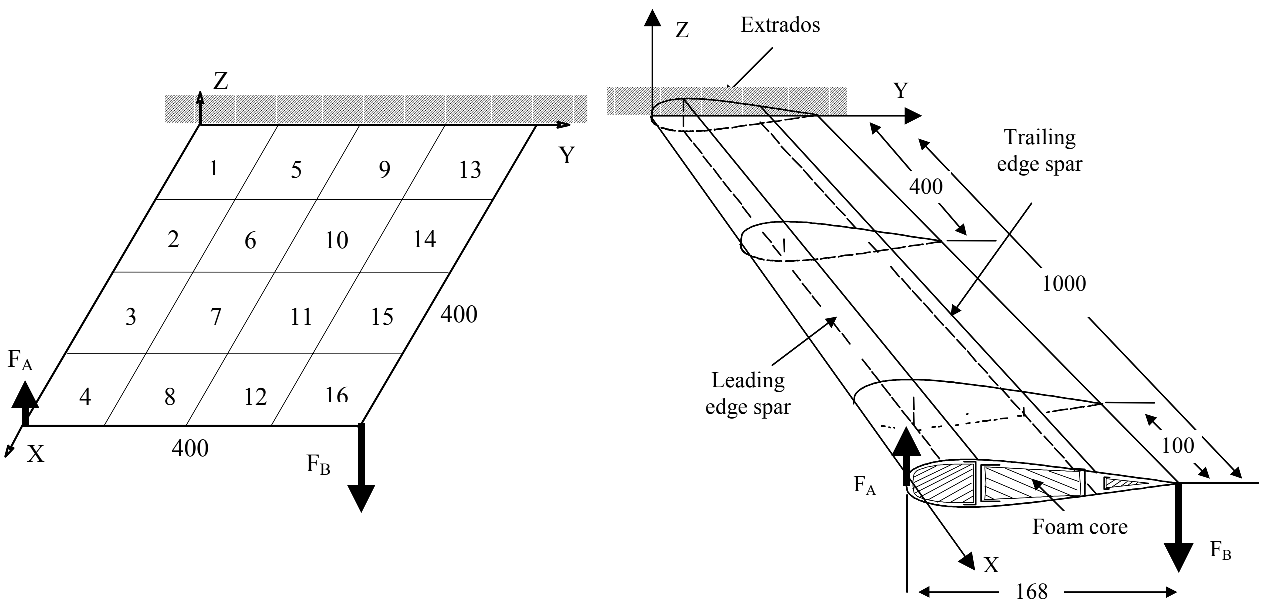

Figure 4 : two types of optimised structures

The three plates were meshed with 144 elements DKQ4 (*ie* 9 elements per subdomain for plates 2 and 3) with the following purposes:

- the first one, called hereafter reference plate, is a whole subdomain and has a balanced stacking sequence;

- the second one is divided into 16 subdomains. The balanced stacking sequence is optimised while the ply continuity is respected;
- the last case is similar to the second one. Only the constraint associated to ply angle symmetry is omitted. The stacking sequences are therefore no longer balanced in this case.

The main objective was to minimise the structure weight and not to exceed, by lower value (between 1 and 15 %) the failure strength for the most stressed plies using the Tsai-Hill failure criterion. The angle ply variation step was equal to 15°, with a minimal ply number of 8. The maximum difference between the ply numbers of two adjacent subdomains was taken equal to 4 (for the first two plates) and to 2 for the third. This value can of course be increased, but probably leads to delamination risk along subdomain borders, induced by the interlaminar shear stresses due to a too strong ply number gap. Results are given in *table 2*. After optimisation, the stacking sequence obtained on the reference plate is $[60/-60/30/-30/-45/45]_s$. Its weight is 571 g. The half stacking sequences of the two other plates are plotted in *figure 5*. The stresses being prevalent in the embedding area (more precisely, at the opposite edge to point B), the GA naturally reinforced this zone for the second plate, by enforcing ply angles at +15° and -15°. One will also notice the presence of two plies at 0° and +45°/-45° angles, supporting the stresses induced by the bending moment on one hand, and by the torque on the other hand.

The weight saving in this case is 25 % lower than that obtained with the reference plate. By suppressing the constraint concerning the angle ply symmetry in the third case, the first two plies are extended and the weight saving then reaches 28 %. One will notice that the computing time is twice and one half more important than for the second plate because of a greater number of potential solutions. For the second plate the computing time is 26 times more important than for the first one because the number of variables was roughly speaking multiplied by the number of subdomain (*$N_{SD}=9$*, instead of *$N_{SD}=1$* for the reference plate). After manufacturing the three optimized plates without major difficulty, displacements at loading points A and B were measured for moderate loads (F_A=3 N and F_B=4,5 N, see *table 2*). Their weights were also measured and are very close to those numerically estimated. The comparison between measured and calculated displacements, at these two points, shows that the predictions given by the finite element programme remain correct in terms of stiffness behaviour. In the case of linear elastic behaviour until failure, it is thus possible to extrapolate these results to the theoretical loading case considered here. However, because of the too great bending flexibility, it would undoubtedly be necessary to use finite element programme including a geometrical non linear formulation. Under the linear elastic behaviour assumption of the FE programme, measurements thus bring a beginning of experimental validation. It should be validated thereafter in the non-linear case using a more suitable finite elements formulation.

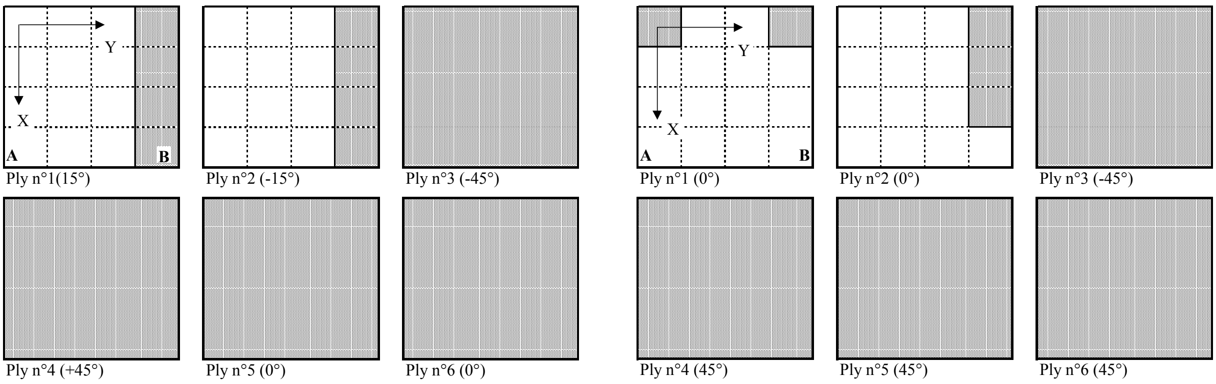

Figure 5 : stacking sequences of plates 2 and 3 obtained after optimisation

The two blade models were meshed with 266 elements DKQ4, with the following purposes:

- the first one (blade 1), called the reference blade, is composed of 4 subdomains : one for each skin and each spar. The ply continuity was ensured between the extrados and intrados skins on leading edge border. The stacking sequences were optimised independently for the two spars;
- the second one (blade 2) is composed of 24 subdomains (*figure 4*): 18 for the two skins with ply continuity ensured on the leading edge border, and 3 for each spar, with the ply continuity ensured for each one.

The main objective was, as previously, to save weight and not to exceed, by lower value (between 1 and 5 %) the failure strength of the most stressed plies, using the Tsai-Hill failure criterion. The ply angle variation step was 15°, with a minimal number of plies equal to 1. The maximum difference between the ply numbers of two adjacent subdomains was equal 2. After optimisation of the first model, one obtains the following stacking sequences:

- $[60/0]_s$ for the skins;
- $[(-45)_2]_s$ and $[- 45/15]_s$ for the leading and trailing edge spars respectively.

For the second model, the stacking sequences obtained after optimisation are shown in *figure 6*. The respect of the optimised stacking sequences, during the manufacturing of the two blade models, did not raise a major difficulty. However, after release from the mould, the second model left slightly twisted (approximately 15° at 1000 mm), likely due to unbalanced internal stresses induced by curing thermal dilatation, associated with the fact that the airfoil does not have a symmetry plane between its intrados and extrados sides. Displacements were measured at the two loading application points A and B (*table 3*) and, for the first model, the measured values are in good agreement with the computed ones, within the limit of the inherent FE model assumption. For the second model, one obtains weight saving (foam not included) of 43 %, but measured displacements are more important than those calculated. A single finite elements calculation, carried out with a more important number of elements (4000) shows that the numerical results obtained on the first model are not influenced by the mesh size, contrary to the second one (see *table 3,* model number 2a), for which the computed values converge towards the measured ones. This is due to a numerical locking phenomenon which occurs when the element thickness due to the ply number reduction imposed by the GA) becomes too small compared to its size. Consequently, our experiments allow us to highlight some problems related to the FE formulations employed, guiding sometimes AG towards false solutions. Rather than initially meshing the structure with a too important number of elements, it would be possible in further developments to include in the flow chart optimisation in *figure 2* an automatic meshing routine to avoid this problem. For instance, it could compute the element size in function of its thickness given by the ply numbers of the stacking sequences in order to save computing time.

In order to clearly highlight the potential weight savings, we thus carried out a new optimisation, but only in a pure numerical way. Considering the basic ply thickness (0,6 mm), we decided to subject the model to more important loads (F_A=4500 N, F_B=-9000 N) in order to obtain more significant ply number gaps than those obtained in the previous experimental study. The subdomain partition of the new reference model (blade 3) remains the same as previously, while the new second model (blade 4) is divided into 10 subdomains for the skins (5 on each side) and 5 for each spar. The numerical precision is guaranteed by meshing each model with 1034 DKQ4 elements. For the first model (blade 3), the following stacking sequences are obtained:

- $[0_4/15/-15]_s$ and $[15/-15/0_4/15/-15]_s$ for the two skins;
- $[60/-60/-75/75]_s$ and $[75/-75/0_2]_s$ for the trailing leading edge spars respectively.

Figure 6 shows the stacking sequences for the second model (blade 4). At point A, the calculated deflexion increases from 100 mm for the first model to 140 mm for the second, with a weight saving of 26,7 % (foam not included). The suppression of a certain number of plies thus resulted in a significant weight saving with a moderate stiffness loss. Computed displacements remain here in agreement with the FE programme assumptions used. The laminate thicknesses in each subdomain remains sufficiently important compared to the size of the elements. It is therefore not necessary to use a more important number of elements. The last step would now consist in manufacturing and performing mechanical tests on these two optimised models to finally check the agreement between experimental and numerical results.

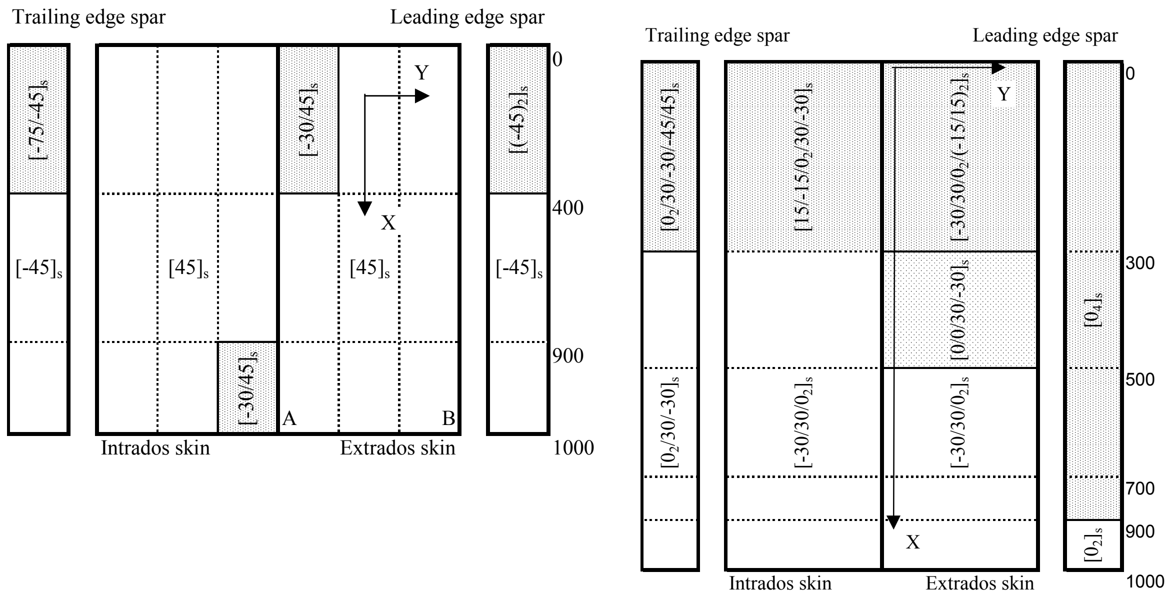

Figure 6 : optimised stacking sequences for blade models 2 and 4

6. CONCLUSION

The coupling between genetic algorithm and finite element programme, associated with an automatic ply continuity subroutine allows us to find feasible solutions for structures having a sufficient degree of complexity and representative of industrial structural components. The plates and blade model manufacturing did not raise major difficulties and significant weight saving were obtained (about 25%). When the solution converges towards a more flexible structure, the classical assumptions related to the FE programme are not respected and erroneous results can be obtained. However the method can become more reliable by integrating an automatic meshing routine in order to adapt the mesh size, thus allowing the calculation to converge towards correct solutions. In the case of non symmetrical and not balanced stacking sequences, the thermal dilation phenomenon would make the solutions found by the genetic algorithm ineffective because of unbalanced curing residual stresses. A parameter calculated by the finite element programme could be added to the fitness function to avoid such a risk. Further extension of this work will concern the structure internal geometry optimisation, as for example: spar positions, subdomain sizes and so on, by the simple addition of an automatic meshing programme to the global optimisation procedure. In the near future, the decrease of computer cost, the increase in the memory sizes as well as processors computing power will undoubtedly allow the designers to deal with much more complex structures, thus putting such methods in user hands not specially confined to research laboratories. That would open large prospects to the rational use of composite materials in all application fields.

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Materials	Elastic properties				Strengths					Ply	Curing
	E_1 GPa	E_2 GPa	E_6 GPa	ν_{12}	X_t MPa	X_c MPa	Y_t MPa	Y_c MPa	S MPa	thickness mm	cycle
UD carbon fiber T800/5245	170	9,9	5,1	0,35	3460	1730	50	165	75	0,183	2h at 180 °C
UD carbon fiber T300/M10	105	9,7	7,5	0,30	2350	930	40	155	63	0,600	1h at 120 °C

Table 1 : mechanical properties of composite materials used for plates and blade models manufacturing

Plate number	Displacement A		Displacement B		Weight	Weight saving	Optimisation time
	FEC mm	measured mm	FEC mm	measured mm	g	%	ratio
1	-0,20	-0,07	-3,42	-3,94	571		1
2	3,19	3,28	-3,95	-3,65	429	25	26,5
3	0,65	0,42	-6,00	-5,50	411	28	66,9

Table 2 : numerical and experimental results obtained with the three optimised plates

Blade number	Forces		Number of elements	Displacement A		Displacement B		Weight saving %
	F_A N	F_B N		FEC mm	measured mm	FEC mm	measured mm	
1	150	300	266	-16,50	-16,66	-21,41	-21,39	
2	65	130	266	-30,17	-37,33	-34,16	-38,32	43
2a	65	130	4000	-37,38		-37,95		

Table 3 : numerical and experimental results obtained with optimised blade models