# THE INFLUENCE OF PHONON TRANSMISSION IMPEDANCE ON THE THERMAL CONDUCTIVITY IN A COMPOSITE MATERIAL FOR CRYOGENIC STRUCTURES

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## ABSTRACT

Column-type support posts are used in particle accelerators to sustain cryomagnets. The design of these supports involves accurate positioning of the magnets in their cryostats and heat load minimisation. For the Large Hadron Collider (LHC) under construction at the CERN, a glass-fibre/epoxy resin composite woven has been chosen for the design of the supports for both its high stiffness and low thermal conductivity over the 2-293 K temperature range. But a compromise must be found. For example, a higher volume fraction of fibre increases both stiffness and thermal conductivity. The work presented in this paper is part of an overall thermo-mechanical optimisation procedure, which aims to fulfil both requirements.

It is well-known that, at very low temperatures, thermal conductivity of composite materials is highly temperature-dependent. Moreover, phonon transmission at the interface between dissimilar solids like fibre and matrix induces a thermal barrier. Moreover, for a perfectly joined interface, the heat flow is proportional to the difference of the fourth powers of the temperature on each side of the interface.

In this paper, a non-linear periodic homogenisation method with a two-scale asymptotic expansion is proposed in order to determine the effective transverse conductivity of an unidirectional composite while taking into account a phonon transmission impedance at the interface between matrix and fibres.

This procedure is applied to a unidirectional glass-fibre/epoxy resin where the thermal conductivity of each constituant is extracted from the literature.

Numerical results show that the model with phonon transmission impedance gives a good estimation of the transversal thermal conductivity for temperatures ranging between 2K and 70K.

## 1. INTRODUCTION

The LHC (Large Hadron Collider) under construction at the CERN consists of 1700 cryomagnets for orbit bending or focusing/defocusing of the circulating high-energy proton beam. Column-type support posts are used to sustain these cryomagnets and achieve thermal insulation. Thermal and mechanical requirements therefore dictated materials selection. An example of a performance index used is the ratio $I = \frac{\sigma}{\rho\lambda}$, the specific strength per average thermal conductivity being necessarily high. Typical values of ratio $I$ are given in [1] for metals and polymer matrix composites in the 4.2-77 K temperature range. It appears that for composite material, this ratio is many times higher than in metallic material. Consequently, organic composite materials are attractive alternatives to metals for the design of the support posts as they allow substantial savings in refrigeration costs. Composite support posts may be either filament wound or pultruded in a continuous process or high-pressure laminated with a cloth reinforcement. It is obvious that the spatial arrangement of the reinforcement and their size strongly influence the engineering properties of the support posts. The material used in the LHC project is a triaxially braided composite with a longitudinal tow and two braid tow at ± 45°. Therefore, it must be considered as a three-dimensional assembly of a transverse isotropic unidirectional composite. As a consequence, the first step of our research whose results are presented here concern the study of the thermal conductivity in a unidirectional composite for cryogenic structures.

It is well known that at very low temperatures, thermal conductivity of material is highly temperature-dependent. Moreover, phonon transmission at the interface between dissimilar solids like fibre and matrix induces a thermal barrier. For a perfectly joined interface, Little [2] showed that the heat flow is proportional to the difference of the fourth powers of the

temperature on each side of the interface. So homogenisation of the thermal behaviour of composite materials for cryogenic application must account for non-linearity and phonon transmission impedance. Few papers in the literature deal with non-linear homogenisation with an interfacial barrier. Auriault and Ene [3] show that according to the Biot number, the description could lead to various models: perfect contact, imperfect contact with one temperature field or insulated constituents with two temperature fields. The Biot number is represented by the ratio of the impedance of the interface barrier to the conductivity of the components. Galka and al. [4] proposed a study of the heat equation with temperature-dependent conductivity coefficients using a two-scale asymptotic expansion. For microperiodic layered composites, analytical formulae are derived. Then, a method to compute bounds on the effective conductivity using Padé approximants is proposed for the general case. Laschet [5] studied the non-linear heat transport problem and developed a specific FE program to investigate numerically the influence of design of transpiration cooling channels on the effective thermal properties.
We propose in this paper a non-linear homogenisation of thermal behaviour of a unidirectional composite with a phonon transmission impedance at the interface, what is new in the litterature to our knowledge. Before developing the two-scale asymptotic expansion method, thermal conductivity of the constituents and heat transmission conditions are presented. Then numerical simulations and their results will be discussed.

## 2. THERMAL CONDUCTIVITY OF GLASS FIBRE, EPOXY RESIN AND GLASS/EPOXY COMPOSITE

One of the main characteristics of a glass/epoxy composite is its insulating quality. Consequently, its use is very interesting to reduce energy loss within a system, especially if the surrounding temperature is very low. Indeed, the composite's thermal conductivity decreases in a nonlinear way as temperature fails.
In order to predict its thermal conductivity in relation to temperature, a study on the thermal behaviour of each component is required. However, in our case, components are amorphous solids and thus, they have a similar thermal behaviour as one can see it in Fig. 1.

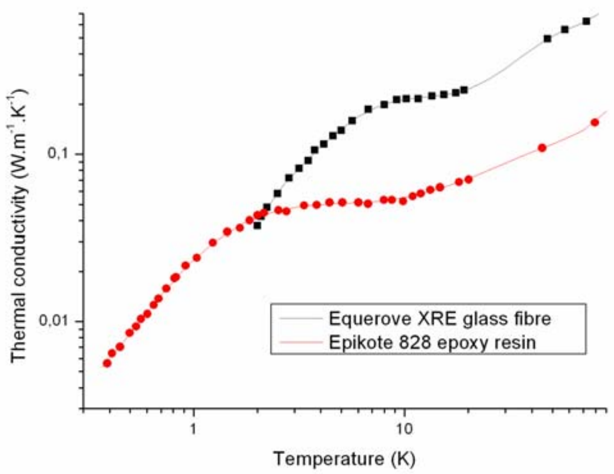


**Fig. 1.** Thermal conductivity evolution of Equerove XRE glass fibre and Epikote 828 epoxy resin charted against the temperature [8][9]

Each curve can be separated into three parts. In the first part, which is defined for $T$<3K for epoxy and $T$<8K for glass, thermal conductivity is proportional to $T^2$ [6]. Then, in the second part, which is defined for 3K<$T$<8K for epoxy and 8K<$T$<20K for glass, thermal conductivity is nearly constant. Finally in the last part, which is defined for $T$>8K for epoxy and $T$>20K for glass, thermal conductivity is proportional to $T$. These physical phenomena can be explained by the following kinetic law [7]:

$$\lambda=\frac{1}{3}\upsilon_{ph}\Lambda_{ph}C_{ph}\rho_{ph} \quad (1)$$

where $\upsilon_{ph}$ is the mean velocity of phonons, $\Lambda_{ph}$ is the mean free path of phonons, $C_{ph}$ is the specific heat of Debye and $\rho_{ph}$ is the density of a solid.

Now, the aim is to estimate the thermal conductivity of a unidirectional glass/epoxy composite, in the direction perpendicular to fibers, using thermal conductivity of each component. Classical models such as mixing law [10] (thermal resistances in series), Rayleigh [11] or Springer and Tsaï model [12] give an effective thermal conductivity in agreement with experiments (Fig. 2) for temperatures ranging between 10K and 80K.

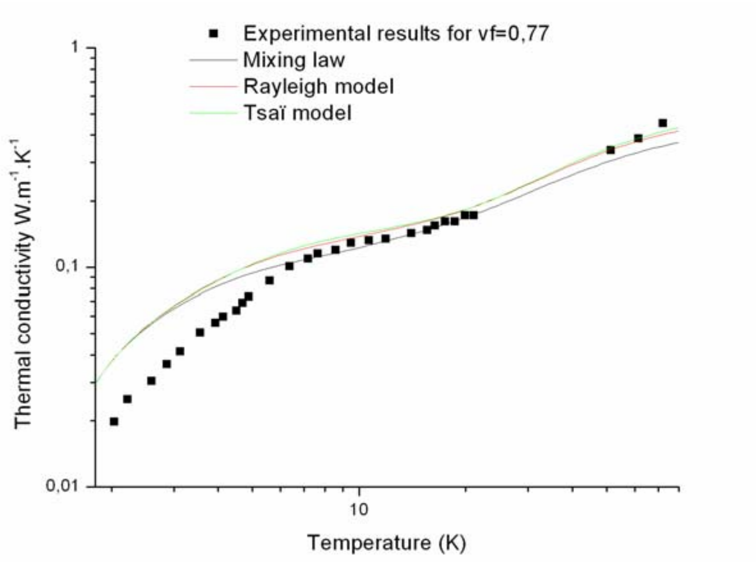


**Fig. 2.** Comparison of the transversal thermal conductivity obtained by theoritical models and experiments [8] for a unidirectional glass/epoxy composite ($v_f$=0.77)

For temperatures below 10K, models diverge because they do not take account of the thermal barrier at the fiber/matrix interface induced by acoustic mismatch [2][13]. This thermal barrier can be interpreted mathematically by the following equation:

$$\begin{cases} \vec{q}_f \vec{n}_f = -\vec{q}_m \vec{n}_m & \text{: flow conservation} \\ \vec{q}_f \vec{n}_f = h(T_f^4 - T_m^4) & \text{: thermal discontinuity} \end{cases} \quad (2)$$

where $h$ is the resistance of Kapitza:

$$h=\frac{2\pi^5 k_B^4 \Gamma}{15 h_p^3 c_1^2} \quad (3)$$

where $k_B$ is Boltzmann constant, $h_p$ is Planck constant, $\Gamma = \Gamma(\rho_1, \rho_2, c_1, c_2)$ is a complex function which depends on density and sound velocity of each component and $c_1$ is the sound velocity in medium 1.

In order to obtain a more accurate effective thermal conductivity of a unidirectional composite for low temperatures, a periodical homogenization using asymptotic developments will be described taking into account Eq. 2 at the fiber/matrix interface.

## 3. PERIODICAL HOMOGENIZATION METHOD

Periodical composite is composed of cells (RVE) distributed periodically in the ($X_1$, $X_2$) plane (Fig. 3).

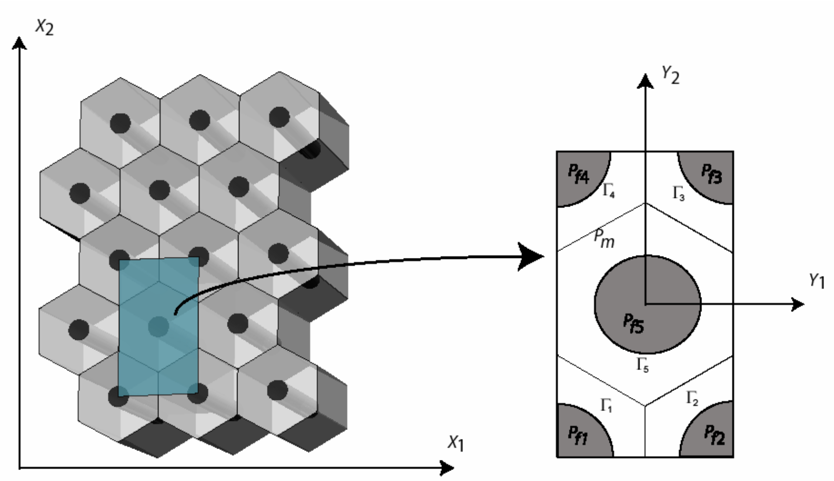


**Fig. 3.** Schematic diagram of homogenization method: (left) periodic structure, (right) RVE

Characteristic length of the RVE is small ($l$) compared with that of a composite sample ($L$). Their ratio is noted $\varepsilon$:

$$\varepsilon=\frac{l}{L}<<1 \tag{4}$$

So in the RVE, stationary heat equation and thermal transfer at fiber/matrix interfaces are written:

$$\frac{\partial}{\partial \mathrm{Y}_i}\left[\lambda_{ij}^{f\varepsilon}\left(\underline{Y},T^{f\varepsilon}(\underline{Y})\right)\frac{\partial T^{f\varepsilon}(\underline{Y})}{\partial Y_j}\right]=0 \quad \text{in } P_{f\theta} \;\; \theta=1,..,5$$

$$\frac{\partial}{\partial \mathrm{Y}_i}\left[\lambda_{ij}^{m\varepsilon}\left(\underline{Y},T^{f\varepsilon}(\underline{Y})\right)\frac{\partial T^{m\varepsilon}(\underline{Y})}{\partial Y_j}\right]=0 \quad \text{in } P_m$$

$$\left.\begin{aligned}-\lambda_{ij}^{f\varepsilon}\left(\underline{Y},T^{f\varepsilon}(\underline{Y})\right)\frac{\partial T^{f\varepsilon}(\underline{Y})}{\partial Y_j}n_i^f&=\lambda_{ij}^{m\varepsilon}\left(\underline{Y},T^{m\varepsilon}(\underline{Y})\right)\frac{\partial T^{m\varepsilon}(\underline{Y})}{\partial Y_j}n_i^m\\ -\lambda_{ij}^{f\varepsilon}\left(\underline{Y},T^{f\varepsilon}(\underline{Y})\right)\frac{\partial T^{f\varepsilon}(\underline{Y})}{\partial Y_j}n_i^f&=h\left[\left(T^{f\varepsilon}\right)^4-\left(T^{m\varepsilon}\right)^4\right]\end{aligned}\right\} \text{on } \Gamma_\theta \tag{5}$$

where $\lambda_{ij}^{f\varepsilon}$ (respectively $\lambda_{ij}^{m\varepsilon}$) is thermal conductivity tensor of the fiber (respectively matrix) and $T^{\varepsilon}$ is the temperature field. Moreover, we assume that the composite is strictly periodic:

$$\lambda_{ij}^{\alpha\varepsilon}\left(\underline{Y},T^{\varepsilon}(\underline{Y})\right)=\lambda_{ij}^{\alpha}\left(\frac{\underline{Y}}{\varepsilon},T^{\alpha\varepsilon}(\underline{Y})\right) \quad \alpha=m \text{ or } f \tag{6}$$

It is necessary to add conditions of periodicity on each edge of the RVE, which will be described later. In order to avoid dimensional problems between the terms of Eq. (5), we introduce the following dimensionless numbers [3]:

$$\underline{y} \equiv \frac{Y}{l} \quad \underline{x} \equiv \frac{Y}{L} \tag{7}$$

Consequently, temperature $T^{\alpha\varepsilon}$ depends on the two preceding variables and can be searched in the following shape:

$$T^{\alpha\varepsilon}(\underline{y}) = T^{\alpha}(\underline{x}, \underline{y}) = T_0^{\alpha}(\underline{x}, \underline{y}) + \varepsilon T_1^{\alpha}(\underline{x}, \underline{y}) + \varepsilon^2 T_2^{\alpha}(\underline{x}, \underline{y}) + o(\varepsilon^3) \tag{8}$$

Then, the Taylor expansion near $T_0$ is applied on the tensor of thermal conductivity [4][5] that becomes

$$\lambda_{ij}^{\alpha}(\underline{y}, T^{\alpha\varepsilon}(\underline{y})) = \lambda_{ij}^{\alpha 0} + \varepsilon \lambda_{ij}^{\alpha 1} + \varepsilon^2 \lambda_{ij}^{\alpha 2} + o(\varepsilon^3) \tag{9}$$

with $\lambda_{ij}^{\alpha 0} = \lambda_{ij}^{\alpha}(\underline{y}, T_0^{\alpha}(\underline{x}, \underline{y}))$

$$\lambda_{ij}^{\alpha 1} = T_1^{\alpha} \left( \frac{\partial \lambda_{ij}^{\alpha}(\underline{y}, T^{\alpha\varepsilon}(\underline{y}))}{\partial T^{\alpha\varepsilon}(\underline{y})} \right)_{T^{\alpha\varepsilon}(\underline{y}) = T_0^{\alpha}(\underline{x}, \underline{y})}$$

$$\lambda_{ij}^{\alpha 2} = T_2^{\alpha} \left( \frac{\partial \lambda_{ij}^{\alpha}(\underline{y}, T^{\alpha\varepsilon}(\underline{y}))}{\partial T^{\alpha\varepsilon}(\underline{y})} \right)_{T^{\alpha\varepsilon}(\underline{y}) = T_0^{\alpha}(\underline{x}, \underline{y})} + \frac{1}{2} T_1^{\alpha 2} \left( \frac{\partial^2 \lambda_{ij}^{\alpha}(\underline{y}, T^{\alpha\varepsilon}(\underline{y}))}{\partial T^{\alpha\varepsilon}(\underline{y})^2} \right)_{T^{\alpha\varepsilon}(\underline{y}) = T_0^{\alpha}(\underline{x}, \underline{y})}$$

In our computations, independence between $\underline{x}$ and $\underline{y}$ must be taken into account as well as the expression of the following derivation operator :

$$\frac{\partial}{\partial y_j} = \frac{\partial}{\partial y_j} + \varepsilon \frac{\partial}{\partial x_j} \tag{10}$$

By introducing Eqs. 8, 9 and 10 into 5, we obtain :

$$\begin{aligned}
&\left( \frac{\partial}{\partial y_i} + \varepsilon \frac{\partial}{\partial x_i} \right) \left[ \left( \lambda_{ij}^{f0} + \varepsilon \lambda_{ij}^{f1} + \varepsilon^2 \lambda_{ij}^{f2} \right) A_j^f \right] = 0 \quad \text{in } P_{f\theta} \\
&\left( \frac{\partial}{\partial y_i} + \varepsilon \frac{\partial}{\partial x_i} \right) \left[ \left( \lambda_{ij}^{m0} + \varepsilon \lambda_{ij}^{m1} + \varepsilon^2 \lambda_{ij}^{m2} \right) A_j^m \right] = 0 \quad \text{in } P_m \\
&-A_j^f \left( \lambda_{ij}^{f0} + \varepsilon \lambda_{ij}^{f1} + \varepsilon^2 \lambda_{ij}^{f2} \right) n_i^f = A_j^m \left( \lambda_{ij}^{m0} + \varepsilon \lambda_{ij}^{m1} + \varepsilon^2 \lambda_{ij}^{m2} \right) n_i^m \\
&-A_j^f \left( \lambda_{ij}^{f0} + \varepsilon \lambda_{ij}^{f1} + \varepsilon^2 \lambda_{ij}^{f2} \right) n_i^f = hl \left[ \left( T_0^f + \varepsilon T_1^f + \varepsilon^2 T_2^f \right)^4 - \left( T_0^m + \varepsilon T_1^m + \varepsilon^2 T_2^m \right)^4 \right] \quad \text{on } \Gamma_\theta
\end{aligned} \tag{11}$$

where $A_j^{\alpha} = \left[ \frac{\partial T_0^{\alpha}}{\partial y_j} + \varepsilon \left( \frac{\partial T_0^{\alpha}}{\partial x_j} + \frac{\partial T_1^{\alpha}}{\partial y_j} \right) + \varepsilon^2 \left( \frac{\partial T_1^{\alpha}}{\partial x_j} + \frac{\partial T_2^{\alpha}}{\partial y_j} \right) \right]$

Continuation of the method consists in collecting terms of identical power in ε. Thus for terms of order $\varepsilon^0$, Eq. 11 becomes :

$$\begin{aligned}
&\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{f0}\frac{\partial T_0^f}{\partial y_j}\right)=0 \quad \text{in } P_{f\theta}\\
&\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{m0}\frac{\partial T_0^m}{\partial y_j}\right)=0 \quad \text{in } P_m\\
&\left.\begin{aligned}
-\lambda_{ij}^{f0}\frac{\partial T_0^f}{\partial y_j}n_i^f&=\lambda_{ij}^{m0}\frac{\partial T_0^m}{\partial y_j}n_i^m\\
-\lambda_{ij}^{f0}\frac{\partial T_0^f}{\partial y_j}n_i^f&=hl\left[\left(T_0^f\right)^4-\left(T_0^m\right)^4\right]
\end{aligned}\right\} \quad \text{on } \Gamma_\theta
\end{aligned} \tag{12}$$

The first two expressions of Eq. 12 have respectively the following obvious solutions

$$T_0^f(\underline{x},\underline{y})=T_0^f(\underline{x}) \text{ and } T_0^m(\underline{x},\underline{y})=T_0^m(\underline{x}) \tag{13}$$

By introducing these results in the fourth expression, we obtain :

$$T_0^f(\underline{x})=T_0^m(\underline{x})=T_0(\underline{x}) \tag{14}$$

Consequently, $T_0$ corresponds to the temperature field depending only on $\underline{x}$ ($T_0$ is constant within the RVE).
By collecting terms of order $\varepsilon^1$ and using results of preceding order, Eq. 11 becomes :

$$\begin{aligned}
&-\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{f0}\frac{\partial T_1^f}{\partial y_j}\right)=\frac{\partial T_0}{\partial x_j}\frac{\partial \lambda_{ij}^{f0}}{\partial y_i} \quad \text{in } P_{f\theta}\\
&-\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{m0}\frac{\partial T_1^m}{\partial y_j}\right)=\frac{\partial T_0}{\partial x_j}\frac{\partial \lambda_{ij}^{m0}}{\partial y_i} \quad \text{in } P_m\\
&\left.\begin{aligned}
-\lambda_{ij}^{f0}\left(\frac{\partial T_0}{\partial x_j}+\frac{\partial T_1^f}{\partial y_j}\right)n_i^f&=\lambda_{ij}^{m0}\left(\frac{\partial T_0}{\partial x_j}+\frac{\partial T_1^m}{\partial y_j}\right)n_i^m\\
-\lambda_{ij}^{f0}\left(\frac{\partial T_0}{\partial x_j}+\frac{\partial T_1^f}{\partial y_j}\right)n_i^f&=4hlT_0^3\left(T_1^f-T_1^m\right)
\end{aligned}\right\} \quad \text{on } \Gamma_\theta
\end{aligned} \tag{15}$$

Here, the lemma of Lions [14] can be applied so as to affirm existence and unicity of $T_1^f(\underline{x},\underline{y})$ and $T_1^m(\underline{x},\underline{y})$. Moreover, as the second member of the first two expressions is a linear function of the gradient $\frac{\partial T_0}{\partial x_j}$, the field solution $T_1^\alpha(\underline{x},\underline{y})$ can be expressed in the following way:

$$T_1^\alpha(\underline{x},\underline{y})=\chi^{\alpha k}(\underline{y})\frac{\partial T_0}{\partial x_k}+\widetilde{T}_1^\alpha(\underline{x}) \tag{16}$$

By using Eqs. 13, 14 and by factorizing using $\frac{\partial T_0}{\partial x_j}$, Eq. 15 becomes :

$$\begin{gathered}
\frac{\partial}{\partial y_i}\left[\lambda_{ij}^{f0}\left(\delta_{jk}+\frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\right)\right]=0 \quad \text{in } P_{f\theta} \\
\frac{\partial}{\partial y_i}\left[\lambda_{ij}^{m0}\left(\delta_{jk}+\frac{\partial \chi^{mk}(\underline{y})}{\partial y_j}\right)\right]=0 \quad \text{in } P_m \\
\left.\begin{aligned}
-\lambda_{ij}^{f0}\left(\delta_{jk}+\frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\right)n_i^f=\lambda_{ij}^{m0}\left(\delta_{jk}+\frac{\partial \chi^{mk}(\underline{y})}{\partial y_j}\right)n_i^m \\
-\lambda_{ij}^{f0}\left(\delta_{jk}+\frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\right)n_i^f=4hlT_0^3\left(\chi^{fk}-\chi^{mk}\right)
\end{aligned}\right\} \quad \text{on } \Gamma_\theta
\end{gathered} \tag{17}$$

This equation will be solved by finite element method in the next chapter.
Now, the last step consists in collecting terms of order $\varepsilon^2$ of Eq. 11 and taking into account the results of preceding orders:

$$\begin{gathered}
\frac{\partial}{\partial y_i}\left[\lambda_{ij}^{f0}\frac{\partial T_1^f}{\partial x_j}+\lambda_{ij}^{f1}\left(\frac{\partial T_0^f}{\partial x_j}+\frac{\partial T_1^f}{\partial y_j}\right)\right]+\frac{\partial}{\partial x_i}\left[\lambda_{ij}^{f0}\left(\frac{\partial T_0^f}{\partial x_j}+\frac{\partial T_1^f}{\partial y_j}\right)\right]=-\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{f0}\frac{\partial T_2^f}{\partial y_j}\right) \quad \text{in } P_{f\theta} \\
\frac{\partial}{\partial y_i}\left[\lambda_{ij}^{m0}\frac{\partial T_1^m}{\partial x_j}+\lambda_{ij}^{m1}\left(\frac{\partial T_0^m}{\partial x_j}+\frac{\partial T_1^m}{\partial y_j}\right)\right]+\frac{\partial}{\partial x_i}\left[\lambda_{ij}^{m0}\left(\frac{\partial T_0^m}{\partial x_j}+\frac{\partial T_1^m}{\partial y_j}\right)\right]=-\frac{\partial}{\partial y_i}\left(\lambda_{ij}^{m0}\frac{\partial T_2^m}{\partial y_j}\right) \quad \text{in } P_m
\end{gathered} \tag{18}$$

By using properties of *P*-periodicity [14], Eq. 18 becomes :

$$\frac{\partial}{\partial x_i}\left[k_{ik}^{\text{hom}}(\underline{x},T_0)\frac{\partial T_0(\underline{x})}{\partial x_k}\right]=0 \tag{19}$$

where

$$k_{ik}^{\text{hom}}(\underline{x},T_0)=\frac{1}{|P|}\left[\int_{P_{f\theta}}\lambda_{ij}^f(\underline{y},T_0)\left(\delta_{jk}+\frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\right)dV+\int_{P_m}\lambda_{ij}^m(\underline{y},T_0)\left(\delta_{jk}+\frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\right)dV\right] \tag{20}$$

$|P|$ is the term which represents the area of RVE

Eq. 19 corresponds to the non-linear heat equation in stationary regime of equivalent material while Eq. 20 corresponds to the homogeneous thermal conductivity tensor. In order to define Eq. 19 completely, we compute $\chi^{fk}$ and $\chi^{mk}$ using the finite element method.

## 4. NUMERICAL RESOLUTION AND RESULTS

The numerical model is separated in two parts. The first part wich corresponds to geometrical model (Fig. 3 (right)) and mesh of the RVE is carried out on ANSYS 8.0 while the second part consists in solving the following variational formulation, deduced from Eq. 17, using MATLAB 6.5 (see Fig. 4 and Fig. 5):

$$\chi^{mk}, \chi^{fk} \text{ are } P\text{-periodic and sufficient continuous on } P$$

$$\sum_{\theta=1}^{5}\int_{P_{f\theta}} \lambda_{ij}^{f} \frac{\partial \chi^{fk}(\underline{y})}{\partial y_j}\frac{\partial \varphi^k}{\partial y_i} dS + \sum_{\theta=1}^{5}\int_{\Gamma_\theta^f} 4hlT_0^3\left(\chi^{fk}-\chi^{mk}\right)\varphi^k d\Gamma + \int_{P_f} \lambda_{ik}^{f}\frac{\partial \varphi^k}{\partial y_i} dS +$$
$$\int_{P_m} \lambda_{ij}^{m} \frac{\partial \chi^{mk}(\underline{y})}{\partial y_j}\frac{\partial \varphi^k}{\partial y_i} dS + \sum_{\theta=1}^{5}\int_{\Gamma_\theta^m} 4hlT_0^3\left(\chi^{mk}-\chi^{fk}\right)\varphi^k d\Gamma + \int_{P_\theta} \lambda_{ik}^{m}\frac{\partial \varphi^k}{\partial y_i} dS = 0 \quad (21)$$

where $\varphi^k$ is a virtual field which is identical on two opposite faces of the RVE. Moreover, its derivative compared to $y_i$ takes values of the same modulus but of contrary signs. It should be noted that interface $\Gamma_\theta$ can be divided in two parts (unfolding of nodes) $\Gamma_\theta^f$ and $\Gamma_\theta^m$ which are associated respectively to fibers and matrices.

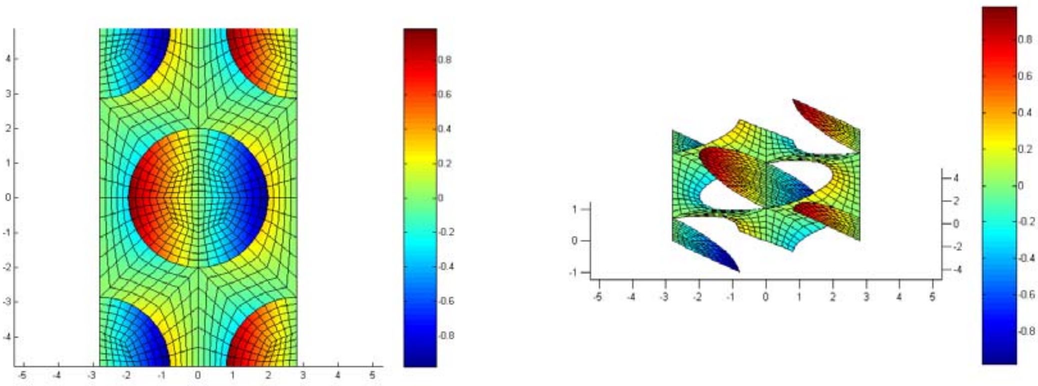

**Fig. 4.** Representation of $\chi^1$ within the RVE for *h*=450 and $T_0$=2K

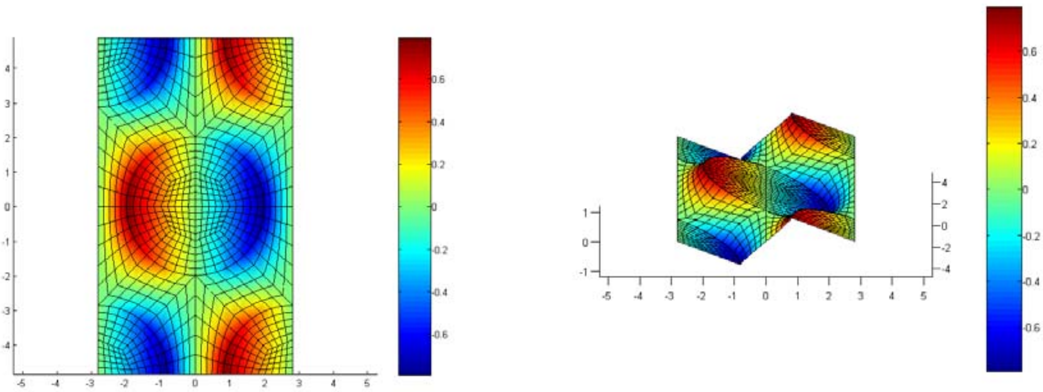

**Fig. 5.** Representation of $\chi^1$ within the RVE for *h*=450 and $T_0$=15K

In order to solve Eq. 21, we must know the thermal resistance *h* that is determined in the following way:

(1) $T_0$ and $v_f$ are respectively fixed at 2K and 0.77;
(2) $\lambda_{ij}^{f}$ and $\lambda_{ij}^{m}$ are calculated for $T_0$=2K using curves on Fig. 1;
(3) *h* is adjusted in Eq. 21 until $k_{ik}^{\hom}$=0.02K (Fig. 2);

(4) finally, we obtain $h$=450 $W.m^{-2}.K^{-4}$;
(5) this value is checked by increasing $T_0$ from 2 to 70K in Eq. 21 and by comparing these results with experimental results (Fig. 6);
(6) another means of checking $h$, is to modify $v_f$ and repeat the procedure since the step (5);

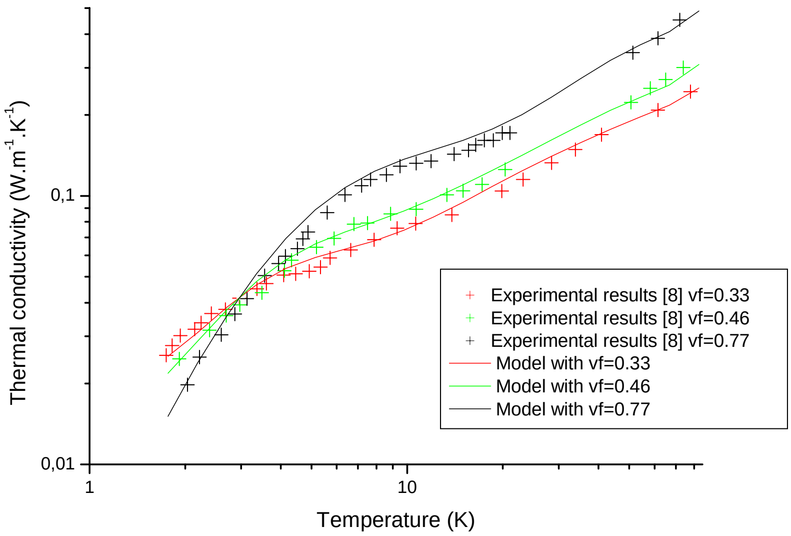


**Fig. 6.** Transversal conductivity of glass-fiber/epoxy composite as a function of temperature for vf=0.77, 0.46, 0.33 and $h$=450

Fig. 6 shows that for a given value of $h$, numerical curves fit well with experimental results for various volumic fractions of fibers.

## 5. CONCLUSIONS

A theoritical and numerical model was developed in order to take account of Kapitza resistance between fiber and glass for temperatures below 10K. This model is also available for higher temperatures. Indeed, when temperature increases, the jump of temperature at interfaces decreases and consequently, the second and fifth term of Eq. 21 become negligeable. In order to validate results obtained in Fig. 6 and check the value of $h$, it would be interesting to create a set-up to measure the resistance of Kapitza and carry out thermal tests on other unidirectional glass/epoxy composites.
Currently, we are developing a model to compute thermal conductivity of a woven-fabric composite by using preceding model to find the equivalent thermal conductivity of each yarn. Using similar reasoning, mechanical behaviour of a woven-fabric composite would be determined completed by an optimization method in order to find the best trade-off between high tensile modulus and low thermal conductivity.

References


1. **Horiuchi, T.** and **Ooi, T.,** "Cryogenic properties of composite materials", *Cryogenics*, **35**/11 (1995), 677-679.
2. **Little, W.A.,** "The transport of heat between dissimilar solids at low temperatures", *Can. J. Phys.* , **37** (1959), 334-349.
3. **Auriault, J.L.** and **Ene, H.I.,** "Macroscopic modelling of heat transfer in composites with interfacial thermal barrier", *Int. J. Heat Mass Transfer*, **37**/18 (1994), 2885-2892.
4. **Galka, A., Telega, J.J**. and **Tokarzewski, S.,** "Heat Equation with temperature-Dependent Conductivity Coefficients and Macroscopic Properties of Microheterogeneous Media", *Mathematical and Computer Modelling* , **33**/8-9 (2001), 927-942.

5. **Laschet, G.,** “Homogenization of the thermal properties of transpiration cooled multilayer plates”, *Comput. Methods Appl. Mech. Engrg.*, **191**/41-42 (2002), 4535-4554.
6. **Hunklinger, S.**, “Tunneling in amorphous solids”, *Cryogenics*, **28**/4 (1988), 224-229.
7. **Greig, D.,** “Low temperature thermal conductivity of polymers”, *Cryogenics*, **28**/4 (1988), 243-247.
8. **Radcliffe, D.J.** and **Rosenberg, H.M.,** “The thermal conductvity of glass-fibre and carbon-fibre/epoxy composites from 2 to 80 k”, *Cryogenics*, **22**/5 (1982), 245-249.
9. **Kelham, S.** and **Rosenberg, H.M.,** “The thermal conductivity and specific heat of epoxy-resin from 0.1-80k”, *J. Phys. C: Solid State Phys.*, **14** (1981), 1737-1749.
10. **Tavman, I.H.** and **Akinci, H.,** “Transverse thermal conductivity of fiber reinforced polymer composites”, *Int. Comm. Heat Mass Transfer*, **27**/2 (2000), 253-261.
11. **Rayleigh, L.,** “On the influence of obstacles arranged in rectangular order upon the properties of medium”, *Phil. Mag*., **34** (1892), 481-502.
12. **Springer, G.S.** and **Tsaï, S.W.,** “Thermal conductivities of unidirectional materials”, *J. Comp. Mater.*, **1** (1967), 166-173.
13. **Peterson, R.E.** and **Anderson, A.C.,** “The Kapitza Thermal Boundary Resistance”, *J. Low Temp. Phys.*, **11** (1973), 631-665.
14. **Bensoussan, A., Lions, J.L.** and **Papanicolaou, G.,** “Asymptotic analysis for the periodic structures”, *Studies in Mathematics and its Application*, **5** (1978).