Internal Stresses Induced by Cyclical Environmental Conditions in Laminated Composite Plates

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ABSTRACT

A tool for calculating cyclical transient hygrothermal stress in laminated composite plates is presented. It relies on a detailed calculation of the hygrothermal fields, described by the classical Fick's law, obtained through analytical and finite difference methods and the classical lamination theory, extended for taking into account transient conditions. The state of stress generated by cyclical environmental conditions due to a supersonic flight is simulated and some discussion about the relevance of such stress is provided.

1. INTRODUCTION

Environmental conditions induce internal stresses in composite materials. At the mesoscopic scale, these stresses are mainly induced by the mismatch in the coefficients of hygroscopic expansion between adjacent plies and are usually calculated assuming uniform and constant hygrothermal fields.

Real hygrothermal conditions are of variable amplitude and, due to the diffusion processes, produce internal distributions which are far from being uniform. There are many applications, for example aeronautical structures subject to real flight conditions, in which the external hygrothermal conditions are approximately cyclical, the period of a cycle being the time of a flight. These conditions produce transient internal concentrations and transient internal stresses which deserve to be detailed [1].

Calculation of internal stresses due to transient conditions has been performed for cylindrical and plate structures [2,3], the problem of a composite pipe subjected to cyclical hygrothermal conditions has been also solved [4].

In the present paper both analytical and difference finite approaches are provided to determine the hygrothermal concentration fields inside a composite plate subject to cyclical conditions. Stresses are then calculated by using the classical lamination theory in a formulation adapted for taking into account non-uniform moisture concentrations [2]. Residual curing stresses are also taken into account by considering the thermal differential between cure and room conditions. The uncoupled hygrothermoelastic model is presented in section 2.

The model is then used to simulate environmental induced stress in supersonic flight. The cycle consists of a mixed cycling of external temperature and moisture, conditioning due to maintenance is also taken into account. Discussion of results with emphasis on the complexity of the resulting state of stress is provided, some qualitative consideration on the effects of such stresses on the resulting damage, which has been experimentally observed [5], is also given.

2. MODEL APPROACH

The thermal and hygroscopic fields are in some sense coupled, the coefficient of moisture diffusivity depends on the temperature by an Arrhenius law, as it will be detailed later. Once the hygrothermal field is obtained, strains and stresses are calculated by assuming hygrothermoelastic behaviour. Attention is paid to plate structures which are sufficiently thin to enable us to formulate the following hypotheses:

- the thermal field is *uniform* along the entire structure. This hypothesis relies also on the fact that the time for thermal diffusion is much smaller than the time for moisture diffusion,
- moisture diffusion is unidirectional along the thickness of the plate and follows the Fick's law [6],
- stresses and strains are calculated according to the classical lamination theory [7].

2.1 HYGROTHERMAL FIELD

We consider the problem of a composite plate in a Euclidean reference frame (*x,y,z*) subjected to *cyclical* hygrothermal conditions at its boundaries. By cyclical we mean that they vary in a periodic manner as a function of time. The plate can be considered as infinite along two directions, *x,y* for instance, so that the hygrothermal diffusion is essentially unidirectional along the *z* direction. As mentioned introducing the model, the thermal equilibrium is supposed instantaneous, the temperature *T* inside the plate is *uniform,* even if it may vary with time *T=T(t)*. The moisture diffusion coefficient *D* depends on the temperature through the Arrhenius law:

$$D(t) = A\exp\left(\frac{B}{T(t)}\right) \tag{1}$$

where *A* and *B* are material constants.

The moisture concentration *c*, which is the density of solute absorbed by the composite plate, is obtained by solving the following *field* equation:

$$c(z,t)_{,t} = D(t)c(z,t)_{,zz} \quad \forall \;\; 0 < z < e, \quad t > 0 \tag{2}$$

$$\begin{cases} c(0,t) = c(e,t) = c_\infty(t) \\ c(z,0) = c_i(z) \end{cases} \tag{3}$$

where *e* is the thickness of the plate, $c_i(z)$ is the initial concentration and $c_\infty(t)$ is the concentration at the lateral surfaces, which depends on the external relative humidity *HR(t)* by the law:

$$c_\infty(t) = C \cdot HR(t)^b \tag{4}$$

C and *b* are constants. All the above functions have the same period τ.

The problem is relatively complex, as boundary conditions and diffusion coefficient depend explicitly on the time.

As already noted in [8], a diffusion/conduction problem along a semi-infinite region with periodic boundary conditions is characterised by a typical dimension, indicated by λ, which measure the distance from the surface where a fluctuating regime exists. λ is given by:

$$\lambda = 2\sqrt{\pi\ k\tau} \tag{5}$$

where k is a conduction/diffusion coefficient and τ is the period. λ can be exactly defined as the distance, taken from the surface, at which the amplitude of all oscillations is reduced by a factor of $e^{-2\pi} = 0.0019$. By analogy with this problem, a length e_0 can be defined as (see also [4,9]):

$$e_0 = 2\sqrt{\pi \int_0^{\tau} D(t)dt} \tag{6}$$

which has the same meaning as λ, but it takes into account the effect of a varying diffusion coefficient. Some important considerations need to be addressed concerning e_0:

- e_0 practically measures the distance from the surfaces in which hygrothermal oscillations exist. During the absorption phase, internal zones outside e_0 are in a transient regime. When "saturation" is reached, spatial regions *inside* e_0 *keep fluctuating* with period τ, while spatial regions *outside* e_0 reach a permanent (thus uniform) regime,
- e_0 depends on the diffusion coefficient and the period of the cycle.

An important conclusion is that a plate with $e < 2e_0$ *is in a fluctuating regime all along its thickness.*

An analytical solution of equation (2) with boundary conditions (3) can be obtained by employing a change of variable, in order to eliminate the explicit time dependence, and a recursive technique, in order to express the concentration as a function of the number of cycles. Details of such technique can be found in reference [4] for laminated cylinders and in reference [9] for laminated plates.

In the following equations the solution for a laminated plate is presented, by assuming that the diffusion coefficient and the moisture concentration are not discontinuous at the interfaces between adjacent plies. In other terms, concerning the diffusion problem, the plate behaves as if it was homogeneous.

By indicating as N the number of cycles at which the internal regions of the plate reach a permanent state, the concentration field after (N-k) cycles is finally given by:

$$c(z,N-k) = \hat{c}_\infty + \frac{\Delta(\tau)\pi^2}{2\left(\frac{e}{2}\right)^3}\sum_{n=0}^{\infty}\sum_{i=0}^{k-1}\left((2n+1)^2\exp\left(-\frac{(2n+1)^2\pi^2\Delta(\tau)(N-i)}{4\left(\frac{e}{2}\right)^2}\right)\right)\cos\left(\frac{(2n+1)\pi z}{e}\right)$$
$$\left\{\frac{2\left(\frac{e}{2}\right)(-1)^{n+1}\hat{c}_\infty}{(2n+1)\pi} + \int_0^{\frac{e}{2}} c_i(z')\cos\left(\frac{(2n+1)\pi z'}{2\left(\frac{e}{2}\right)}\right)dz'\right\} \quad \forall\ -\frac{e}{2} \le z \le \frac{e}{2} \tag{7}$$

where:

$$\Delta(\tau) = \int_0^{\tau} D(t)dt, \qquad \hat{c}_\infty = \frac{1}{\Delta(\tau)}\int_0^{\tau} D(t)c_\infty(t)dt \tag{8}$$

When $c_i(z) = 0$, equation (7) becomes:

$$c(z, N-k) = c_\infty - \frac{4\, c_\infty \pi}{\beta^2} \sum_{n=0}^{\infty} \sum_{i=0}^{k-1} \left((-1)^n (2n+1) \exp\left(-\frac{(2n+1)^2 \pi^2 (N-i)}{\beta^2} \right) \right) \cos\left(\frac{(2n+1)\pi z}{e} \right) \qquad (9)$$

$$\forall \quad -\frac{e}{2} \leq z \leq \frac{e}{2}$$

where:

$$\beta = \sqrt{\frac{e^2}{\Delta(\tau)}} \qquad (10)$$

Equations (7) and (9) employ average concentration values at the boundaries (eq.8), thus they describe correctly the solution in the interior regions of the plate only, that is, outside e_0. The finite difference method is employed besides the analytical one, to better represent the concentration distribution in regions close to the lateral surfaces.

2.2 STRESS FIELD

The stress field is obtained by employing the classical lamination theory, adapted for taking into account transient moisture concentration fields, such as those given by equations (7) and (9). In the following paragraph we present the main features of the model, see reference [9] for further details.

The main assumption of the model is to approximate the hygrothermal fields given by equations (7) and (9) by equivalent fields, which are linear over spatial sub-regions of a physical ply, called sub-plies. This is done so that the total strains, including the free expansion strains of hygrothermal nature, satisfy automatically the compatibility equations which otherwise are not satisfied, at least in the context of a plate theory. The employment of sub-plies has some shortcomings. As the number of sub-plies is finite, the concentration and the stress fields are approximate: the accuracy increases as the number of sub-plies increases, but the maximum number of sub-plies itself is limited by some physical consideration. In the present communication a macroscopic approach is proposed and, physically, a sub-ply must be able to represent the homogenised behaviour of the material, this imposes a limitation on its minimum dimensions.

3 EXAMPLE : APPLICATION TO SUPERSONIC FLIGHT

In order to illustrate the technique presented in section 2, we consider the hygrothermal cycle illustrated in figure 1. This cycle simulate the environmental conditioning encountered by skin aircraft structures during a flight. In the cycle some peculiar zones can be distinguished:

- point **A**, at room temperature and ground humidity conditions, which simulate ground environmental conditions
- point **B**, at –55°C and 0% HR, which simulate the subsonic phase of the flight
- point **C**, at 130°C and 0% HR, which simulate the supersonic phase of the flight
- point **D**, with the same conditions as in point **A**, which is located at the end of a flight

An initial conditioning at 23°C and 50% HR for about 3 months is included in the calculations, in order to simulate a long term ground maintenance, which leads to a pseudo-humid state.

A 4 mm thick $[0_4/90_4]_s$ composite plate is considered, each ply "block" being 1 mm thick.

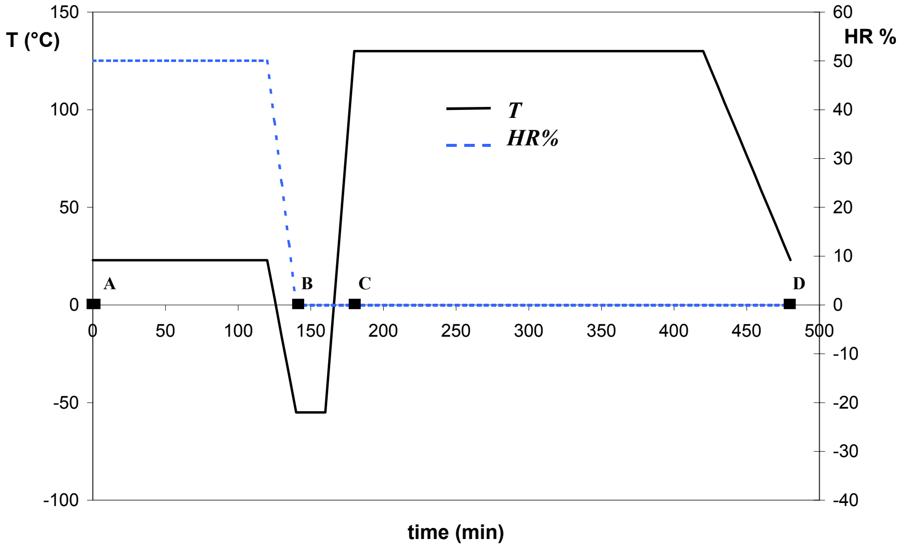

Fig. 1: Hygrothermal cycle

Inside each ply block the spatial domain is divided into sub-domains 0.1 mm thick, such subdivision is sufficient to give a good accuracy of the calculated concentration fields. Figure 2 presents the concentration field at each point of cycle 1: point A for such cycle corresponds to the pseudo-humid state. For cycle 1 only the finite difference solution is employed.

Figure 3 presents the concentration field at each point of cycle 50. For such cycle, the analytical solution is fast to be estimated, as few terms in the series (7) are required, and such a solution is presented in figure 3 for comparison with the finite difference one.

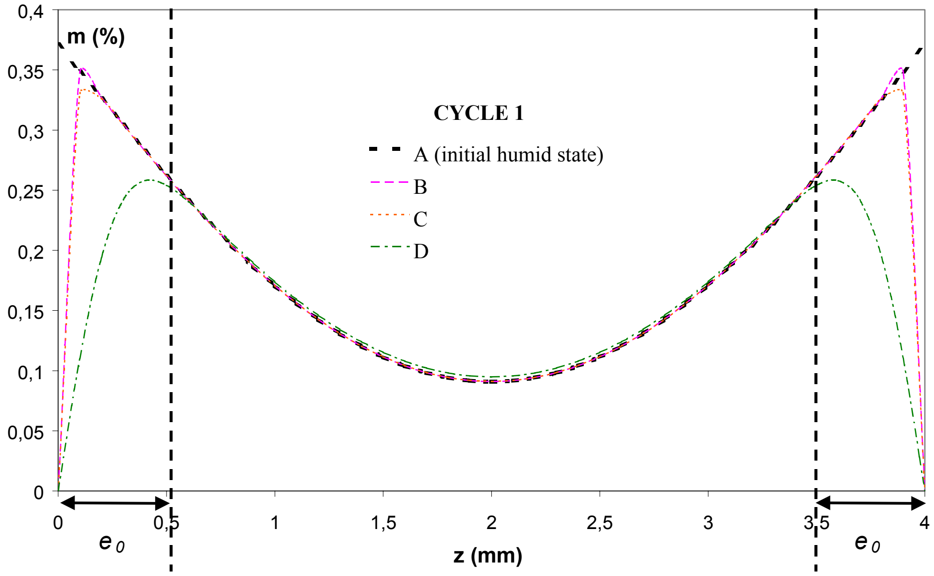

Figure 2: Moisture concentration during cycle 1

In both figures the symbol *m(%)* is employed, which indicates the percentage of solute density divided by the density of the reference dry solvent material.

Figures 2 and 3 show that hygrothermal fluctuations are close to the lateral surfaces and extend over spatial regions of value e_0, in this case equal to around 0.5 mm.

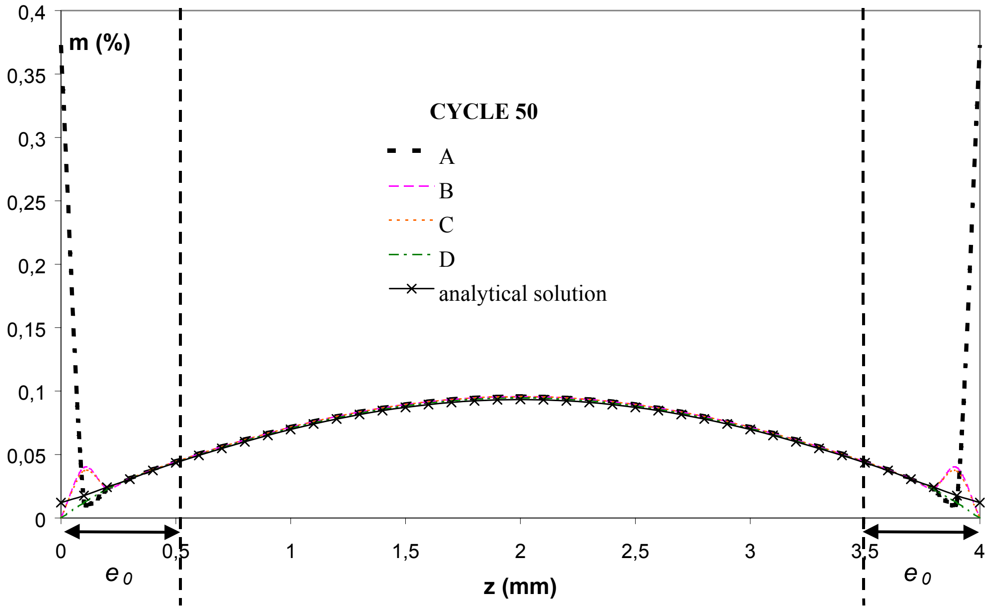

Figure 3: Moisture concentration during cycle 50

The average effect of the supersonic cycles consists of drying the plate until a quasi-dry state, which is reached after around 300 cycles. The analytical solution, as already mentioned in section 2, respects average boundary conditions and is adequate for the internal regions only.

The initial mechanical state, before any conditioning, is not free of stress, residual stresses due to manufacturing are present in the structure. The *stress free state* is supposed to be achieved at a temperature of 210°C, which is around the final T_g of a high-temperature composite system, the temperature differential is then given by ΔT= -187°C.

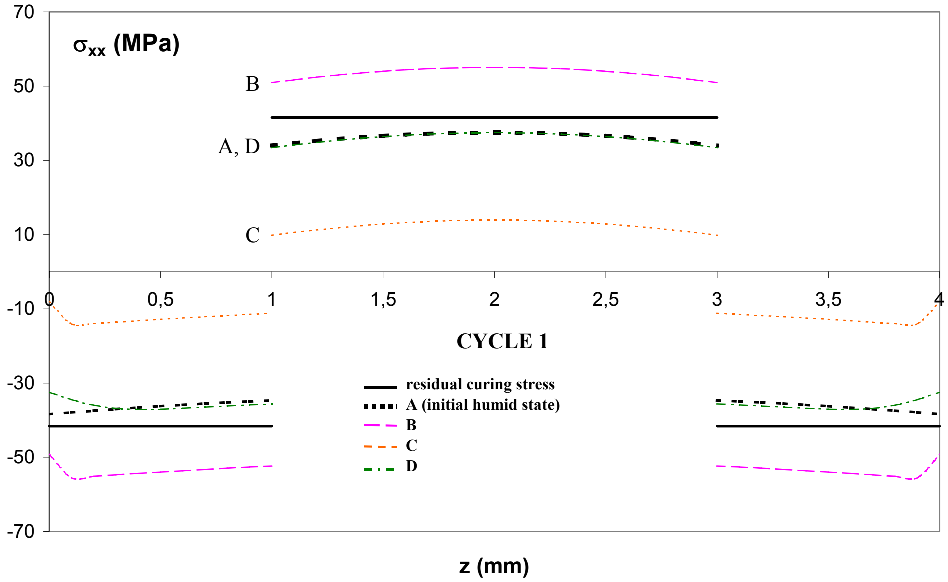

Figure 4: Internal stresses σ_{xx} during cycle 1

Figures 4 and 5 illustrate stresses σ_{xx} and σ_{yy} at points A, B, C and D of cycle 1. Figures 6 and 7 show the evolution of internal stresses with increasing number of cycles.

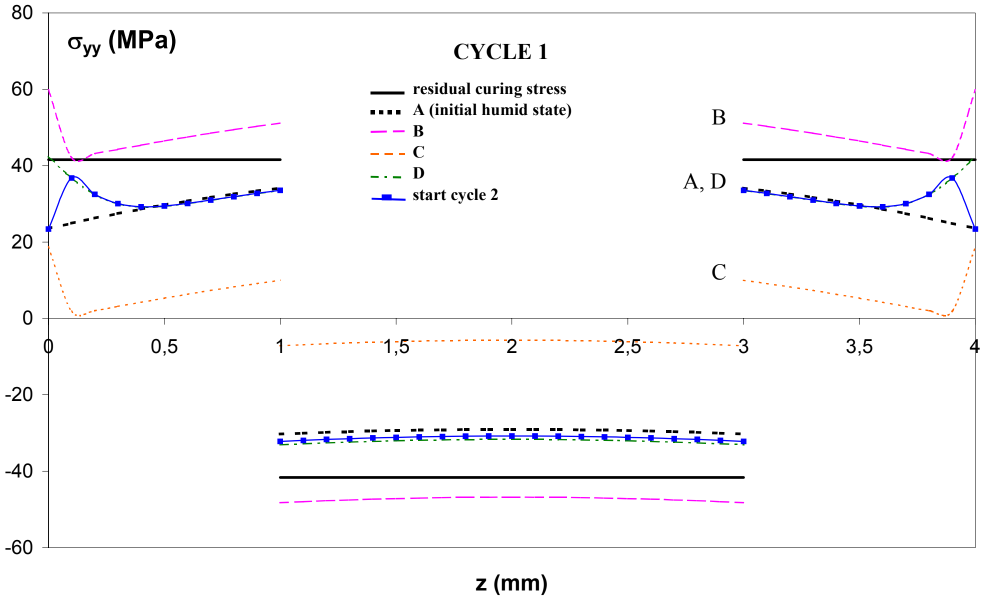

Figure 5: Internal stresses σ_{yy} during cycle 1

Mechanical properties used for simulations are given in table 1, they are representative of a IM7/977-2 carbon fibre-epoxy ply and are given in the usual orthotropic material axes.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	α_1 $(\varepsilon^{o}C^{-1})$	α_2 $(\varepsilon^{o}C^{-1})$	β_1	β_2
152	8.4	4.2	0.35	$0.09\cdot10^{-6}$	$28.8\cdot10^{-6}$	0	0.6

Table 2: Material properties used in the simulations (IM7/977-2)

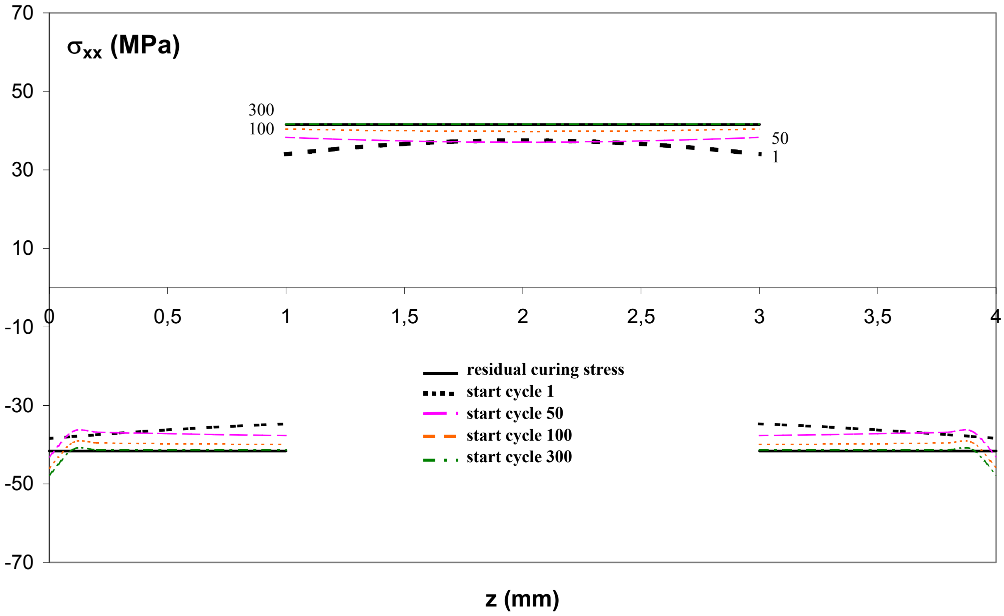

Figure 6: Internal stresses σ_{xx} at the start of cycles 1, 2, 50, 100, 300

4 DISCUSSION

It can be noted from simulations that stresses are quite high after manufacturing. Residual curing stresses, which are presented as a reference in all figures, are uniform through plies, by the assumption of uniform ΔT, and take a value of around 42 MPa.

The internal stresses which arise after a long period of maintenance due to the absorption of moisture, point A in cycle 1, counteract only partially the residual curing stress and have an average value of 35 MPa, which is quite high when compared to the reference dry state.

Thermal fluctuations from –55°C to 130°C, points B and C respectively, extend over the whole thickness of the plate and exhibit an average amplitude of around 40 MPa, the mean stress is about 30 MPa. Low temperature conditions are quite critical, transverse stresses can reach, over the cycling, values as high as 60 MPa close to the lateral surfaces.

Hygroscopic fluctuations induce stress gradients over a region close to the lateral surfaces and of thickness e_0 and fluctuating stresses whose magnitude can be as high as 40 MPa.

Drying induced by supersonic cycles promotes a progressive resurgence of the residual curing stresses, the mean of all fluctuations increase with increasing cycles.

The calculated stresses, which values are quite high especially at low temperatures, may potentially damage the structure. In particular, the mixed hygrothermal cycling close to the lateral surface, whose extent is around 0.5mm (two or more physical plies), could be particularly aggressive.

5 CONCLUSIONS

A methodology for simulating hygrothermal fields and internal stresses induced by cyclical environmental conditions in laminated composite plates is presented. This methodology assumes uniform temperature distribution over the plate, employs the Fick’s law for calculating the moisture concentration profiles and the classical lamination theory, adapted for transient conditions, for simulating the induced stresses.

Simulations are provided for a plate structure subjected to almost realistic supersonic flight conditions and give values of fluctuating stresses which can potentially induce damage in the structure without any external load.

The model is quite simple, based on straight and plausible hypotheses and is supported by analytical calculations. It can be adapted to constitutive laws different from the elastic ones, it can be generalised for taking into account the effect of damage.

The model is also intended as a tool for preliminary design and for conceiving experimental tests.

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